NUMERICAL MODEL WITH FREE BOUNDARY OF UPPER AIR MASSES INTENDED TO INVESTIGATE DYNAMICS OF HEAT ISLAND

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A mathematical model is described of atmospheric hydrothermodynamics in the quasistatic approximation with a free upper boundary of air masses for a bounded region. The coordinates affixed to the Earth's surface relief are used. Discrete approximations and algorithm implement the variational principle and the splitting method. Some examples are given of modeling of mesoclimate of an industrial region under the joint effect of the relief and urban heat island.

1. INTRODUCTION

A problem on the formation of atmospheric circulation with a free upper boundary of air masses above the Earth's surface with inhomogeneous orography and temperature is considered. The height of the upper boundary of air masses varies with time and is the sought–after parameter. To construct the numerical model, we introduce the coordinates affixed to the Earth's surface relief (so-called σ-coordinates) with the vertical coordinate

$$\sigma = (z - \delta(x, y))/h(x, y, t), \qquad (1)$$

where the function $\delta(x, y)$ describes the surface relief, $h(x, y, t) = H(x, y, t) - \delta(x, y)$, $H(x, y, t)$ is the height of the upper boundary of air mass, t is time, x and y are the horizontal coordinates, x is directed eastward, and y is directed to the north.

This class of problems arises in the study of climatic changes of industrial regions under the impact of anthropogenic changes of the parameters of large surface areas of the Earth. In this case, the dynamics of mesoclimate formation is examined considering the interaction of the urban heat island with the background heat flux. This is necessary for estimation of the level of atmospheric pollution under specific conditions of urban circulation, when the heat island, inhomogeneous relief, and other factors of the Earth's surface act simultaneously. The model with the free upper boundary in the Cartesian coordinates was considered in Ref. 1. The main point of its implementation scheme was the use of the border method to construct the equation for the upper boundary.

Here, the problem to be solved is formulated in an analogous way. To solve this problem, explicit – implicit noniterative algorithm[2] is used at a joining stage. This algorithm is very convenient and efficient.

By virtue of energy balanced discrete analogs of the model, the stability of calculations is provided irrespectively of the temperature stratification. The model is constructed for a local region. For convenient presentation, we do not decompose the state function into the background ones and their deviations in accordance with scales of the processes in the explicit form. The background atmospheric conditions are considered only when we formulate the initial conditions and close the algorithm on the upper boundary and side boundaries of the domain of model definition.

2. PROBLEM FORMULATION

The basic equations of hydrothermodynamics in the selected coordinate system are written as

$$\frac{1}{h}\left(\frac{\partial \rho h u}{\partial t} + \mathrm{div}\, \rho h u \mathbf{u}\right) - l\rho v = -\frac{\partial p}{\partial x} + B_x + F_u, \qquad (2)$$

$$\frac{1}{h}\left(\frac{\partial \rho h v}{\partial t} + \mathrm{div}\, \rho h v \mathbf{u}\right) + l\rho u = -\frac{\partial p}{\partial y} + B_y + F_v, \qquad (3)$$

$$\frac{\partial p}{\partial \sigma} = -g\rho h, \qquad (4)$$

$$\frac{A}{h}\left(\frac{\partial \rho h T}{\partial t} + \mathrm{div}\, \rho h T \mathbf{u}\right) + \frac{\gamma_a p}{gh}\left(\frac{\partial h}{\partial t} + \mathrm{div}\, h\mathbf{u}\right) = F_T, \qquad (5)$$

$$\frac{\partial \rho h}{\partial t} + \mathrm{div}\, \rho h \mathbf{u} = 0, \qquad (6)$$

$$w = h\omega + \sigma\frac{\partial h}{\partial t} + u\left(\sigma\frac{\partial h}{\partial x} + \frac{\partial \delta}{\partial x}\right) + v\left(\sigma\frac{\partial h}{\partial y} + \frac{\partial \delta}{\partial y}\right), \qquad (7)$$

where

$$B_s = \frac{\partial p}{\partial \sigma}\left(\frac{\sigma}{h}\frac{\partial h}{\partial s} + \frac{1}{h}\frac{\partial \delta}{\partial s}\right);$$

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$$F_\phi = \frac{\partial}{\partial\sigma}\left(\chi_\phi \frac{\partial\phi}{\partial\sigma}\right) + \mathrm{div}_s\, \rho\mu_s\, \mathrm{grad}_s\, \phi;$$

$$s = (x, y);\ \phi = (u, v, T);\ A = 1 - \gamma_a R/g.$$

Here u, v, and w are the components of the wind velocity vector along the x, y, and axes, respectively; $\mathbf{u} = (u, v, \omega)$; T is the temperature; p is the pressure; ρ is the density; g is the acceleration due to gravity; l is the Coriolis parameter; μ_s is the horizontal turbulence exchange coefficient; $\chi_\phi = \nu + \mu|\mathrm{grad}_s\,\sigma|^2$; ν is the vertical turbulence exchange coefficient; R is the universal gas constant; γ_a is the dry-adiabatic gradient; and ω is the analog of the vertical velocity related with w (the vertical wind velocity in the Cartesian coordinates) by Eq. (7).

To calculate the function ω, we use the continuity equation (6) integrated over the vertical coordinate with boundary conditions for $\omega = 0$ at $\sigma = 0$ and $\sigma = 1$:

$$\omega = -\frac{1}{\rho h}\left(\frac{\partial h}{\partial t}\int\limits_0^\sigma \rho \mathrm{d}\sigma + \int\limits_0^\sigma \left(\frac{\partial \rho h u}{\partial x} + \frac{\partial \rho h v}{\partial y}\right)\mathrm{d}\sigma\right). \quad (8)$$

At $\sigma = 1$, we obtain the equation for h

$$\frac{\partial h}{\partial t} = -\left(\int\limits_0^1 \rho \mathrm{d}\sigma\right)^{-1} \int\limits_0^1 \left(\frac{\partial \rho h u}{\partial x} + \frac{\partial \rho h v}{\partial y}\right)\mathrm{d}\sigma. \quad (9)$$

The system of equations (1)–(6) is solved for time interval $[0, t]$ in the domain $D = \{0 \le x \le X, 0 \le y \le Y, 0 \le \sigma \le 1\}$; X, Y, and t are the input model parameters.

The boundary conditions for the flux of momentum and the thermal flux at $\sigma = 0$ are specified with the use of the surface layer parameterization. The background conditions of atmospheric circulation are considered on the upper boundary at $\sigma = 1$ and on the side boundaries of the domain D_t. These conditions of model closing can be conveniently prescribed in terms of the first derivatives of the corresponding state functions. The state of the system at $t = 0$ is assumed known. It is prescribed on the basis of the background conditions when we examine the model scenarios.

3. COMPUTER ASPECTS OF MODEL REALIZATION

Discrete approximations for model (2) – (9) are constructed on the basis of the variational principle in combination with the splitting method.[3] To this end, the variational model formulation is written in the form of the integral identity that considers all model equations, initial and boundary conditions, and relations describing the input parameters and the external sources in addition to the system of fundamental equations (2) – (9). The main functional of this identity is constructed based on the conditions of the energy balance.

To consider the temporal dependence, the weak approximation with fractional steps is used. As a result, the splitting schemes are obtained with energy balance, in which two stages are tentatively distinguished: transport and matching.

Without writing down the integral identity of the variational model, we note only that the following conditions should be satisfied to balance energetically the discrete approximations in spatial variables:

1. The discrete operators of substance transport along trajectories of air mass motion should be antisymmetric. The turbulent exchange operators should preserve their symmetry.[3]

2. The components of the vector gradient of the atmospheric pressure entering into equations of motion (2) and (3) as well as into Eq. (7) for ω should be calculated from the same formulas.

3. The approximations of the gradient operator in Eqs. (2) – (4) should match with the approximation of the divergence operator in continuity equation (6) and should preserve their antisymmetry peculiar to the transport operators in differential form.

4. Approximations of $\partial p/\partial\sigma$ in hydrostatic equations and of the coefficient σ in the expression for calculating w should be selected to meet the relations analogous to

$$\frac{\partial p}{\partial \sigma} + p = \frac{\partial \sigma p}{\partial \sigma}, \quad (10)$$

$$\int\limits_0^1 \left(\sigma \frac{\partial p}{\partial \sigma} + p\right)\mathrm{d}\sigma = \sigma p\Big|_0^1 = p_H. \quad (11)$$

Now we consider the way of approximation of the gradients of functions h, σ and p.

First, we examine the term of the balance equation of the total system energy that describes the exchange between the kinetic and potential energy at $[t_j, t_{j+1}]$:

$$I(w) = \int\limits_{D_t^j} g\rho w\, \mathrm{d}D\, \mathrm{d}t. \quad (12)$$

After simple transformations, this expression assumes the form

$$\int\limits_{D_t^j} \rho w\, \mathrm{d}D\, \mathrm{d}t = \int\limits_{D_t^j} \frac{1}{h}(\rho z h)\Big|_{t_j}^{t_{j+1}} \mathrm{d}D +$$

$$+ \int\limits_{\Omega_t^j} \rho z h\, u_n\, \mathrm{d}\Omega\, \mathrm{d}t, \quad (13)$$

where Ω_t^j is the side boundary of the domain D_t^j, $D_t^j = D \times [t_j, t_{j+1}]$; u_n is the component of the wind velocity vector **u** perpendicular to the boundary, $d\Omega$ is the element of area of the side boundary Ω of the domain D_t^j. To satisfy this relation in its discrete form, we should take an appropriate approximation of w that matches in the spatial variables with the continuity equation. In this case, gradients should be approximated so that the corresponding terms of the integral identity were antisymmetric. Then they will be canceled from the energy balance equation. From these conditions we obtain the difference analogs for the gradient of the function $h(x, y, t)$.

To obtain matched approximations for gradients of the function p and to preserve their divergent character, we take the expression[2]

$$\int\limits_{D_t^j} \left[u^{j+1} \frac{\partial p^j}{\partial x} + v^{j+1} \frac{\partial p^j}{\partial y} + \omega^{j+1} \frac{\partial p^j}{\partial \sigma} + \right.$$

$$\left. + \frac{p^j}{h} \left(\frac{\partial h u^{j+1}}{\partial x} + \frac{\partial h v^{j+1}}{\partial y} + \frac{\partial h \omega^{j+1}}{\partial \sigma} \right) \right] \mathrm{d}D\mathrm{d}t =$$

$$= \int\limits_{D_t^j} \frac{1}{h} \operatorname{div} hp^j \mathbf{u}^{j+1} \, \mathrm{d}D \, \mathrm{d}t = \int\limits_{\Omega_t^j} hp^j u_n^{j+1} \, \mathrm{d}\Omega \, \mathrm{d}t, \qquad (14)$$

which is employed for matching of the fields of wind velocity vector and pressure and is used to construct the energy balance equation at $[t_j, t_{j+1}]$, where the superscript adjacent to symbols of functions indicates the instant of time at which these functions are taken. Matched divergence schemes for the gradients of the function p are constructed in the same way as the approximations for the gradients of the function h, which allows the family of the consistent approximations to be obtained in both cases when the same schemes are selected.

Now we formulate the sequence of main operations to implement the matching algorithm.

1. We assume that at $t = t_j$ the functions u, v, ω, h, p, and T are preset. The function p^j is determined from the hydrostatic equation, ρ^j – from the equation of state and function h^j – from Eq. (9) for the preset wind velocity field.

2. The equations of motion are solved for the functions u^{j+1} and v^{j+1} by the matrix pass technique for the vertical variable.

3. From Eq. (8), we determine the function ω^{j+1} and from Eq. (9) – $\partial h / \partial t$.

4. The equation of heat flux is solved for T^{j+1}.

5. Equation (9) is solved for h^{j+1}.

6. The value of the function p^{j+1} is calculated on the upper boundary of the air mass by the formula

$$(\ln p^{j+1}) = [(\ln p^j) - g\,(h^{j+1} - h^j)]/(R \cdot T^{j+1}).$$

At the stage of transport, the integral identity is taken with symmetric turbulent exchange operators and gradient representation of the transport operators to construct the discrete approximations. The weighting functions are specified by solving the local conjugate problems. Finally, stable calculation algorithms are obtained for the approximation of the advective diffusion operators that have the properties of monotony and transportation.[2] Here the following comments are necessary. Because the model is considered in the domain with variable relief, the turbulent exchange operators can be preset in two ways, namely, in the Cartesian coordinates on surfaces z = const and directly in σ coordinates on surfaces σ = const.

The transport operator is invariant under coordinates. In the first case, the turbulent operator should be transformed to σ-coordinates. In so doing, additional terms appear in the equations caused by the difference of z-surfaces from σ-surfaces, which makes the construction of monotonic numerical algorithms difficult. To overcome this problem, the effective transport vector is introduced with components having diffusive terms due to change of coordinate axes.[4]

It should be noted that characteristic scales differ at the stages of transport and matching of the fields. Therefore, to approximate on the basis of physical meaning of the phenomena to be modeled, temporal steps for modeling should be chosen differently. The procedure of matching of scales by itself creates no problems, because it is implemented based on the variational principle.

4. NUMERICAL EXPERIMENTS ON MODELING OF MESOCLIMATES

Here, we present some results of numerical experiments on modeling of mesoclimate in an industrial region. By way of example, we examine the region with the relief and characteristics of the underlying surface corresponding to the Tomsk region with the city in its central part. The schematic map of the Tomsk region was given in Ref. 5.

Characteristics of the underlying surface were prescribed in accordance with land capabilities: water surface, marshes, forest, localities, and city. The surface temperature was specified in accordance with the surface type. In this case, the temperature increased monotonically from the water surface to urban buildings. This temperature distribution was observed in the daytime in spring–summer. The city plays a role of a heat island. The surface relief elevations increase from the northwest to the south, southeast, and east from 60 to 260 m. The model scenario was organized so that at the initial instant, the state assumed stationary, the surface temperature remained unchanged with time, the wind velocity at the western boundary of this region did not change over a period of modeling. The atmospheric circulation developed under the impact of temperature contrasts and relief inhomogeneities.

It gets stationary as time passes and acquires all the typical features of mesoclimate. The time of establishing the stationary circulation regime was 5–6 h in our case. Calculations were done for the following input model parameters: $X = Y = 100$ km, $H = 3000$ m, $\Delta t = 10$ s, $\Delta x = \Delta y = 4000$ m, $\Delta\sigma = 0.1$, $\sigma = 1$, $\mu_x = \mu_y = 1500$ m^2/s, and $\nu = 10$ m^2/s. The figures shown below illustrate effects of orography and temperature contrast on the atmospheric circulation above the city and its environs and on the change of the position of the upper air mass boundary.

Figure 1 illustrates the stationary field of the wind velocity vector on the surface $\sigma = 0.1$. Lengths of arrows are proportional to the wind speed in the nodes of the calculation grid. Small squares at the origins of arrows mean that the wind speed is smaller by an order of magnitude than the peak wind speed for this region. In the environs and at the center of the region, air masses circulated more intensively. From the side exposed to the wind the circulation intensified due to the temperature contrast "river–city" that engendered the breeze. From the leeward side, the air flow with vortices whose directions were opposite to the prevailing air flow direction, markedly weakened.

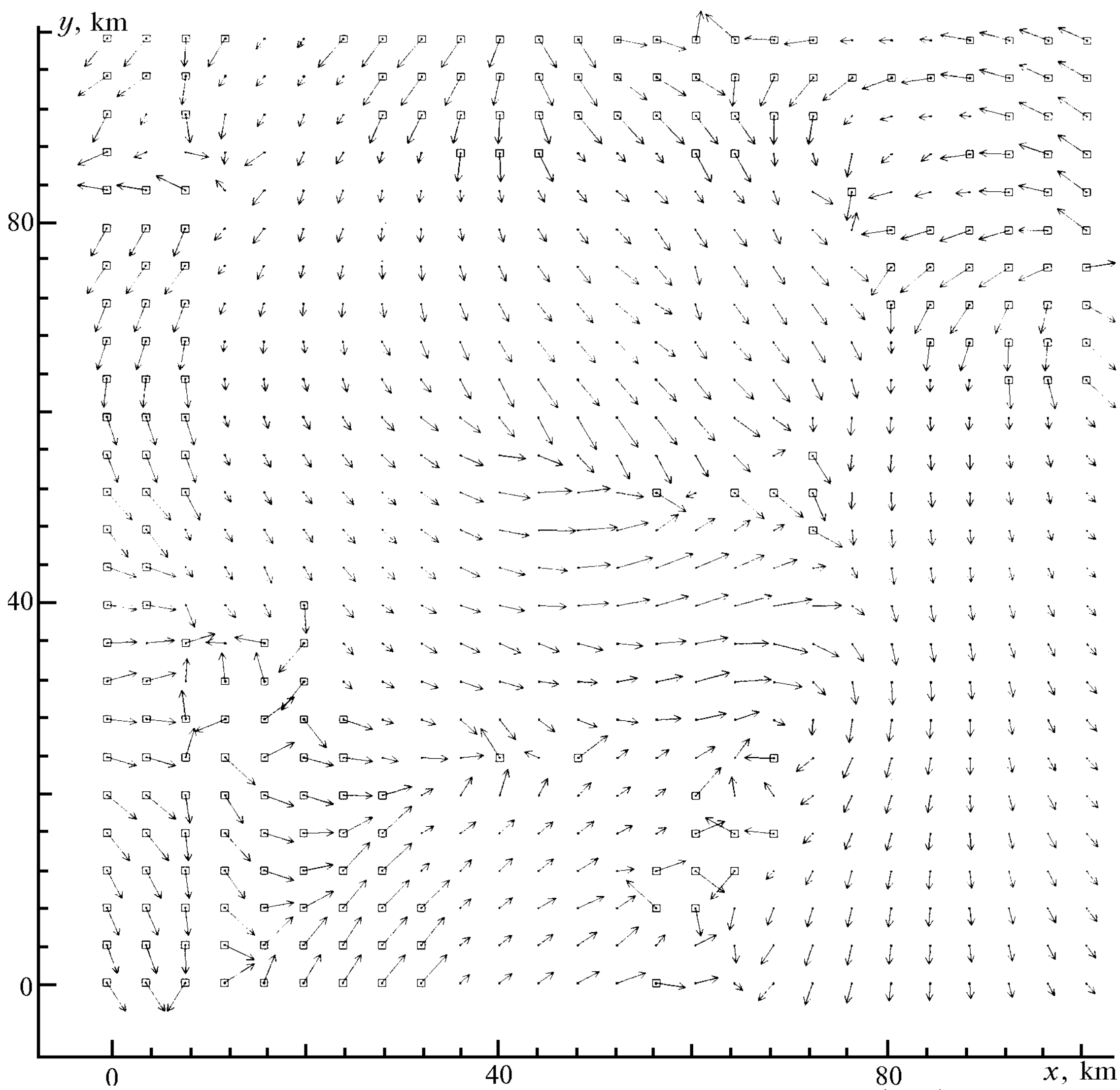

FIG. 1. Wind velocity vector field at $\sigma = 0.1$ *in the plane (x, y).*

Figure 2 shows heights of the free upper boundary of the air mass at the final time moment. Its gradient was most vividly pronounced above sections of the underlying surface with high temperature contrasts and it attained 83 m for the entire region. Figure 3 shows the wind behavior in the vertical cross section parallel to the OX axis near the central part of the region. Ascending air motions are seen in the environs and descending flow is seen above the river and floodplain. When the background air flow was strong, the effects

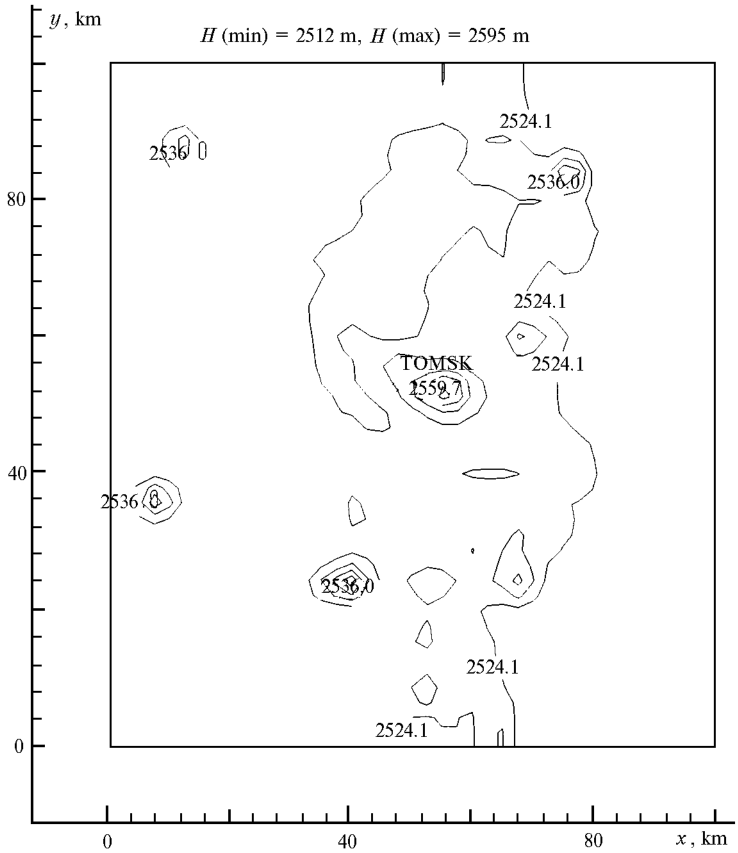

FIG. 2. Contour lines of the heights of the free upper air mass boundary.

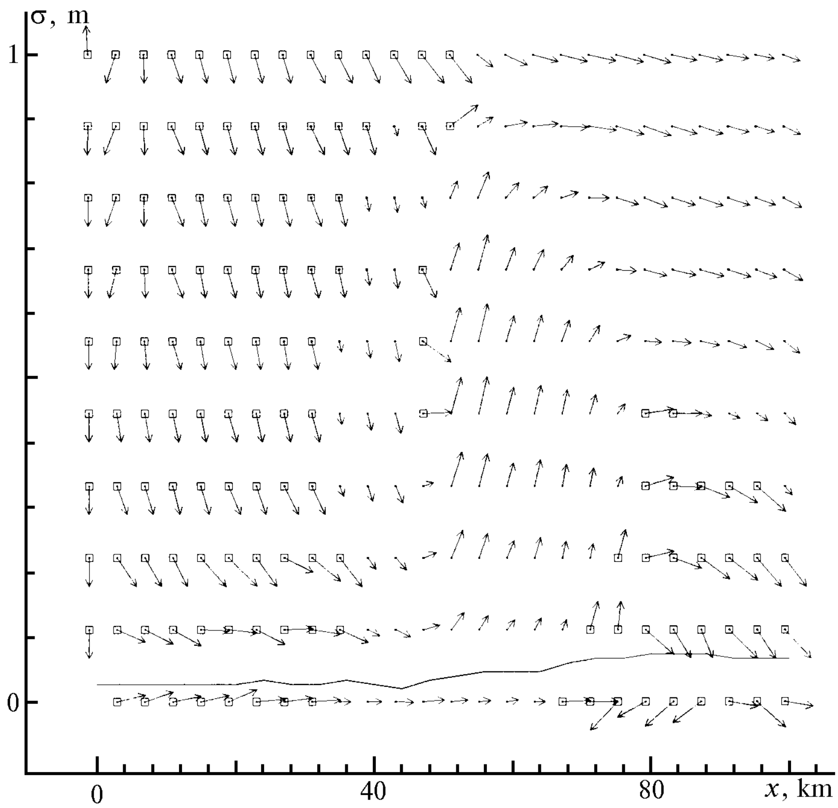

FIG. 3. Vector field of wind velocity in the plane (x, σ) *for* y = 13 km. (*Solid line shows the surface relief and* $\Delta\sigma$ = 0.1 *corresponds to* Δz = 300 m.)

of orography and temperature contrast were less pronounced. Thus, our numerical experiments demonstrated that the relief and the temperature contrast resulted in the formation of the air circulation, most intensive above the heat island where the height of the free boundary increased.

5. CONCLUSION

The mathematical model described in the present paper is oriented toward the study of joint effects of local relief and distributed natural and anthropogenic heat sources on the local circulation against the background of large-scale atmospheric motions. By its structure, the model is a part of the model complex of interrelated problems on ecology and climate developed at the Computing Center of the Siberian Branch of the Russian Academy of Sciences. The use of this model in the Cartesian coordinates and transition to the coordinates affixed to the local relief and free upper boundary of air masses obviates the necessity of the hydrostatic approach. These situations arise in practice under the strong anthropogenic thermal impacts accompanied with emissions of pollutants.

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