DIFFERENTIAL OPTICAL METER OF THE PARAMETERS OF ATMOSPHERIC TURBULENCE

L.V. Antoshkin, N.N. Botygina, O.N. Emaleev, L.N. Lavrinova, and V.P. Lukin

Institute of Atmospheric Optics,
Siberian Branch of the Russian Academy of Sciences, Tomsk
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We describe here a differential optical meter of the parameters of atmospheric turbulence. It reduces the contribution of relative shifts of the radiation source and receiving system to the error of the parameters' determination. Determination of the structure constant of the refractive index and of the Fried radius is based on detecting random image jitter caused by the atmospheric turbulence. We estimate the degree of suppression of the angular displacements of the source in the spectrum of the difference signal compared to the same component in the signal spectrum at the output of a single-channel meter. The values C_n^2 are measured under angular displacements of the source in the frequency range 0–130 Hz by the single-channel and differential methods. The contribution coming from these displacements to the error of C_n^2 determination from the differential measurements is shown to be reduced by approximately 50 times at the amplitude of angular fluctuations of the source being 3 seconds of arc.

INTRODUCTION

Compact mobile meters of the parameters of atmospheric turbulence are used to estimate the influence of atmospheric turbulence on the visibility of extra-terrestrial sources through the atmosphere when choosing places for mounting astronomical instruments, and minimization of the distorting effect of the turbulence upon operation of complex optical systems. The value of the structure constant of the refractive index C_n^2 averaged over the propagation path and the Fried radius, r_0, of a plane wave can be determined by measuring the variance of random shifts of the power center of gravity (PCG) of a light source's image.[1] In such meters, the source's and the receiver's vibrations bring a significant contribution to the error of turbulence parameters' determination. Differential meters of the image jitters[2] are free from this shortcoming.

DESCRIPTION OF THE DEVICE AND MEASUREMENT TECHNIQUE

In this paper, we describe the differential optical meter of turbulence parameters with a laser source of radiation. It has been designed on the base of the Maksutov mirror-lens telescope AZT–7 and an IBM compatible computer.

Block-diagram of the meter is presented in Fig. 1. A He–Ne laser *1* is used as a radiation source. The diverging beam is formed at the transmitter by use of small-size optics *2* (what is important when designing mobile units). By the end of the path, the beam diameter exceeds the diameter of the input lens of the telescope. The meter is calibrated using an angular controllable mirror *3*, with known angular sensitivity. Radiation propagates along an atmospheric path *4* of the length L.

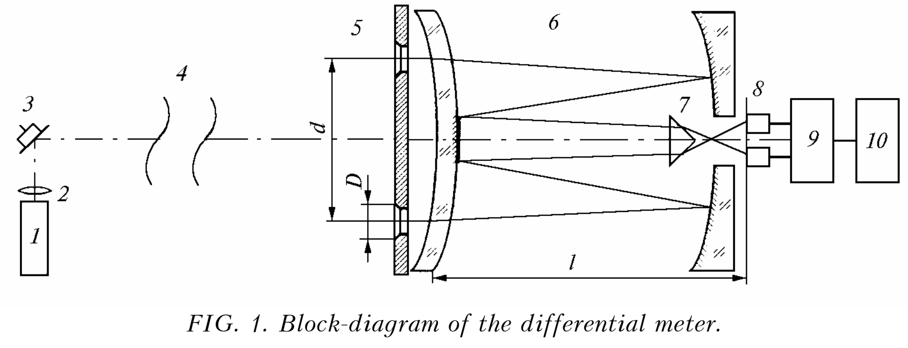

FIG. 1. Block-diagram of the differential meter.

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The changeable masks *5* are placed in front of the entrance lens *6* of the telescope. Two round sub-apertures in the masks are symmetric with respect to the optical axis of the receiving system. The sub-apertures' diameter D is 4 mm, the distance between those, d, may be varied from 68 to 90 mm. Two images of the source which are formed by the optical system of the telescope and by the entrance sub-apertures are spatially separated in the plane of sharp image with a Fresnel biprism *7*.

Image displacements are detected by two quadrant coordinate-sensitive detectors *8* of FD–19KK type. Signals from the detectors are entered into the two-coordinate meters of angular displacements *9* of the image's power center of gravity.[3]

Recording and processing of analog signals from the meters are performed by use of the hardware-software complex *10* which includes an IBM compatible computer, an 8-channel system of analog information acquisition, a 12-bit ADC, and a specialized software package.[4]

The construction of the receiving part of the meter guarantees sufficient robustness of the adjustment units. This is a critical point in the differential measurements.

The measurement technique used is as follows. One measures the variance of the difference in angular displacements of the images' PCG from two sub-apertures and calculates parameters of the atmospheric turbulence.

To determine the structure constant of the refractive index and the Fried radius, we use the calculation scheme proposed in Ref. 2. However, we perform calculations for a diverging laser beam.

Let us write the variance of the difference between the angular PCG displacements, $\sigma^2_{\alpha_1-\alpha_2}$, in terms of the variance of angular PCG displacements of each image $\sigma^2_{\alpha_1}$, $\sigma^2_{\alpha_2}$ and the cross correlation function B_α under condition that $\sigma^2_{\alpha_1} = \sigma^2_{\alpha_2} = \sigma^2_\alpha$:

$$\sigma^2_{\alpha_1-\alpha_2} = 2\,(\sigma^2_\alpha - B_\alpha); \tag{1}$$

$$B_\alpha = \langle\alpha_1\,\alpha_2\rangle = \langle\alpha(x,\,y)\,\alpha(x+\xi,\,y+\eta)\rangle, \tag{2}$$

where $\alpha(x, y)$, $\alpha(x + \xi, y + \eta)$ are fluctuations of the arrival angles at the center of entrance sub-apertures.

Let us make use of the expressions for the correlation function of the arrival angles B_{α_x}, correlation function B_φ, and the structure function of phase fluctuations D_φ in the following form:

$$B_{\alpha_x}(\xi,\,\eta) = -\frac{\lambda^2}{4\pi^2}\frac{\partial^2}{\partial\xi^2}B_\varphi(\xi,\,\eta)\,, \tag{3}$$

$$B_\varphi(\xi,\,\eta) = -\frac{D_\varphi(\xi,\,\eta)}{2} + \sigma^2_\varphi\,, \tag{4}$$

$$D_\varphi = 6.88\left(\frac{r}{r_c}\right)^{5/3}, \tag{5}$$

where α_x is the xth component of the fluctuation part of the arrival angles for radiation with the wavelength λ, σ^2_φ is phase fluctuations' variance; $r^2 = \xi^2 + \eta^2$; r_c is the Fried radius of the beam. So, if the directions of separation of the observation points and the wave front tilts coincide, the correlation function of fluctuations of the arrival angles is as follows ($\xi = d$, $\eta = 0$ longitudinal correlation):

$$B_{\alpha_x} = 0.097\left(\frac{\lambda}{r_c}\right)^{5/3}\left(\frac{\lambda}{d}\right)^{1/3}, \tag{6}$$

If the directions of separation of the observation points and the wave front tilts at these points are perpendicular, we have ($\xi = 0$, $\eta = d$ transverse correlation):

$$B_{\alpha_x} = 0.145\left(\frac{\lambda}{r_c}\right)^{5/3}\left(\frac{\lambda}{d}\right)^{1/3}, \tag{7}$$

Using general expression for the variance of displacements of the images' PCG,[5] we obtain formula for the variance of angular PCG displacements in the plane of sharp image under Kolmogorov turbulence and concrete parameters of the beam ($\Omega = ka^2/L = 4.90$; $F = -4.5$ m, where k is the wave number; a is the beam radius; F is the wave front curvature) and receiving system ($\Omega_t = ka_t^2/L = 0.1$, where a_t is the efficient radius of the entrance aperture):

$$\sigma^2_\alpha = 2.187\,C_n^2\,LD^{-1/3} = 0.314\,r_c^{-5/3}\,\lambda^2\,D^{-1/3}, \tag{8}$$

where

$$r_c = 1.69\,r_0 = 1.69\,[0.423\,k^2\,C_n^2\,L]^{-3/5}; \tag{9}$$

r_0 is the Fried radius of a plane wave. The factor 1.69 is obtained from analysis of the expression for the coherence radius of a laser beam.[6]

Without the allowance for anisotropy ($\sigma^2_{\alpha_x} = \sigma^2_{\alpha_y}$), it follows from Eq. (8) that

$$\sigma^2_{\alpha_x} = 0.157\,r_c^{-5/3}\,\lambda^2\,D^{-1/3}. \tag{10}$$

Substituting Eqs. (6), (10), and (7), (10) consequently into Eq. (1), we obtain variance of the difference of images' angular displacements:

– in the direction of separation of entry subapertures:

$$\sigma^2_l = 2r_c^{-5/3}\,\lambda^2\,[0.157\,D^{-1/3} - 0.097\,d^{-1/3}]; \tag{11}$$

– in the perpendicular direction:

$$\sigma^2_t = 2r_c^{-5/3}\,\lambda^2\,[0.157\,D^{-1/3} - 0.145\,d^{-1/3}]. \tag{12}$$

It follows from Eq. (11) that

$$r_c = \left\{ \frac{\sigma_l^2}{2\lambda^2\left[0.157D^{-1/3} - 0.097d^{-1/3}\right]} \right\}^{-3/5} . \quad (13)$$

Taking into account Eq. (9), we have

$$r_0 = 0.592\left\{ \frac{\sigma_l^2}{2\lambda^2\left[0.157D^{-1/3} - 0.097d^{-1/3}\right]} \right\}^{-3/5} ; \quad (14)$$

$$C_n^2 = \frac{\sigma_l^2}{1.411\pi^2 L\left[0.157D^{-1/3} - 0.097d^{-1/3}\right]} . \quad (15)$$

Similarly, as follows from Eqs. (12) and (9),

$$r_c = \left\{ \frac{\sigma_t^2}{2\lambda^2\left[0.157D^{-1/3} - 0.145d^{-1/3}\right]} \right\}^{-3/5} , \quad (16)$$

$$r_0 = 0.592\left\{ \frac{\sigma_t^2}{2\lambda^2\left[0.157D^{-1/3} - 0.145d^{-1/3}\right]} \right\}^{-3/5} , \quad (17)$$

$$C_n^2 = \frac{\sigma_t^2}{1.411\pi^2 L\left[0.157D^{-1/3} - 0.145d^{-1/3}\right]} . \quad (18)$$

When determining C_n^2 from measurements with a single-channel scheme, using the formula (8) and the isotropy condition, we obtain

$$C_n^2 = \sigma^2_{\alpha_x}/(1.094\ LD^{-1/3}). \quad (19)$$

TESTS OF THE DIFFERENTIAL METER

The tests of the differential meter have been performed in October, 1997 at a 100-m-long horizontal path between the buildings of the Institute of Atmospheric Optics. The path run at the height of 10 m over the underlying surface.

To calibrate the meter, a laser beam was deviated by a controllable mirror at the entrance to the turbulent medium along the direction of separation of the entrance sub-apertures and in the perpendicular direction at a rate of 700 Hz. This rate considerably exceeds the upper boundary of the temporal spectrum of the arrival angles' fluctuations caused by the turbulence. Sensitivity of the meter was determined with the account of the experiment geometry by comparing the mirror fluctuations' amplitude and the amplitude of the input signals from single-channel meters.

To provide the operation of quadrant detectors of the meter in the linear portion of the position characteristic, random displacements of the image must not exceed 1/3 of the diffraction image radius. At the diameter of the entrance aperture of 4 mm, maximum permissible angular displacements of PCG are
$6.4\cdot10^{-5}$ rad. Therefore, the variance of the angular displacements must not exceed $4.0\cdot10^{-10}$ rad^2. This condition determines the limiting values of turbulence parameters to be measured. At the path length of 100 m, the limiting value of C_n^2 is $6.3\cdot10^{-13}$ m$^{-2/3}$, and the minimum Fried radius of a plane wave is 9 mm.

Minimal values of C_n^2 are determined by noises in the measuring system. For the signal-to-noise ratio 3, the threshold value of C_n^2 is $1.4\cdot10^{-15}$ m$^{-2/3}$.

Simultaneous measurements of the turbulence parameters by the differential technique and by the single-channel scheme demonstrated good coincidence of the results. The measurements have been performed under different conditions of atmospheric turbulence and at different time. Figure 2 presents a fragment of data obtained on the 16th of October, 1997.

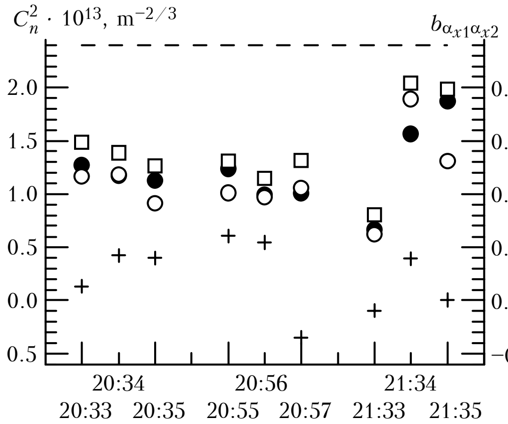

FIG. 2. Comparison of C_n^2 values measured by the single-channel scheme and by the differential technique: differential scheme (□); simultaneous measurements by meters I and II (●,); coefficients of cross correlation between the signals from the I and II meters of the angular displacements of the image (+ –); the calculated value is shown by the dashed line. Realization length is 40 seconds. The distance between the sub-apertures on the X axis is 68 mm.

The observed excess of C_n^2 values that are measured by the differential technique can be explained by the fact that the cross correlation coefficients obtained from the measurements of the images' PCG displacements differ from the values obtained by the formula $b_{\alpha_{x1}\,\alpha_{x2}} = 0.618\ (d/D)^{-1/3}$ that follows from Eq. (11). This is probably connected with the specific features of the path of radiation propagation, i.e., closely situated buildings.

The relative measurement error for C_n^2 is determined mainly by the systematic error and is 55% in measurements by the differential technique and 35% for the single-channel scheme.

The differential meter has been tested relative to the noise stability under conditions of considerable vibrations of the transmitter's and receiver's base. At the entrance to the turbulent atmosphere, a light beam was deviated by the mirror *3* controlled by a variable signal. The noise stability of the differential meter was estimated by the degree of suppression of the controlled mirror's fluctuations in the spectrum of the difference signal. Spectra of the signals from the differential meter were compared with those of the meter of the image's PCG displacements from one sub-aperture (single-channel measurement scheme).

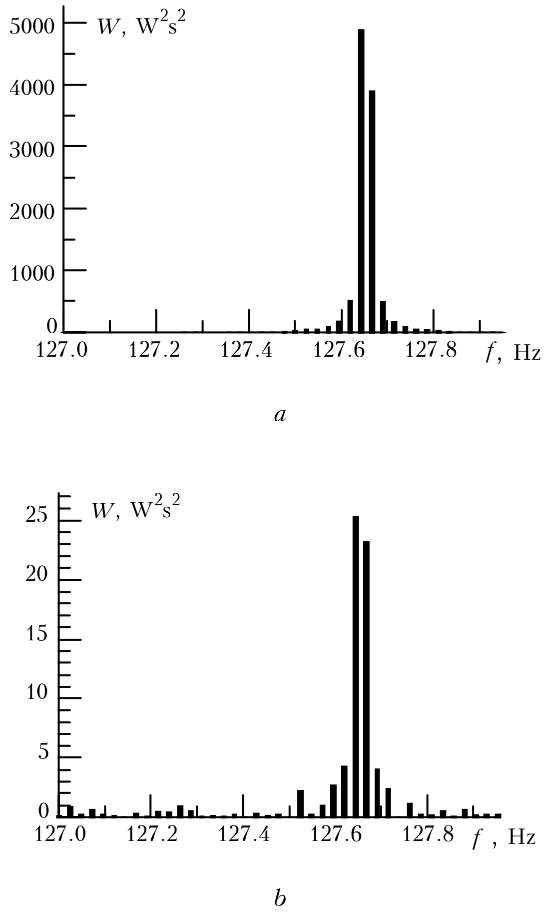

FIG. 3. Power spectra of signals near the fluctuation frequency of the controlled mirror: from the single-channel meter of the image jitter (a) and from the output of the differential meter (b).

Figure 3 presents power spectra of signals in the neighborhood of the frequency of fluctuations of the controlled mirror (amplitude of angular displacements of the source is 3 seconds of arc). To compare the spectra, realizations were simultaneously taken from three channels: signals at the outputs of the meters of image displacements from every aperture and the difference signal. The volume of input data is 2^{14} values for every channel, discretization frequency is 390 Hz. The distance between the sub-apertures' centers is 90 mm.

The degree of suppression (defined as relative decrease in the spectral component at the frequency of mirror fluctuation in the compared spectra) is 0.9948 in the presented case.

To illustrate the advantage of the differential measurement technique under conditions when relative displacements of the source and receiver occur, we present in the Table I the structure constant of the refractive index obtained from simultaneous measurements of the image jitter along one coordinate and from measurements of the difference in the images' displacements along the direction of the entrance apertures separation.

As follows from the Table, contribution of relative displacements of the source and receiver to the error of C_n^2 determination decreases approximately by 50 times in the differential measurement technique under conditions of the experiment (amplitude of angular displacements is 3 seconds of arc).

TABLE I.

Frequency of mirror fluctuations, Hz	C_n^2, m$^{-2/3}$, from measurements of $\sigma^2_{\alpha_x}$	C_n^2, m$^{-2/3}$, from measurements of σ^2_l
130	$6.88 \cdot 10^{-13}$	$1.45 \cdot 10^{-13}$
30	$6.30 \cdot 10^{-13}$	$1.14 \cdot 10^{-13}$
10	$6.18 \cdot 10^{-13}$	$1.05 \cdot 10^{-13}$
1.5	$6.88 \cdot 10^{-13}$	$1.20 \cdot 10^{-13}$
Stationary source	$1.06 \cdot 10^{-13}$	$1.16 \cdot 10^{-13}$

The tests conducted have demonstrated a good stability of the method of differential measurement to relative displacements of the source and receiver.

CONCLUSION

The differential meter of turbulence parameters designed can be used as a mobile unit in determining turbulence parameters along the atmospheric paths, when operatively estimating the contribution brought by the optical state of the atmosphere to the measurement error of optical systems operating in turbulent atmosphere.

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