DIFFERENCES AND SIMILARITY OF TWO SCHEMES TO FORM THE LASER-INDUCED GUIDE STARS

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The paper considers some problems connected with one of the most promising tendencies in the development of modern ground-based adaptive telescopes, in particular, with its fitting with additional optical system for a laser-induced guide star formation. The calculated results are presented for a "general" scheme of the laser-induced star formation when an arbitrary correlation between the random angular displacements of the image due to the laser beam fluctuations along the paths of forward and backward propagation may be achieved.

It is known that the use of the laser-induced guide stars for improving the quality of the image constructed with the ground-based telescopes may significantly extend the possibilities of applying the adaptive correction to astronomical observations. However, the use of signals from the laser-induced guide stars to correct total tilts of the wave front directly is impossible because of the necessity to isolate such signals from the data of optical observations on the jitter of an artificial star image.

In his most recent papers[1–4] Roberto Ragazzoni tried to systematize numerous approaches to the solution of the problem on determining the full tilt of the wave front for an adaptive optical system operating based on the signal from a laser-induced guide star. Different approaches are being used to solve this problem, what essentially complicates the technical aspects of its solution. Those may, for example, use simultaneous measurements of the overall angular jitter of a sufficiently bright natural star,[3,4] two-color laser-induced guide stars,[5] auxiliary telescopes[6,7] or auxiliary laser searchlights.[8–10] In the latter two approaches, given definite optical geometry, the laser-induced guide star can no longer be considered as a point reference source of light.

It should be noted that these approaches (with the use of auxiliary telescopes or auxiliary laser searchlights) may be implemented using quite simple optical arrangements. In one of his papers[4] R. Ragazzoni has proposed the following two versions of the optical arrangement that are shown in the Figure 1 (*a* and *b*).

In Ref. 4 R. Ragazzoni has even used an idea of "symmetry" to underline the identity of these two different approaches (from the viewpoint of their efficiency in correcting the overall tilt of the wave front). In this paper I make an attempt to show that no complete "symmetry" of the two optical arrangements presented in the figure occurs.

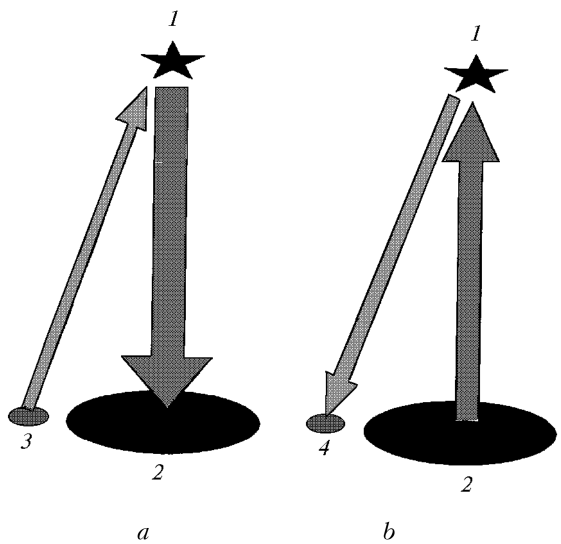

FIG. 1. Two optical arrangements for forming the laser-induced reference stars with an auxiliary laser searchlight (a) and an auxiliary telescope (b): the laser-induced star (1), the aperture of the basic telescope (2), the aperture of an auxiliary laser searchlight (3), and the aperture of an auxiliary telescope (4). The axes of the basic and auxiliary telescopes (laser searchlight) are spatially separated.

The signal that bears information on the instantaneous angular position of the laser-induced reference star image in the focal plane of a measuring telescope, in the case *a*, is described by the following expression:

$$\varphi_a = \varphi_{lb}(\rho_0) + \varphi_F^{ss}(0),$$

whereas in the case *b* the same signal has different form:

$$\varphi_b = \varphi_{lb}(0) + \varphi_F^{ss}(\rho_0).$$

Here we use the following designations: $\varphi_{lb}(\rho_0)$ are the random angular displacements of the energy center of gravity of the focused (at an altitude X) laser beam formed with an auxiliary laser searchlight whose optical axis is shifted from the origin of the coordinate system by the vector $\boldsymbol{\rho}_0$ and tilted from the zenith direction by the angle ρ_0/X; $\varphi_{lb}(0)$ are the random angular displacements of the energy center of gravity of a focused (at an altitude X) laser beam, formed with the basic telescope whose optical axis is directed exactly along the zenith direction; $\varphi_F^{ss}(\rho_0)$ is the vector characterizing the random angular tilts of the wave front, formed by the secondary light source at the altitude X due to scattering on atmospheric inhomogeneities, when observing the jitter of the secondary source image in the focal plane of the auxiliary telescope (whose optical axis is shifted from the origin of the coordinates by the vector $\boldsymbol{\rho}$ and is tilted from the zenith direction by the angle ρ_0/X); $\varphi_F^{ss}(0)$ is the vector characterizing the random angular tilts of the wave front, formed by the secondary source at the altitude X (due to light scattering on atmospheric inhomogeneities exactly backwards), when observing the jitter of the secondary source image in the focal plane of the basic telescope.

It is expected that spacing between the apertures of the basic and auxiliary telescopes (or an auxiliary laser light source) is such that the correlations $\langle\varphi_{lb}(\rho_0)\,\varphi_F^{ss}\rangle$ and $\langle\varphi_{lb}(0)\,\varphi_F^{ss}(\rho_0)\rangle$ are practically equal to zero.[9,10,19] The task to be achieved with these schemes is to correct the wave front from a natural star for random tilts (it is well known that such a star forms the plane wave front in the telescope focal plane), when the laser-induced reference star and the natural star have the same zenith and azimuth positions, that means, that the task is to provide correction for the random function $\varphi_F^{pl}(0)$.

Throughout this paper we shall use the algorithm of optimal[11–13] correction, that is known as providing for the lowest level of residual distortions. Now we assess the level of residual distortions, normalized to the value of uncorrected variance of the natural star image jitter for these two schemes of the reference star formation. The variance of the residual angular distortions for the scheme a, when using an optimal correction algorithm,[14–19] is given by the following expression:

$$\langle\beta^2\rangle_a = \langle(\varphi_F^{pl} - \varphi_a)^2\rangle/\langle(\varphi_F^{pl})^2\rangle =$$

$$= 1 - \frac{\langle\varphi_F^{pl}\ \varphi_a\rangle^2}{\langle(\varphi_F^{pl})^2\rangle\,\langle(\varphi_a)^2\rangle},$$

where

$$\langle(\varphi_a)^2\rangle = \langle\varphi_{lb}^2\rangle + \langle(\varphi_F^{ss})^2\rangle;$$

$$\langle\varphi_F^{pl}(0)\ \varphi_a\rangle = \langle\varphi_F^{pl}(0)\ \varphi_F^{ss}(0)\rangle.$$

Whereas for the scheme b it is

$$\langle\beta^2\rangle_b = \langle(\varphi_F^{pl} - \varphi_b)^2\rangle/\langle(\varphi_F^{pl})^2\rangle =$$

$$= 1 - \frac{\langle\varphi_F^{pl}\ \varphi_b\rangle^2}{\langle(\varphi_F^{pl})^2\rangle\,\langle(\varphi_b)^2\rangle},$$

where

$$\langle(\varphi_b)^2\rangle = \langle\varphi_{lb}^2\rangle + \langle(\varphi_F^{ss})^2\rangle;$$

$$\langle\varphi_F^{pl}(0)\ \varphi_b\rangle = \langle\varphi_F^{pl}(0)\ \varphi_{lb}(0)\rangle.$$

As a result we obtain that the relative efficiency of correction with these two approaches is presented by the following expressions:

$$\langle\beta^2\rangle_a = 1 - \frac{\langle\varphi_F^{pl}(0)\ \varphi_F^{ss}(0)\rangle^2}{\langle(\varphi_F^{pl})^2\rangle\,[\langle\varphi_{lb}^2\rangle + \langle(\varphi_F^{ss})^2\rangle]}, \tag{1}$$

$$\langle\beta^2\rangle_b = 1 - \frac{\langle\varphi_F^{pl}(0)\ \varphi_{lb}(0)\rangle^2}{\langle(\varphi_F^{pl})^2\rangle\,[\langle\varphi_{lb}^2\rangle + \langle(\varphi_F^{ss})^2\rangle]}. \tag{2}$$

The difference between the variance values of the residual angular fluctuations in the schemes a and b (see the figure) is in different numerators of the second terms of the expressions (1) and (2): $\langle\varphi_F^{pl}(0)\ \varphi_F^{ss}(0)\rangle^2$ and $\langle\varphi_F^{pl}\ \varphi_{lb})\rangle^2$.

Having used the results from Refs. 17–19 we can essentially simplify Eq. (2) by using the following representation:

$$\langle\varphi_F^{pl}(0)\ \varphi_{lb}(0)\rangle = K(0)\,\sqrt{\langle\varphi_{lb}^2\rangle\,\langle(\varphi_F^{pl})^2\rangle}.$$

As a result Eq. (2) reduces to the following formula:

$$\langle\beta^2\rangle_b = 1 - \frac{K(0)}{[1 + \langle(\varphi_F^{ss})^2\rangle/\langle\varphi_{lb}^2\rangle]}. \tag{3}$$

The function $K(0)$ is the cross-correlation between the angular shifts of the center of gravity of the focused laser beam and the shifts of the center of gravity of a plane wave in the focal plane of the telescope. This function has been calculated and tabulated in Ref. 19. The remaining quantities, of the Eq. (3), have also been calculated earlier,[14–18] for example:

$$\langle\varphi_{LB}^2\rangle = \left(2\pi^2\ 0.033\ \Gamma\left(\frac{1}{6}\right)\right)\,2^{1/6}\,R_0^{-1/3}\ \times$$

$$\times \int_0^X \mathrm{d}\xi\ C_n^2(\xi)\ \{[b^2\,(1 - \xi/X)^2]^{-1/6} -$$

$$- [b^2\,(1 - \xi/X)^2 + 4c^2]^{-1/6}\}, \tag{4}$$

where R_0 is the size of the aperture of the basic telescope; X is the altitude where the laser reference star occurs; $C_n^2(\xi)$ is the altitude profile of the structure characteristic of the refractive index of the turbulent atmosphere; $b = a_0/R_0$, a_0 is the size of a laser beam focused at the distance X; $c = \varkappa_0^{-1} R_0^{-1}$, $\varkappa_0^{-1}(\xi)$ is the outer scale of turbulence at the current altitude ξ;

$$<(\varphi_F^{ss})^2> = <(\varphi_F^{sp})^2> \left(\frac{a_{\log s}}{R_0}\right)^{-1/3} \times \frac{\int\limits_0^X \mathrm{d}\xi\, C_n^2(\xi)\, (1 - \xi/X)^2\, (\xi/X)^{-1/3}}{\int\limits_0^X \mathrm{d}\xi\, C_n^2(\xi)\, (1 - \xi/X)^{5/3}}. \quad (5)$$

The latter expression has been obtained in Ref. 19 based on the generalization of the results from Refs. 9 and 20.

The variance of the jitter, $<(\varphi_F^{ss})^2>$, of the "secondary" source image in Eq. (5) is expressed as the product of the variance of the image jitter of a point reference source (at the altitude X), $<(\varphi_F^{sp})^2>$, the averaging coefficient $(a_{\log s}/R_0)^{-1/3}$, where $a_{\log s}$ is the apparent size of the reference star and of the ratio between the two integrals:

$$\frac{\int\limits_0^X \mathrm{d}\xi\, C_n^2(\xi)\, (1 - \xi/X)^2\, (\xi/X)^{-1/3}}{\int\limits_0^X \mathrm{d}\xi\, C_n^2(\xi)\, (1 - \xi/X)^{5/3}}. \quad (6)$$

It is worth noting that the latter two factors in equation (5) produce a counteracting effects: if the factor $(a_{\log s}/R_0)^{-1/3}$ decreases the quantity $<(\varphi_F^{sp})^2>$, which is the variance of the image jitter of a point source, then the second factor is the ratio between the two integrals (6), that increases with the growth of X. There are no doubts that, such, seemingly quite different quantities, as $a_{\log s}$, that is, the apparent size of the reference star and its altitude X, are linearly related to each other. The ratio between the integrals (6) is denoted as

$$\frac{\int\limits_0^X \mathrm{d}\xi\, C_n^2(\xi)\, (1 - \xi/X)^2\, (\xi/X)^{-1/3}}{\int\limits_0^X \mathrm{d}\xi\, C_n^2(\xi)\, (1 - \xi/X)^{5/3}} = 1(X) \quad (7)$$

and it is given in the Table I (second column) as the function of the star altitude X (first column) for the model of $C_n^2(\xi)$ from Ref. 21. The table is arranged as three sections corresponding to calculations for the cases of a "medium", the "best", and the "worst" conditions of the atmospheric turbulence. As a result we obtain a simple formula for calculating the residual jitters of a natural star image:

$$<\beta^2>_b = 1 - \frac{K^2(0)}{1 + \dfrac{R_0^{-1/3}}{R_0^{-1/3}} \left(\dfrac{a_{\log s}}{R_0}\right)^{-1/3} 1(X)} = $$

$$= 1 - \frac{K^2(0)}{1 + \left(\dfrac{a_{\log s}}{a_0}\right)^{-1/3} 1(X)}. \quad (8)$$

TABLE I.

u, km	$1(u)$	$2(u)$
	$C_n^2(\xi)$ − medium	
1	2.260	0.863
10	3.671	2.650
20	4.329	3.469
30	4.826	4.024
40	5.240	4.455
50	5.602	4.815
60	5.921	5.134
70	6.211	5.415
80	6.472	5.666
90	6.720	5.897
100	6.947	6.112
	$C_n^2(\xi)$ − the best	
1	2.107	0.915
10	3.661	2.884
20	4.452	3.740
30	5.000	4.319
40	5.444	4.780
50	5.826	5.161
60	6.167	5.493
70	6.468	5.793
80	6.747	6.062
90	7.005	6.310
100	7.242	6.537
	$C_n^2(\xi)$ − the worst	
1	2.411	0.618
10	3.292	1.999
20	3.692	2.717
30	4.073	3.194
40	4.406	3.561
50	4.697	3.865
60	4.957	4.133
70	5.194	4.367
80	5.410	4.578
90	5.615	4.771
100	5.802	4.949

The estimates of the quantity $<\beta^2>_b$ by Eq. (8) show that for the case of $a_{\log s} = 10^3$ m, $a_0 = 10^0$ m, $R_0 = 8$ m, and the reference star at the altitude $X = 90 - 100$ km and at the parameter

$c = \varkappa_0^{-1} R_0^{-1} = 5 - 10$, the value $1(X) = 7.0$, and the coefficient of correlation $k(0) = 0.9$. Then

$$<\beta^2>_b = 1 - 0.81/[1 + 0.7] = 0.52.$$

That means that the actual correction achieved using the scheme b, decreases the variance of a natural star image jitter by about two times.

In the case a of the reference star formation scheme, the variance of the residual angular jitters of a natural star image, after the correction, is given by the following expression:

$$<\beta^2>_a = 1 - \frac{<\varphi_F^{pl}\ \varphi_F^{ss}>^2}{<(\varphi_F^{pl})^2>\ [<\varphi_{lb}^2> + <(\varphi_F^{ss})^2>]} .$$

If following the same way as with equations (7) and (8), we may write the cross-correlation function in the form

$$<\varphi_F^{pl}\ \varphi_F^{ss}> = K_1(0) \sqrt{<(\varphi_F^{pl})^2>\ <(\varphi_F^{ss})^2>}. \qquad (9)$$

Then we obtain that for the scheme a

$$<\beta^2>_a = 1 - \frac{K_1^2(0)}{[1 + <\varphi_{lb}^2>/<(\varphi_F^{ss})^2>]} , \qquad (10)$$

where the correlation coefficient

$$K_1(0) = <\varphi_F^{pl}\ \varphi_F^{ss}>/\sqrt{<(\varphi_F^{pl})^2>\ <(\varphi_F^{ss})^2>}. \qquad (11)$$

Let us now consider the cross-correlation presented by the expression (9). Let us calculate this cross-correlation as in the case with the function $K(0)$. As a result we obtain that

$$<\varphi_F^{pl}\ \varphi_F^{ss}> = <\varphi_F^{pl}\ \varphi_F^{ss}>\ (a_{\log s}/R_0)^{-1/6}\ 2(X),$$

where the function $2(X)$ is given in the third column of the table. As a result we have for the scheme a that

$$<\beta^2>_a = 1 - \frac{K_1^2(0)\ (a_{\log s}/R_0)^{-1/3}\ 2(X)}{b^{-1/3} + (a_{\log s}/R_0)^{-1/3}\ 1(X)} . \qquad (12)$$

If we use the same values of the parameters as those used to calculate $<\beta^2>_b$ by expression (8), then we obtain that $K_1^2(0) = 0.793$. For the altitude X of the laser-induced reference star at 90 to 100 km, we obtain from the table that the value $2(X) = 6$. In all these calculations we took the size of the basic telescope to be $R_0 = 8$ m and that of the laser lighter (the auxiliary telescope) $a_0 = 1$ m, that means that the parameter $b = 1/8$, and the apparent size of the reference star $a_{\log s}$ was taken to be 1 km. Thus, for the scheme a the level of the residual angular star image jitters is

$$<\beta^2>_a \approx 0.58 .$$

Thus, the scheme b provides for a lower level of the residual distortions as compared to those achievable with the scheme a that makes this scheme preferable. At the same time it should be noted that the scheme a is technically more easy to perform than the scheme b. The point is that the scheme b assumes, in addition to the basic telescope, the use of an auxiliary (two) small telescope, whereas the scheme a needs only for an auxiliary laser lighter. It is understandable that the laser sources are the less expensive instruments then auxiliary telescopes.

So, we have shown that no symmetry occurs between the two new schemes proposed in Refs. 4, 6, 7 and 8, 9, 10, for the formation of the extended laser-induced guide stars. These two types of the formation schemes of the off-axial laser-induced guide stars were intended for creating a sufficiently extended guide star instead of the point one. However, as estimates show,[22] it has been impossible to obtain an essential decrease in the contribution of angular fluctuations to the guide star jitters when averaging over an extended guide source, both in the schemes with a auxiliary telescopes[8,9] and in the schemes with the use of auxiliary laser lighters.[4,6,7]

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REFERENCES

1. S. Esposito and R. Ragazzoni, Adapt. Opt. At the Telescop. Nazionale Galileo, No. 12, 1–20 (1997).
2. R. Ragazzoni and S. Esposito, Adapt. Opt. at the Telescop. Nazionale Galileo, No. 12, (1997).
3. R. Ragazzoni, Astron. Astrophys. **319**, L9–L12 (1997).
4. R. Ragazzoni, Adapt. Opt. at the Telescop. Nazionale Galileo, No. 12, (1997).
5. R. Foy, A. Migus, F. Biraben, G. Grynberg, P.R. McCullough and M. Tallon, Astron. Astrophys. Suppl. **111**, 569–578 (1995).
6. R. Ragazzoni, S. Esposito, and E. Marchetti, Mon. Not. R. Astron. Soc. **276**, L76–L78 (1995).
7. S. Esposito, R. Ragazzoni, and A. Riccardi, Adaptive Optics in the Telescop. Nazionale Galileo, No. 12, (1997).
8. M.S. Belen'kii, Proc. SPIE **2471**, 289–296 (1995).
9. M.S. Belen'kii, Proc. SPIE **2956**, 206–217 (1996).
10. R. Ragazzoni, Astron. Astrophys. **305**, L13–L16 (1996).

11. V.P. Lukin and V.F. Matyukhin, Kvant. Elektron. **10**, 2465–2473 (1983).
12. V.P. Lukin, *Atmospheric Adaptive Optics* (Nauka, Novosibirsk, 1986), 286 pp.
13. V.P. Lukin, in: *Proceedings of ICO–16 on Active and Adaptive Optics,* (1993), p. 521–524.
14. V.P. Lukin and B.V. Fortes, OSA Techn. Digest **23**, 192–194 (1995).
15. V.P. Lukin, Adaptive Optics, Techn. Digest Series **13**, 35-1–35-5 (1996).
16. V.P. Lukin, Atmos. Oceanic Opt. **8**, 152–173 (1995).
17. V.P. Lukin and B.V. Fortes, Atmos. Oceanic Opt. **10**, 34–41 (1997).
18. V.P. Lukin, Atmos. Oceanic Opt. **10**, 609–612 (1997).
19. V.P. Lukin, Appl. Opt. (1998).
20. M.A. Kalistratova and A.I. Kon, Izv. Vyssh. Uchebn. Zaved., Ser. Radiofizika **9**, No. 6, 1100–1107 (1966).
21. M.A. Gracheva and A.S. Gurvich, Izv. Akad. Nauk SSSR, Fizika Atmos. Okeana **16**, 1107–1111 (1980).
22. V.P. Lukin, Atmos. Oceanic Opt. **11**, No. 5, 402–412 (1998).