# Optical model of a plate crystal as applied to bistatic polarization laser sensing of crystal clouds 

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An optical model of a particle is developed as applied to bistatic laser sensing of crystal clouds. A round water ice plate is considered as a scatterer. Within the framework of physical optics, the equations are derived for cross sections of scattering into the backward scattering hemisphere taking into account polarization state of incident radiation.

## Introduction

At present lidar sensing methods are being widely used in the studies of crystal clouds. As a rule, as a linearly polarized optical radiation interacts with nonspherical particles, the cross-polarized component appears in the reflected signal. So, the use of polarization lidars along with non-polarization ones yields more informative results.

The polarization state of scattered radiation can differ from that of the incident radiation due to variations in microphysical, optical, and orientation properties of the scatterers. As known, water ice crystals in clouds are characterized by a wide variety of habits and size. Up to now, because of difficulties in numerical simulation of the process of light scattering by oriented particles, only some particular models have been developed which represent only that or another optical phenomenon observed in crystal clouds. The domain of their applicability is mostly limited to only few problems.

Remote sensing with a bistatic polarization scanning lidar shows good promises for studying the upper and middle atmosphere. Researchers note great advantages of this method over the traditional monostatic one. ${ }^{1,2}$

It should be noted that, in spite of the advent of bistatic lidars, there are no satisfactory theoretical grounds developed for interpreting the data of sensing crystal clouds. So the theories applied to monostatic laser sensing are mainly used for this purpose.

This paper describes a numerical model of a scatterer as an oriented plate crystal as applied to the bistatic sensing scheme.

## Experimental optical arrangement of a bistatic sensing

Let us imagine the optical arrangement of bistatic polarization laser sensing of crystal clouds to be as follows. Let a source of radiation be at the point $n_{1}$, the receiver be at the point $n_{2}$, and the object under study be at the point $n_{3}$. First let us set the absolute
coordinate system $n x y z$, and then introduce three coordinate systems more: $n_{1} x_{1} y_{1} z_{1}, n_{2} x_{2} y_{2} z_{2}$, and $n_{3} x_{3} y_{3} z_{3}$ in this absolute system. These three coordinate systems are related to the source, receiver, and scatterer, respectively. The coordinate plane $n x y$ is parallel to the ground, and the normal to it is directed along the $O z$ axis. The sensing radiation propagates along the positive direction of the $O_{1} z_{1}$ axis. Electric components of the incident plane wave having the elliptical polarization ( $\mathbf{E}_{1}, \mathbf{E}_{2}$ ) are directed along the $O_{1} x_{1}$ and $O_{1} y_{1}$ axes. The coordinate plane $n_{3} x_{3} y_{3}$ is a plane of the preferred orientation of particles in the ensemble, and the $O_{3} z_{3}$ axis is normal to it. The scattered radiation is received along the direction of the $\mathrm{O}_{2} z_{2}$ axis, and the $\mathrm{O}_{2} y_{2}$ axis is parallel to the horizontal plane.

To further define the normalized characteristics of light scattering, it is sufficient to define the angular position of the unit vectors corresponding to the components of the scattered field. Therefore, at this stage, set the origins of all the four coordinate systems at the same point $n$ and define the angular dependence of the unit vectors ( $\mathbf{x}, \mathbf{y}, \mathbf{z}$ ) of the absolute coordinate system with respect to the corresponding unit vectors ( $\mathbf{x}_{i}, \mathbf{y}_{i}, \mathbf{z}_{i}$ ) of each of the rest three systems. Then the following equation is valid:

$$
\left(\begin{array}{c}
\mathbf{x}  \tag{1}\\
\mathbf{y} \\
\mathbf{z}
\end{array}\right)=S_{i}\left(\begin{array}{c}
\mathbf{x}_{i} \\
\mathbf{y}_{i} \\
\mathbf{z}_{i}
\end{array}\right), i=1,2,3
$$

where

$$
S_{i}=\left(\begin{array}{ccc}
\cos \varphi_{i} \cos \vartheta_{i} & -\sin \varphi_{i} & \cos \varphi_{i} \sin \vartheta_{i} \\
\sin \varphi_{i} \cos \vartheta_{i} & \cos \varphi_{i} & \sin \varphi_{i} \sin \vartheta_{i} \\
-\sin \vartheta_{i} & 0 & \cos \vartheta_{i}
\end{array}\right) .
$$

The angles $\vartheta_{i}$ and $\varphi_{i}$ obviously determine the position of the basis vectors $\mathbf{x}_{i}, \mathbf{y}_{i}, \mathbf{z}_{i}(i=1,2,3)$ of each of the three corresponding coordinate systems $n x_{i} y_{i} z_{i}(i=1,2,3)$ relative to the absolute coordinate system $n x y z$.

## Statement of the problem

As known, plate crystals of water ice in the atmosphere have, as a rule, a hexagonal shape. When considering a system of oriented hexagonal plates, it is needed to average the light scattering characteristics of an individual particle over a set of orientations determined by rotation of a plate around its axis. It is clear that averaging significantly smooths out peculiarities due to the fine geometrical features in the scattering characteristics. Therefore, because calculation of the characteristics of light scattering by a single crystal is followed by transition to integral characteristics of a polydisperse medium, it is worth simplifying first the geometry of a particle. In this connection, the round plate is the best geometrical approximation of an individual particle of a polydisperse ensemble consisting of plate crystals. ${ }^{3}$

Let us consider a disc-shaped particle with the complex refractive index $\tilde{n}=n+i \chi$, radius $a$, and thickness $d$ (see Fig. 1) as an object when studying light scattering by a single crystal.


Fig. 1. Geometry of formation of refracted beams.
According to the laws of crystal growth, the following dependence between the diameter and thickness of a plate is valid ${ }^{4}: d=L(2 a)^{\mu}$, where $L=2.020$ and $\mu=0.449$. Let a plane polarized wave ( $\mathbf{E}_{1}, \mathbf{E}_{2}$ ) be incident on a particle at an angle $\beta$ to the normal to the plate base. The angle $\gamma$ sets orientation of the polarization plane relative to the incidence plane. The rays reflected from the plate base or coming out of it after several internal reflections form the beams in the direction $\pi-\beta$ relative to the normal. The major part of the energy of the field scattered in the backward hemisphere is obviously concentrated about the direction of specular reflection. When talking about the space containing scattered power, we shall call as the backward hemisphere the part of the sphere, which is bounded by the plate base and contains the incident and reflected rays. Note that the area of the side surface
of a plate is much less than the area of the projection of the base in the direction of reflection. Besides, as the wave interacts with the side surface of a scatterer, the major part of energy of the electromagnetic field is concentrated within the forward hemisphere. It should also be noted that according to the earlier reported calculations, ${ }^{3,5}$ the internal reflections can be neglected when estimating the radiation scattered into the backward hemisphere.

For thus described particle, let us now determine the light scattering characteristics which are important for the bistatic laser sensing. Namely, for the case of an arbitrary direction from which a polarized signal in the backward hemisphere is received, consider the set of scattering cross sections $\sigma_{\pi_{i}}$, each proportional to the corresponding Stokes vector $I_{\pi_{i}}$, that is,

$$
\begin{equation*}
\sigma_{\pi_{i}}=\omega I_{\pi_{i}} \quad(I=1,2,3,4) \tag{2}
\end{equation*}
$$

Note that the whole set of cross sections can be considered as the scattering cross sections for the polarized signal, only keeping in mind that in the classical understanding of this term only one of them, $\sigma_{\pi_{i}}$, has this meaning. We omit here the detailed presentation of the coefficient of proportionality $\omega$, because for all four equations (2) it is the same. Besides, the light scattering characteristics considered below do not depend on it. In derivation of the sought equations for $\sigma_{\pi_{i}}$, we use the following scheme. First, we determine the radiation field reflected in the direction $\pi-\beta$ (see Fig. 1) and then the components of the field scattered in the direction of reception. The next step is calculation of the Stokes vector parameters from the complex amplitude of this field followed by reduction of the Stokes vector parameters to the scattering cross sections.

## Scattered field in the backward hemisphere

Let us set the electric and magnetic components of the field of an incident plane wave and the direction of its propagation in the coordinate system $n x_{1} y_{1} z_{1}$ : $\mathbf{E}=\mathbf{x}_{1} E_{1}+\mathbf{y}_{2} E_{2}, \mathbf{H}=\mathbf{x}_{1} H_{2}+\mathbf{y}_{1} H_{1}, \mathbf{k}=k \mathbf{z}_{1}, \mathbf{k}$ is the wave vector, $H_{1}=E_{1} / z_{1}, H_{2}=E_{2} / z_{1}$, where $z_{1}$ is the resistance of the free space. Because the complex amplitudes $\left(H_{1}, H_{2}\right)$ and $\left(E_{1}, E_{2}\right)$ of the magnetic and electric components of the incident field are related to each other, equations for the magnetic components are omitted in this paper.

Determine the position of the components of the incident $\left(\mathbf{E}_{1}, \mathbf{E}_{2}\right)$ field in the system of coordinates related to the scatterer. The unit vectors of the coordinate system $n x_{1} y_{1} z_{1}$ and $n x_{3} y_{3} z_{3}$ given by Eq. (1) can be presented by the following linear transformation:

$$
\left(\begin{array}{l}
\mathbf{x}_{3}  \tag{3}\\
\mathbf{y}_{3} \\
\mathbf{z}_{3}
\end{array}\right)=A\left(\begin{array}{l}
\mathbf{x}_{1} \\
\mathbf{y}_{1} \\
\mathbf{z}_{1}
\end{array}\right)
$$

where

$$
A=\left(\begin{array}{ccc}
\cos \alpha \cos \beta \cos \gamma-\sin \alpha \sin \gamma & -\cos \alpha \cos \beta \sin \gamma-\sin \alpha \cos \gamma & \cos \alpha \sin \beta  \tag{3}\\
\sin \alpha \cos \beta \cos \gamma+\cos \alpha \sin \gamma & -\sin \alpha \cos \beta \sin \gamma+\cos \alpha \cos \gamma & \sin \alpha \sin \beta \\
-\sin \beta \cos \gamma & \sin \beta \sin \gamma & \cos \beta
\end{array}\right)
$$

$\alpha, \beta$, and $\gamma$ are Euler angles. Based on relations (1) and (3) for the coordinate systems defined above, the matrix $A$ can be written as $A=S_{3}^{-1} S_{1}$. Then Euler angles can be calculated as some combinations of the angles $\vartheta_{1}, \varphi_{1}$ and $\vartheta_{3}, \varphi_{3}$. Introduce one more matrix $B=S_{3}^{-1} S_{2}$, which establishes the following relation:

$$
\left(\begin{array}{l}
\mathbf{x}_{3}  \tag{4}\\
\mathbf{y}_{3} \\
\mathbf{z}_{3}
\end{array}\right)=B\left(\begin{array}{l}
\mathbf{x}_{2} \\
\mathbf{y}_{2} \\
\mathbf{z}_{2}
\end{array}\right) .
$$

The matrices $A$ and $b$ will allow us to obtain, in the below discussion, the components of scattered field in the coordinate system related to the plate.

Elements of the matrix $A$ determine the positions of the vectors $\mathbf{k}, \mathbf{E}_{1}$, and $\mathbf{E}_{2}$ in the coordinate system $n x_{3} y_{3} z_{3}$. It should be noted that at an arbitrary angle of orientation of the plane of polarization $\gamma$ the components $\mathbf{E}_{1}$ or $\mathbf{E}_{2}$ do not lie in the plane of wave incidence. Therefore, for further calculations of the scattering characteristics and application of Fresnel formulas to them, the components $\mathbf{E}_{1}$ and $\mathbf{E}_{2}$ should be transformed so that one of them is normal to the plane of incidence, while the other one being in this plane. To do this, let us use the following linear transformation:

$$
\left(\begin{array}{c}
\mathbf{E}_{\|} \\
\mathbf{E}_{\perp} \\
\mathbf{k}
\end{array}\right)=F\left(\begin{array}{c}
\mathbf{E}_{1} \\
\mathbf{E}_{2} \\
\mathbf{k}
\end{array}\right),
$$

where

$$
F=\left(\begin{array}{ccc}
-\cos \gamma & \sin \gamma & 0 \\
\sin \gamma & \cos \gamma & 0 \\
0 & 0 & 1
\end{array}\right) .
$$

The vectors $\mathbf{E}_{\perp}$ and $\mathbf{E}_{\|}$in the coordinate system related to the plate depend on the elements of the first and second columns of the matrix $A_{F}=A F$.

Upon reflection from the plate, the amplitudes of the components of the electric field can be found as $E_{\perp}^{R}=E_{\perp} R_{\perp}$ and $E_{\|}^{R}=E_{\|} R_{\|}$. Here $R_{\perp}, R_{\|}$are Fresnel reflection coefficients having the following form:

$$
\begin{gather*}
R_{\|}=\frac{\tilde{n} \cos \beta-\cos \tilde{\vartheta}}{\tilde{n} \cos \beta+\cos \tilde{\vartheta}}, \quad R_{\perp}=\frac{\cos \beta-\tilde{n} \cos \tilde{\vartheta}}{\cos \beta+\tilde{n} \cos \tilde{\vartheta}} ; \\
\sin \tilde{\vartheta}=\frac{\sin \beta}{n+i \chi} . \tag{5}
\end{gather*}
$$

Let $\mathbf{k}_{R}$ be the direction of propagation of the reflected beam. The position of each of the vectors $\mathbf{E}_{\|}^{R}$,
$\mathbf{E}_{\perp}^{R}$, and $\mathbf{k}_{R}$ in the coordinate system related to the plate is determined by the elements of the corresponding columns of the matrix $A_{R}$ :

$$
A_{R}=A_{F} R
$$

where

$$
R=\left(\begin{array}{ccc}
-\cos 2 \beta & 0 & -\sin 2 \beta  \tag{6}\\
0 & 1 & 0 \\
\sin 2 \beta & 0 & -\cos 2 \beta
\end{array}\right)
$$

Now determine the angles defining the position of $\mathbf{z}_{2}$ relative to the vectors $\mathbf{E}_{\|}^{R}, \mathbf{E}_{\perp}^{R}$, and $\mathbf{k}_{R}$. If $B_{i j}$ and $A_{R_{i j}}$ are the matrix elements described by Eqs. (4) and (6), then the projection of the vector $\mathbf{z}_{2}$ onto the corresponding directions is determined as

$$
\begin{gathered}
\cos \psi_{x}=A_{R_{11}} B_{13}+A_{R_{21}} B_{23}+A_{R_{31}} B_{33} \\
\cos \psi_{y}= \\
A_{R_{12}} B_{13}+A_{R_{22}} B_{23}+A_{R_{32}} B_{33} \\
\cos \psi_{z}= \\
A_{R_{13}} B_{13}+A_{R_{23}} B_{23}+A_{R_{33}} B_{33} \\
\cos \psi_{x}=\sin \vartheta \cos \varphi \\
\cos \psi_{y}=\sin \vartheta \sin \varphi \\
\cos \psi_{z}=\cos \vartheta
\end{gathered}
$$

Since $\quad \cos \vartheta=\cos \psi_{x} \quad$ and $\quad \sin \vartheta=\sqrt{1-\cos ^{2} \vartheta}$, $\cos \varphi=\cos \psi_{x} / \sin \vartheta$ and $\sin \varphi=\cos \psi_{y} / \sin \vartheta$. The angles $\vartheta$ and $\varphi$ are counted from the direction $\pi-\beta$. Let us introduce a new coordinate system $O x_{s} y_{s} z_{s}$ related to $O x_{3} y_{3} z_{3}$ :

$$
\begin{equation*}
A_{T}=A_{R} S P \tag{7}
\end{equation*}
$$

where

$$
\begin{gathered}
S=\left(\begin{array}{ccc}
\cos \varphi \cos \vartheta & -\sin \varphi & \cos \varphi \sin \vartheta \\
\sin \varphi \cos \vartheta & \cos \varphi & \sin \varphi \sin \vartheta \\
-\sin \vartheta & 0 & \cos \vartheta
\end{array}\right) \\
P=\left(\begin{array}{ccc}
\cos \varphi & \sin \varphi & 0 \\
-\sin \varphi & \cos \varphi & 0 \\
0 & 0 & 1
\end{array}\right)
\end{gathered}
$$

The electric components of a scattered field are determined by the elements of the first and second columns of the matrix $A_{T}$ given by Eq. (7). Moreover, $\mathbf{E}_{R_{1}}$ and $\mathbf{E}_{R_{2}}$ lie in the plane normal to the given direction of reception $\mathrm{O}_{2}$.

For the amplitudes of the scattered electric field we have

$$
E_{R_{1}}=\frac{\mathrm{e}^{i k r}}{i k r} C_{1}, E_{R_{2}}=\frac{\mathrm{e}^{i k r}}{i k r} C_{2}
$$

$$
\begin{gather*}
C_{1}=\frac{k^{2}}{4 \pi}(1+\cos \vartheta) E_{\|} R_{| |} G_{0}(\vartheta, \varphi) \mathrm{e}^{i \psi_{0}} ;  \tag{8}\\
C_{2}=\frac{k^{2}}{4 \pi}(1+\cos \vartheta) E_{\perp} R_{\perp} G_{0}(\vartheta, \varphi) \mathrm{e}^{i \psi_{0}} ; \\
G_{0}(\vartheta, \varphi)=\pi a^{2} \cos \beta \frac{J_{1}(k a V)}{k a V} ; \\
V=\sin \vartheta \sqrt{\sin ^{2} \varphi+\cos ^{2} \beta \cos ^{2} \varphi} ;
\end{gather*}
$$

$J_{1}$ is the Bessel function of the first order; $k=2 \pi / \lambda$ is the wave number, $\lambda$ is the wavelength. The parameters $\mathbf{E}_{I}$ and $\mathbf{E}_{\text {II }}$ are the components of the field recorded with a receiver. Find now the linear transformation which relates ( $\mathbf{E}_{R_{1}}, \mathbf{E}_{R_{2}}$ ) to ( $\mathbf{E}_{\mathrm{I}}, \mathbf{E}_{\mathrm{II}}$ ). In other words, the unit vectors $\mathbf{E}_{R_{1}}$ and $\mathbf{E}_{R_{2}}$ should be rotated around $\mathbf{z}_{2}$ until full coincidence with $\mathbf{E}_{\mathrm{I}}$ and $\mathbf{E}_{\mathrm{II}}$. Here $A_{R_{i j}}$ are the elements of the matrix (7):

$$
\begin{aligned}
& \cos u=A_{T_{12}} B_{12}+A_{T_{22}} B_{22}+A_{T_{32}} B_{32}, \\
& \sin u=A_{T_{11}} B_{12}+A_{T_{21}} B_{22}+A_{T_{31}} B_{32} .
\end{aligned}
$$

Then we have

$$
\begin{align*}
& E_{\mathrm{I}}=-\cos u E_{R_{1}}+\sin u E_{R_{2}}, \\
& E_{\mathrm{II}}=\sin u E_{R_{1}}+\cos u E_{R_{2}}, \tag{9}
\end{align*}
$$

taking into account that

$$
\begin{aligned}
& E_{\mathrm{I}}=A_{D}\left(a_{11} E_{1}+a_{12} E_{2}\right), \\
& E_{\mathrm{II}}=A_{D}\left(a_{21} E_{1}+a_{22} E_{2}\right),
\end{aligned}
$$

where

$$
\begin{aligned}
& A_{D}=\frac{\mathrm{e}^{i k r}}{i k r} \frac{k^{2}}{4 \pi}(1+\cos \vartheta) G_{0}(\vartheta, \varphi) ; \\
& a_{11}=R_{\|} \cos u \cos \gamma+R_{\perp} \sin u \sin \gamma, \\
& a_{12}=-R_{\|} \cos u \sin \gamma+R_{\perp} \sin u \cos \gamma, \\
& a_{21}=-R_{\|} \sin u \cos \gamma+R_{\perp} \cos u \sin \gamma, \\
& a_{22}=R_{\|} \sin u \sin \gamma+R_{\perp} \cos u \cos \gamma .
\end{aligned}
$$

## Scattering cross sections

The scattering cross sections $\sigma_{\pi_{j}}$ determined in the direction of radiation reception are related to the corresponding parameters of Stokes vector $I_{\pi_{j}}$ as follows:

$$
\begin{equation*}
\sigma_{\pi_{j}}=4 \pi r_{2} I_{\pi_{j}} / I_{1}, \quad j=1,2,3,4 \tag{10}
\end{equation*}
$$

where $I_{1}$ is the intensity of electromagnetic field of the incident wave. The parameters of Stokes vector can be expressed in terms of the amplitudes of the incident field

$$
\begin{aligned}
& I_{1}=\left|E_{1}\right|^{2}+\left|E_{2}\right|^{2}, \\
& I_{2}=\left|E_{1}\right|^{2}-\left|E_{2}\right|^{2},
\end{aligned}
$$

$$
\begin{align*}
& I_{3}=2 \operatorname{Re}\left(E_{1} E_{2}^{*}\right),  \tag{11}\\
& I_{4}=2 \operatorname{Im}\left(E_{1} E_{2}^{*}\right) .
\end{align*}
$$

The equations for determination of the Stokes vector parameters of the scattered radiation are obviously the same. Taking into account Eqs. (8)-(11) and making necessary algebraic operations, we derive the following equations for the scattering cross sections:

$$
\begin{gathered}
\sigma_{\pi_{1}}=W\left\{M_{11}+\frac{I_{2}}{I_{1}} M_{12}+\frac{I_{3}}{I_{1}} M_{13}+\frac{I_{4}}{I_{1}} M_{14}\right\}, \\
\sigma_{\pi_{2}}=W\left\{M_{21}+\frac{I_{2}}{I_{1}} M_{22}+\frac{I_{3}}{I_{1}} M_{23}+\frac{I_{4}}{I_{1}} M_{24}\right\}, \\
\sigma_{\pi_{3}}=W\left\{M_{31}+\frac{I_{2}}{I_{1}} M_{32}+\frac{I_{3}}{I_{1}} M_{33}+\frac{I_{4}}{I_{1}} M_{34}\right\}, \\
\sigma_{\pi_{4}}=W\left\{M_{41}+\frac{I_{2}}{I_{1}} M_{42}+\frac{I_{3}}{I_{1}} M_{43}+\frac{I_{4}}{I_{1}} M_{44}\right\} ; \\
W=\frac{k^{2}}{\pi}\left(\frac{1+\cos \vartheta}{2}\right)^{2} G_{0}^{2}(\vartheta, \varphi)
\end{gathered}
$$

where $M_{i j}$ are the elements of Mueller matrix.
If we denote

$$
\begin{aligned}
& \frac{\left|R_{\|}\right|^{2}+\left|R_{\perp}\right|^{2}}{2}=f_{1}, \frac{\left|R_{\|}\right|^{2}-\left|R_{\perp}\right|^{2}}{2}=f_{2} \\
& \operatorname{Re}\left(R_{\|} R_{\perp}^{*}\right)=g_{1} ; \operatorname{Im}\left(R_{\|} R_{\perp}^{*}\right)=g_{2}
\end{aligned}
$$

$\sin 2 \gamma=s_{1}, \sin 2 y=s_{2} ; \cos 2 \gamma=c_{1}, \cos 2 y=c_{2}$,
then

$$
\begin{align*}
& M_{11}=f_{1}, \quad M_{12}=f_{2} c_{1}, M_{13}=-f_{2} s_{1}, M_{14}=0, \\
& M_{21}=f_{2} c_{2}, \quad M_{22}=f_{1} c_{1} c_{2}+g_{1} s_{1} s_{2}, \\
& M_{24}=-g_{2} s_{2}, \quad M_{23}=-f_{1} s_{1} c_{2}+g_{1} c_{1} s_{2}, \\
& M_{31}=-f_{2} s_{2}, \quad M_{32}=-f_{1} c_{1} s_{2}+g_{1} s_{1} c_{2},  \tag{13}\\
& M_{34}=-g_{2} c_{2}, \quad M_{33}=f_{1} s_{1} s_{2}+g_{1} c_{1} c_{2}, \\
& M_{44}=g_{1}, \quad M_{43}=g_{2} c_{1}, \quad M_{42}=g_{2} s_{1}, \quad M_{41}=0 .
\end{align*}
$$

## Specular reflection

Consider a particular case where the direction of reflection coincides with the direction of reception. To simulate this situation with the known position of the source and receiver, we may determine the angles $\vartheta_{3}$ and $\varphi_{3}$ which characterize the position of a plate in the space relative to the absolute coordinate system, provided that $\vartheta=0$. Having set the pairs of angles $\left(\vartheta_{1}, \varphi_{1}\right)$ and $\left(\vartheta_{2}, \varphi_{2}\right)$, determine the values of $\vartheta_{3}$ and $\varphi_{3}$ corresponding to the case of specular reflection.

Since the matrix $A_{R}$ given by Eq. (6) determines the directions of reflection and of the electric components of the reflected field, and the elements of the third column of the matrix $B$ given by Eq. (4)
determine the direction, relative to the plate, of radiation coming to the receiver, we have the following system (taking into account the conditions of specular reflection):

$$
\begin{align*}
& A_{R_{11}} B_{13}+A_{R_{21}} B_{23}+A_{R_{31}} B_{33}=0, \\
& A_{R_{12}} B_{13}+A_{R_{22}} B_{23}+A_{R_{32}} B_{33}=1 . \tag{14}
\end{align*}
$$

Recall that the elements of the matrices $A_{R}$ and $B$ depend on the angles $\vartheta_{i}$ and $\varphi_{i}(i=1,2,3)$. Then, upon solution of the system (14), we have

$$
\begin{gather*}
\operatorname{tg} \varphi_{3}=\frac{\sin \vartheta_{2} \sin \varphi_{2}-\sin \vartheta_{1} \sin \varphi_{1}}{\sin \vartheta_{2} \cos \varphi_{2}-\sin \vartheta_{1} \cos \varphi_{1}}  \tag{15}\\
\operatorname{tg} \vartheta_{3}=\frac{-\left(\cos \vartheta_{2}+\cos \vartheta_{1}\right)}{\sin \vartheta_{2} \cos \left(\varphi_{3}-\varphi_{2}\right)+\sin \vartheta_{1} \cos \left(\varphi_{3}-\varphi_{1}\right)}
\end{gather*}
$$

Note that the components of the scattered field $\mathbf{E}_{R_{1}}$ and $\mathbf{E}_{R_{2}}$ in the case of reception of specularly reflected radiation are in the same plane as those normal to the direction of reflection (reception).

As known, for non-spherical particles all 16 elements of Mueller matrix may be nonzero. Analyzing Eqs. (12) for the scattering cross sections, it is clear that in the case of specular reflection two elements of 16 are zero (as in the case of arbitrary directions of incidence and scattering). In the case of monostatic sensing, and specular reflection from plates, only diagonal elements of the matrix are nonzero; besides, their absolute values are equal to each other. ${ }^{3}$ Then it becomes clear that the scattering phase matrix we have in bistatic sensing bears more information on the properties of oriented plates as compared with the monostatic sensing.

## Conclusion

To identify a disperse medium and determine its basic parameters from the data of bistatic polarization laser sensing of crystal clouds, the numerical model of
a scatterer is developed. A water ice particle having the shape of a round plate is taken as an object for the study. Within the framework of the physical optics, the equations have been derived for polarization characteristics and scattering cross sections of radiation in the backward hemisphere. The equations are some combinations of the elements of the scattering phase matrix. The equations obtained allow the abovementioned scattering characteristics to be studied numerically depending on the particle size, orientation, and the refractive index for arbitrary scattering angles in the optical wavelength region.

The equations obtained for the elements of scattering phase matrix illustrate the change in the state of polarization of specularly reflected radiation at varying optical and orientation properties of a particle. In this connection, one should expect that the specularly reflected signal bears more information if the bistatic sensing scheme is used along with the parameters of anomalous backscattering recorded in a monostatic sensing optical arrangement.

## Acknowledgments

The author is indebted to Professor, Corresponding Member of the Academy of Sciences of Higher School I.V. Samokhvalov and Professor A.A. Popov for valuable and very helpful criticism and advice.

Support from the Russian Foundation of Basic Research (Project No. 98023031) is also gratefully acknowledged.

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