ASYMPTOTIC ESTIMATES OF THE MIE SERIES PARTIAL WAVE AMPLITUDES WITHIN LARGE SPHERICAL PARTICLES

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The paper presents asymptotic representations of the partial wave amplitudes of the Mie series, which do not involve special functions. The amplitude convergence depending on the refractive index of a large particle is considered. We have also made a comparison between the partial wave amplitudes calculated by asymptotic formulae and those calculated by the Mie theory formulae. The estimates obtained could be useful for testing the computational algorithms constructed using the precise Mie formulae.

Knowledge of spatial distribution of optical fields within spherical particles is necessary in a wide range of problems in the aerosol nonlinear optics. Among them there are evaporation and levitation of aerosol particles in the field of high-power laser radiation, optical breakdown in aerosol, and nonlinear scattering of plane electromagnetic waves on plasma clusters.[1]

The Mie series, describing electromagnetic fields inside the spherical aerosol particles, have a very slow convergence up to the dimensions satisfying the condition $n >> |m\rho|$, where m is the complex refractive index of the substance and ρ is the diffraction parameter. For a correct description of the electromagnetic field within a spherical particle taking account amplitudes of the higher order partial waves is required as well as the possibility of calculating Riccati-Bessel function of the first (FRB1) and third (FRB3) kind with high accuracy. The account of a large number of partial wave amplitudes does not always result in the increase of accuracy of calculations of the Mie series sums. This is connected with the loss of accuracy of standard methods of FRB1 calculation with a complex argument $z = r - i\mu$ when increasing n index and at relatively large values of the diffraction parameter ρ. The accuracy of FRB1 calculation is limited by the computer word capacity and to calculate FRB1 with a complex argument $|Im(m\rho)| > 30$, it is essential to use the double-numbers.[2] In this case the FRB1 calculation with the use of double-precision arithmetic by the method of ascending recursion gives a correctly calculated FRB1 succession at relatively small values of the diffraction parameter of particles.[3] The Miller-Olver-Temme algorithm, based on the three-term recursion dependence, provides a correct procedure of FRB1 calculation for $|z| < 30$.[4] The potentialities of an extended use of the algorithm are considered in Ref. 5. The accuracy of this method depends on the initial iteration number $N_p >> n$ and on the type of the normalizing expression used. The use of the normalizing expression derived in Ref. 6, is demonstrated to provide higher accuracy of FRB1 calculation than in the case with expanding into the continued fraction.[7] The convergence of the continued fraction method when calculating FRB1 used in Ref. 8 for the Mie sum calculation is entirely related to the numerical stability of the three-term recursion relations.[9] Thus, in any method of numerical calculations at large diffraction parameters, a strict control is needed over the calculations of Mie series coefficients by comparing the results obtained with those by approximate formulas. This approach has first been tried in the monograph 10. Later its usefulness was discussed in Ref. 2. This paper presents simple asymptotic expressions obtained by the authors. These expressions do not contain special functions and have been derived for the partial wave amplitudes within the spherical particle with the number $n >> |m\,\rho|$ and taking into account the degree of growth of the diffraction parameter simultaneously with n, in the range where standard methods of FRB1 calculation do not work.

Such an approach provides the analysis of partial wave amplitudes inside large particles, $\rho > 100$, as well as the analysis of an expanding spherical plasma cell of the optical breakdown.[11]

The partial wave amplitudes may be written in a form suitable for calculations[12]

$$c_n = m\frac{\Psi_n(\rho)}{\Psi_n(m\rho)}\left(\frac{D_n(\rho) - C_n(\rho)}{D_n(m\rho) - mC_n(\rho)}\right); \quad (1)$$

$$d_n = m\frac{\Psi_n(\rho)}{\Psi_n(m\rho)}\left(\frac{D_n(\rho) - C_n(\rho)}{mD_n(m\rho) - C_n(\rho)}\right), \quad (2)$$

where $D_n(\rho)$ and $C_n(\rho)$ are the logarithmic derivatives of FRB1 and FRB3, respectively[13]

$$D_n(\rho) = \Psi'_n(\rho) / \Psi_n(\rho); \quad (3)$$

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$$C_n(\rho) = \zeta'_n(\rho) / \zeta_n(\rho) . \quad (4)$$

By the definitions of FRB1 and FRB3 we have

$$\Psi_n(z) = \sqrt{\frac{\pi z}{2}} J_{n+1/2}(z) ; \quad (5)$$

$$\zeta_n(z) = \sqrt{\frac{\pi z}{2}} H^{(2)}_{n+1/2}(z) . \quad (6)$$

To obtain the desired estimate we use the asymptotic expressions for the Bessel and Hankel functions of the second kind when $|z| \le c(n+3/2)^{1/2}$, $z \to \infty$, $n \to \infty$, $c > 0$ (see Ref. 14):

$$J_{n+1/2}(z) = \left(\frac{z}{2}\right)^{n+1/2} \frac{1}{\Gamma(n+3/2)} \times$$

$$\times \exp\left[-z^2/4(n+3/2)\right]\left[1+O(1/n)\right]; \quad (7)$$

$$H^{(2)}_{n+1/2}(z) = \frac{i}{\pi}\left(\frac{z}{2}\right)^{-(n+1/2)} \Gamma(n+1/2) \times$$

$$\times \exp\left[z^2/4(n+1/2)\right]\left[1+O(1/n)\right], \quad (8)$$

where $c(n+1/2)$ is the gamma function, and the relations for logarithmic derivatives are as follows:

$$D_n(z) = (n+1)/z ; \quad (9)$$

$$C_n(z) = -n/z . \quad (10)$$

Substitution of Eqs. (5)–(10) into the initial relations (1)–(4) gives the asymptotic expressions for the partial amplitudes

$$c_n = \frac{(2n+1)}{(n+1)} \frac{1}{(1+m^2)m^{n-1}} \exp\left[(m^2-1)\rho^2/4(n+3/2)\right]; \quad (11)$$

$$d_n = \frac{1}{m^n} \exp\left[(m^2-1)\rho^2/4(n+3/2)\right] . \quad (12)$$

Considering the asymptotic formulas (11) and (12) under condition of a simultaneous increase of ρ and $n \to \infty$ and real $m^2 > 1$, we obtain the that the amplitudes have, in the denominator, an increasing power function that provides for the convergence of the Mie sum.[2]

At real numbers $m^2 \le 1$ (this case corresponds to the plasma sphere with a negligible absorption) the partial wave amplitudes have, in the denominator, a decreasing exponential function and therefore tend to infinity. However, when summing the amplitudes in Mie series are multiplied by FRB1 and its derivatives that, in the final analysis, offers the Mie sum convergence but a very slow one.

The figure shows relative deviations of the partial wave amplitudes, calculated by standard methods (namely, by the Miller-Olber-Temme algorithm and the two-point recursion formulas of the descending recursion for logarithmic derivatives,[15]) from those calculated by Eqs. (11) and (12). The calculations were made for a spherical particle from Al_2O_3 (corundum) with $m = 1.829 - i5.47 \cdot 10^{-3}$, and radius of 20 μm, at the wavelength of 1.06 μm. The numbers *1* and *2* denote the real and imaginary parts of the relative deviation δ for amplitudes c_n, while *3* and *4*, for the amplitudes d_n, respectively. After $n > 340$ a sharp and simultaneous jump occurs in the real and imaginary parts of the amplitudes due to accumulation of the error when calculating by standard methods.

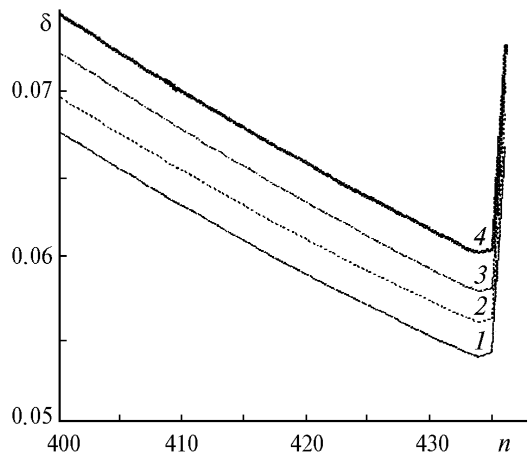

FIG. 1.

Thus, use of the above-mentioned asymptotic forms we show the amplitude convergence at $n \to \infty$ and real $m^2 > 1$ as well as the amplitude divergence in the case of $m^2 \le 1$. Use of the above asymptotic forms together with the Meissel asymptotic representations considered by the authors in Ref. 16, extends the capabilities of the Mie theory for coarse spherical plasma clusters and large absorbing particles with the diffraction parameter $\rho > 100$. The above asymptotic forms may provide a precise control over the calculation process based on standard algorithms.[5,6] Special value must be placed on the fact that in the case of plasma clusters ($m^2 \le 1$) it is necessary to take into account a longer succession of partial wave amplitudes in order to provide correct calculations of the Mie sums.

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