ABOUT THE ECHO-PULSE WAVEFORM IN LASER SENSING OF A ROUGH SEA SURFACE

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Laser echo-pulse waveform for the general scheme of bistatic sensing of a rough sea surface partly covered with foam is investigated. A formula is derived for the signal power recorded by a detector in pulsed sensing of the sea surface in two cases: when the direction toward the detector coincides or is close to the direction of specular reflection and when the direction toward the detector differs significantly from the direction of specular reflection. It is shown that the sensing geometry and the presence or absence of foam on the sea surface may significantly change the signal waveform and the position of laser echo-pulse maximum.

Echo-signal power, delay, and duration of an echo-pulse in laser sensing of a rough sea surface were investigated in many works (see, for example, Refs. 1–5). However, the published works have not yet touched on waveform peculiarities for different sensing schemes. Below we consider the laser echo-pulse waveform for the general scheme of bistatic sensing of the rough sea surface partly covered with foam and investigate its peculiarities in two cases: when the direction toward the detector coincides or is close to the specular reflection direction and when the direction toward the detector differs significantly from the specular reflection direction.

Let a rough sea surface S be sensed with a pulsed signal. Let us assume that the sensing radiation wavelength lies in the IR-range, where the absorption by water is strong so that the main contribution to the echo-signal comes from the light specularly reflected from the air-water interface and the light scattered by foam. The contribution from the radiation diffusely scattered in the water column can be neglected. Shading of some sea surface elements by others is neglected. A model of the echo-signal coming from the sea surface partly covered with foam, because of the incoherent summation of echo-signals coming from the foam-free and foam-covered sections of the sea surface, can be represented in the form

$$P(t) = (1 - S_{\rm f})\, P_{\rm s}(t) + S_{\rm f}\, P_{\rm f}(t)\ , \qquad (1)$$

where $P(t)$, $P_{\rm s}(t)$, and $P_{\rm f}(t)$ are the signal powers recorded by the detector from the sea surface partly covered with foam, without foam, and continuously covered with foam, $S_{\rm f}$ is the relative fraction of the sea surface covered with foam.

In analogy with Refs. 4 and 5, suppose that elevations and slopes of the sea surface are distributed by the lognormal law, and the sections of foam lie on the slopes of the waves and are the Lambertian reflectors. Using Eq. (1) in analogy with Refs. 5 and 6, we write down the power of the echo-signal $P(t)$ for the bistatic sensing scheme (considering that the radiation wavelength is small in comparison with the characteristic curvature radius and roughness elevations of the sea surface; duration of the sensing pulse is large in comparison with the period of the carrying frequency and small in comparison with the period of variations of the sea surface shape; the source, the detector, and their optical axes lie in the plane XOZ; distances from the source and the detector to the sea surface are much greater than roughness elevations and diameter of an illuminated spot on the surface)

$$P(t) \cong \int\limits_{S_0} \frac{d\mathbf{R}_0}{n_z}\, E_{\rm s}(\mathbf{R}'_{0\varsigma})\, E_{\rm r}(\mathbf{R}''_{0\varsigma}) \times$$
$$\times [S_{\rm f} A/\pi + V^2(1 - S_{\rm f})\, \delta\{K_x [q_x + R_{0x} T + \gamma_x q_x]\} \times$$
$$\times\, \delta\, \{K_y [R_{0y}\, s + K_x\, \gamma_y q_z]\}] \times$$
$$\times f\left(t' + \frac{R_{0x}\, q_x}{c} - \frac{\varsigma\,(\mathbf{R}_0)\, q_z}{c} - \frac{R_0^2 + \varsigma^2(\mathbf{R}_0)}{2c}\, s\right), \qquad (2)$$

where

$$q_x = \sin\theta_{\rm s} + \sin\theta_{\rm r}\ , \quad q_z = -\,(\cos\theta_{\rm s} + \cos\theta_{\rm r})\ ,$$

$$K_{x,y} = \frac{n_z}{\sqrt{1 - n_z^2\, \gamma_{y,x}^2}}\ ,$$

$$s = \frac{1}{L_{\rm s}} + \frac{1}{L_{\rm r}}\ , \quad T = \frac{\cos^2\theta_{\rm s}}{L_{\rm s}} + \frac{\cos^2\theta_{\rm r}}{L_{\rm r}}\ ; \quad t' = t - \frac{L_{\rm s} + L_{\rm r}}{c}\ ,$$

$$\mathbf{R}'_{0\zeta} = \{[R_{0x}\ \mathrm{ctg}\theta_{\rm s} - \zeta(\mathbf{R}_0)]\ \sin\theta_{\rm s},\ R_{0y}\}\ ,$$

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$\mathbf{R}''_{0\zeta} = \{[R_{0x}\,\mathrm{ctg}\theta_r - \zeta(\mathbf{R}_0)]\sin\theta_r,\ R_{0y}\}$,

$E_s(\mathbf{R})$ and $E_r(\mathbf{R})$ are illuminations of the sea surface from real and fictitious (with the detector parameters) sources[7]; L_s and L_r are the slant distances from the source and the detector to the sea surface; ζ and $\boldsymbol{\gamma} = \{\gamma_x, \gamma_y\}$ are the elevation and the vector of slopes of the rough sea surface S; $\mathbf{n} = \{n_x, n_y, n_z\}$ is the normal to the element of area S; θ_s and θ_r are the angles between the normal to the surface S_0 (projection of S onto the plane $z = 0$) and the directions toward the source and the detector; $\delta(x)$ is the delta function; V^2 is the Fresnel reflection coefficient of the foam-free sea surface; A is the albedo of foam.

Attempts to obtain analytical formula for $P(t)$ from Eq. (2) for the general scheme of slant bistatic sensing are reduced to very cumbersome mathematical expressions. The main difficulty here is connected with the consideration of quadratic terms in the function $f(t)$. Below the echo-signal power is examined for two cases: when the direction toward the detector is close to the specular reflection direction (θ_r is close to $-\theta_s$ or is equal to $-\theta_s$) and when the direction toward the detector differs significantly from the specular reflection direction (θ_r differs significantly from $-\theta_s$).

1. Direction toward the detector is close to the specular reflection direction

$$R_{0x}\, q_x \ll \zeta q_z + \frac{(R_0^2 + \zeta^2)}{(2c)}\, s .$$

Under this condition, the term $R_{0x}q_x/c$ can be neglected in the integrand $f(t)$ in Eq. (2). We also neglect the term $(\zeta^2/2c)s$ in the integrand $f(t)$, which is true when fairly mild condition $\zeta/L \ll 1$ is satisfied. Then, considering that for the sea surface γ_x^2 and $\gamma_y^2 \ll 1$ and assuming the Gaussian sensing pulse ($f(t) = (2/\sqrt{\pi})\exp(-4t^2/\tau_s^2)$), from Eq. (2), after averaging over ζ and $\boldsymbol{\gamma}$, we derive the following formula for the mean echo-signal power $\bar{P}(t)$:

$$\bar{P}(t) \cong b_{10}\exp[-0.5\, c\,x\,\tau_s\, d2/s + (d5)^2] \times$$
$$\times \{b_2\exp[-0.5\, c\,x\,\tau_s d4/s + (d5)^2\,[(d4)^2 + 2d2\, d4]]\} \times$$
$$\times H\,[d7 + 0.25\,(d3)^{1/2}\, d4d6] + b_3\, H\,[d7]\} , \qquad (3)$$

where

$$H(x) = 1 - \Phi(x) ,$$

$$d1 = 1 + 2\sigma^2\, C_s\sin^2\theta_s + 2\sigma^2\, C_r\sin^2\theta_r , \quad q^2 = q_z^2 + q_x^2 ,$$

$$d2 = C_s(\cos^2\theta_s + 1) + C_r(\cos^2\theta_r + 1) ,$$

$$d3 = 1 + 8\sigma^2 q_z^2/(c^2\tau_s^2\, d1),\quad d6 = c\tau_s/s,$$

$$d4 = \frac{s^2}{2q_z^2\,\bar{\gamma}_y^2} + \frac{T^2}{2q_z^2\,\bar{\gamma}_x^2} ,$$

$$d5 = 0.25d2(d3)^{1/2}d6,\quad d7 = d5 - x(d3)^{-1/2} ,$$

$$x = \frac{2t'}{\tau_s},\quad b_2 = (1 - S_f)\frac{q^4}{q_z^4}\frac{V^2}{8(\bar{\gamma}_x^2\,\bar{\gamma}_y^2)^{1/2}}\exp\left(-\frac{q_x^2}{2\,q_z^2\,\bar{\gamma}_x^2}\right) ,$$

$$b_3 = S_f\, AQ,\quad b_{10} = b_0\,(d1)^{-1/2}\, d6,\quad b_0 = \frac{a_s\, a_r}{L_s^2\, L_r^2} ,$$

$$a = 4\left(\frac{1}{\bar{\gamma}_x^2} + \frac{1}{\bar{\gamma}_y^2}\right)^{-1},\quad \beta = 0.5\,a\Delta,\quad \Delta = 0.5\left(\frac{1}{\bar{\gamma}_x^2} - \frac{1}{\bar{\gamma}_y^2}\right) ,$$

$$Q = \frac{a\exp(1/2a)}{4(\bar{\gamma}_x^2\bar{\gamma}_y^2)^{1/2}}\sum_{k=0}^{\infty}\frac{a^{-k}}{k!}\left(\frac{\beta}{2}\right)^{2k}\{\sin\theta_s\sin\theta_r\, a^{1/4}\frac{\Gamma(2k+2)}{\Gamma(k+1)}\times$$
$$\times W_{-k-3/4,k+3/4}(1/a) - \sin\theta_s\sin\theta_r\, a^{-1/4}\frac{\Gamma(2k+3)}{\Gamma(k+2)}\left(\frac{\beta}{2}\right)\times$$
$$\times W_{-k-5/4,k+5/4}(1/a) + 2\cos\theta_s\cos\theta_r\, a^{-1/4}\frac{\Gamma(2k+1)}{\Gamma(k+1)}\times$$
$$\times W_{-k-1/4,k+1/4}(1/a)\} ,$$

σ^2 and $\bar{\gamma}_{x,y}^2$ are the variances of elevations and slopes of the rough sea surface, $\Phi(x)$ is the probability integral, and $W_{n,m}(x)$ is the Whittaker function.

For the transparent aerosol atmosphere we have[7]

$$C_{s,r} = (\alpha_{s,r}\, L_{s,r})^{-2} ,\quad a_s = \frac{P_0\exp(-\tau_1)}{\pi\,\alpha_s^2} ,$$

$$a_r = \pi r_r^2\exp(-\tau_2) ,$$

where $\tau_{1,2}$ are the optical thicknesses on the paths source-surface and detector-surface, $2\alpha_{s,r}$ are the angle of divergence of the source and the field-of-view angle of the detector, P_0 is the power radiated by the source, and r_r is the effective radius of the receiver aperture.

In the derivation of formula (3), sea surface roughness was assumed to be slightly anisotropic ($\beta \ll 1$) and one of the two conditions: either $\sin^2\theta_{s,r} \ll 1$ (sensing in the direction near the nadir) or $\sigma^2 C_{s,r} \ll 1$ (diameter of the illuminated spot from the source and viewing sector of the detector on the surface are much greater than roughness elevations of the sea surface) was satisfied.

At $\theta_s = \theta_r = 0$ and $L_s = L_r$, Eq. (3) coincides with the formula for the mean power of the echo-signal reflected from the rough sea surface in case of sensing in the nadir.[4]

In Fig. 1, the results of calculations of echo-pulse waveform coming from the sea surface in the direction of specular reflection is shown for different angles of source divergence and different velocities of the driving wind. Calculations of the quantity $\bar{P}(t')/[\bar{P}(t' = 0)] = G(x)$ were done by formula (3) with the following values of the parameters: $\alpha_r = 0.1$,

$\tau_s = 10^{-9}$ s, $L_s = L_r = 5$ km, $\theta_s = \theta_r = 0°$, and $\alpha_s = 5\cdot10^{-3}$ (*1*), 10^{-2} (*2*), and $2\cdot10^{-2}$ (*3*).

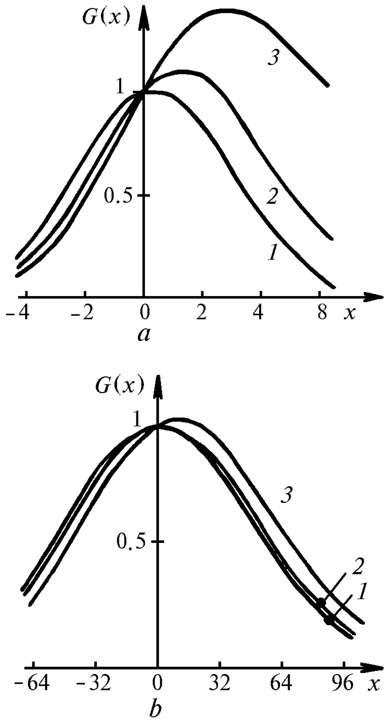

FIG. 1. Waveform of the echo-pulse reflected from the sea surface in case of sensing in the nadir: U = 5 (a) and 15 m/s (b).

Here and further $\overline{\gamma}^2_{x,y}$ were calculated from the Cox and Munk[8] formulas and S_f and σ^2 were calculated from the formulas[2,9] $\sigma = 0.016U^2$, $S_f = 0.009U^3 - 0.3296U^2 + 4.549U - 21.33$, where U is the velocity of the driving wind (m/s).

The albedo of foam was $A = 0.5$ (see Ref. 10).

From Fig. 1, it can be seen that the echo-signal maximum recorded by the detector in the specular reflection direction is observed at $t' > 0$. Its shift with respect to $t' = 0$ is determined by the illuminated spot diameter on the sea surface. With the increase of the illuminated spot diameter (with the increase of the source divergence angle), the shift of the echo-signal maximum with respect to $t' = 0$ is increased. With the increase of the driving wind velocity, the echo-pulse duration is sharply increased (which is connected primarily with the increase of the variance of the sea surface elevations in case of sensing in the nadir) and the shift of the echo-signal maximum with respect to $t' = 0$ becomes less pronounced (Fig. 1*b*).

We note that although Fig. 1 illustrates the most interesting case when $\theta_s = \theta_r = 0$ (sensing in the nadir), the echo-pulse waveform is the same for specular reflection direction at arbitrary θ_s. Dependence on the driving wind velocity and angle of the source divergence is also the same.

2. Direction toward the detector differs significantly from the specular reflection direction ($R_{0x}q_x \gg (R_0^2 + \zeta^2)s/(2c)$).

Under this condition the term $(R_0^2 + \zeta^2)s/(2c)$ can be neglected in the integrand $f(t)$ in Eq. (2). Then considering that for the sea surface $\overline{\gamma}^2_x, \overline{\gamma}^2_y \ll 1$, we derive for the Gaussian sensing pulse, after averaging over ζ and γ (see Refs. 4 and 6)

$$P(t) \cong b_1\{b_2 \exp[-z^2 - zd] + b_3 \exp[-z^2]\} , \qquad (4)$$

where

$$b_1 = b_0 N_r^{-1/2} \frac{2}{\sqrt{\pi}} \nu^{-1/2}\overline{\omega}^{-1/2} ,$$

$$\overline{\omega} = 2\sigma^2\omega; \quad \mu = q_x \varpi - q_z \nu ,$$

$$\tau_M^2 = \frac{\tau_s^2}{8}\left\{1 - \frac{4}{\tau_s^2 c^2 \nu}\left[q_x + \frac{\mu^2}{\omega\nu}\right]\right\}^{-1} ,$$

$$\omega = \frac{d1}{2\sigma^2} + \frac{4q_z^2}{\tau_s^2 c^2} - \frac{\varpi}{\nu} , \qquad z = \frac{t'}{2^{1/2}\tau_M} ,$$

$$\varpi = C_s \sin\theta_s \cos\theta_s + C_r \sin\theta_r \cos\theta_r + \frac{4}{\tau_s^2 c^2} q_z q_x ,$$

$$\nu = \frac{T^2}{2\overline{\gamma}^2_x q_z^2} + C_s \cos^2\theta_s + C_r \cos^2\theta_r + \frac{4q_x^2}{\tau_s^2 c^2} ,$$

$$d = \frac{2^{7/2}\tau_M}{\tau_s^2 c\nu}\left[q_x + \frac{\varpi}{\omega\nu}\mu\right]\frac{q_x T}{q_z^2 2\overline{\gamma}^2_x} , \quad N_r = C_s + C_r .$$

In Fig. 2, the results of calculations of the echo-pulse waveform reflected from the sea surface are shown for different sensing angles and different driving wind velocities. The quantity $\bar{P}(t')/[\bar{P}(t' = 0)] = F(z)$ was calculated by Eq. (4) with the following values of the parameters: $\alpha_r = 0.1$; $\tau_s = 10^{-9}$ s; $L_s = L_r = 10$ km; $\theta_s = 20°$; $\theta_r = -15$ (curves *3* and *6*), 0 (curves *2* and *5*), and 20° (curves *1* and *4*); $\alpha_s = 10^{-3}$ (curves *4*, *5*, and *6*) and $5\cdot10^{-2}$ (curves *1*, *2*, and *3*).

From Fig. 2*a*, it can be seen that at small driving wind velocity for the foam-free surface when the directions toward the detector differ significantly from the specular direction, the echo-signal maximum is shifted to $t' < 0$. Physically, this is connected with the fact that the angle between the specular reflection direction and the direction toward the detector depends on the position of the point within the illuminated spot on the sea surface. Therefore, the echo-signals entering the detector from different points of the illuminated spot on the sea surface differ significantly in their amplitudes. Hence, the echo-signal maximum recorded by the detector is shifted to $t' < 0$ when echo-signals from the points of the surface having maximum angles between the specular reflection direction and the direction toward the detector are coming to the detector. This effect is most strongly manifested in case of slant monostatic sensing and exclusively for sufficiently wide beams producing the illuminated spot. For narrow beams producing the illuminated spot the echo-pulse shift to $t' < 0$ is insignificant (curves 4–6 merge in Fig. 2*a*).

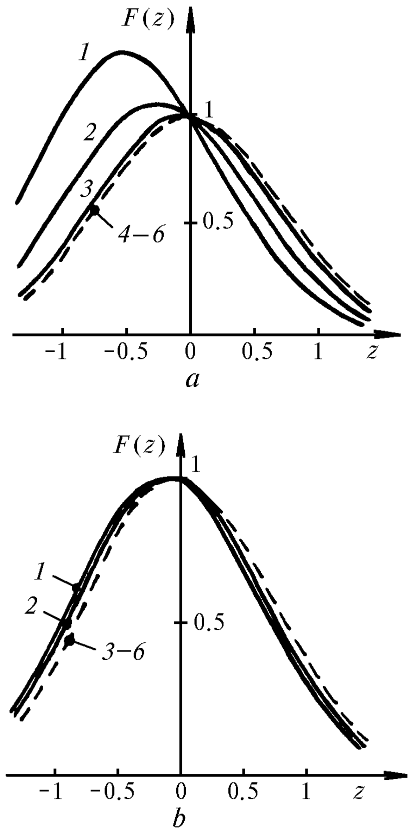

FIG. 2. Waveform of the echo-pulse coming from the sea surface in case of slant bistatic sensing: U = 5 (a) and U = 15 m/s (b).

For large velocity of the driving wind (when foam appears on the sea surface), the echo-signal waveform is largely determined by the radiation scattered by the sections of foam being the Lambertian reflectors and forming the echo-signals symmetric about $t' = 0$. From Fig. 2*b*, it can be seen that the position of the echo-pulse maximum at $U = 15$ m/s is close to $t' = 0$ at any θ_r.

Thus, in the present paper the waveform of the lidar echo-pulse has been investigated in case of sensing of the rough sea surface for a wide range of variations of the driving wind velocities. It is shown that the geometry of sensing and the presence or absence of foam may change the waveform and the position of the laser echo-pulse maximum. For sensing in the nadir, the echo-signal maximum is recorded by the detector at $t' > 0$ and its shift with respect to $t' = 0$ is determined by the illuminated spot diameter on the sea surface. In case of slant monostatic sensing and small velocity of the driving wind, the echo-signal maximum is shifted to $t' < 0$.

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