Mathematical simulation of a lidar under severe noise conditions

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Received June 15, 1998

We have developed a mathematical model of a lidar system for remote study of the near-surface atmospheric layer. The model adequately describes the process of measurements in an actual noise situation and allows correction of a received lidar signal for background to be made.

Introduction

The paper is devoted to mathematical simulation of a lidar system intended for remote investigation of the near-surface atmospheric layer and determination, in particular, of the range profiles of the volume extinction coefficient $\alpha(R)$. If the sounding depth is large, the level of received signals may prove very low in comparison with the intense signal due to background noise.[1–4] The existing methods of noise control, such as compensation for the background, or the methods of amplitude and time adjustment to prevent the detector overload at high-intensity noise, do not provide for a significant effect.[1,5,6]

One of the most important sources of errors in lidar measurements in the visible and near IR spectral regions is the influence of high-intensity background on a photodetector. This influence distorts the characteristics of the photodetector and directly affects the reliability of reconstructed optical profiles. This is especially true for PMTs. The approach we have developed allows the changes in the amplitude characteristics of photodetector due to high-intensity background to be taken into account. For this reason this approach has an advantage over the above-mentioned methods. We have studied the changes in the characteristics of photodetectors under the action of a high-intensity background mostly from three sources: the sun, the solar radiation scattered in the atmosphere when propagating through it, and the radiation reflected from the Earth's surface. The developed model of a PMT-based lidar receiver takes into account the background changes in the slope of the response characteristic of a photodetector depending on the state of the atmosphere, its transmittance, spectral brightness of the sky background, mutual angular position, and others (Fig. 1).

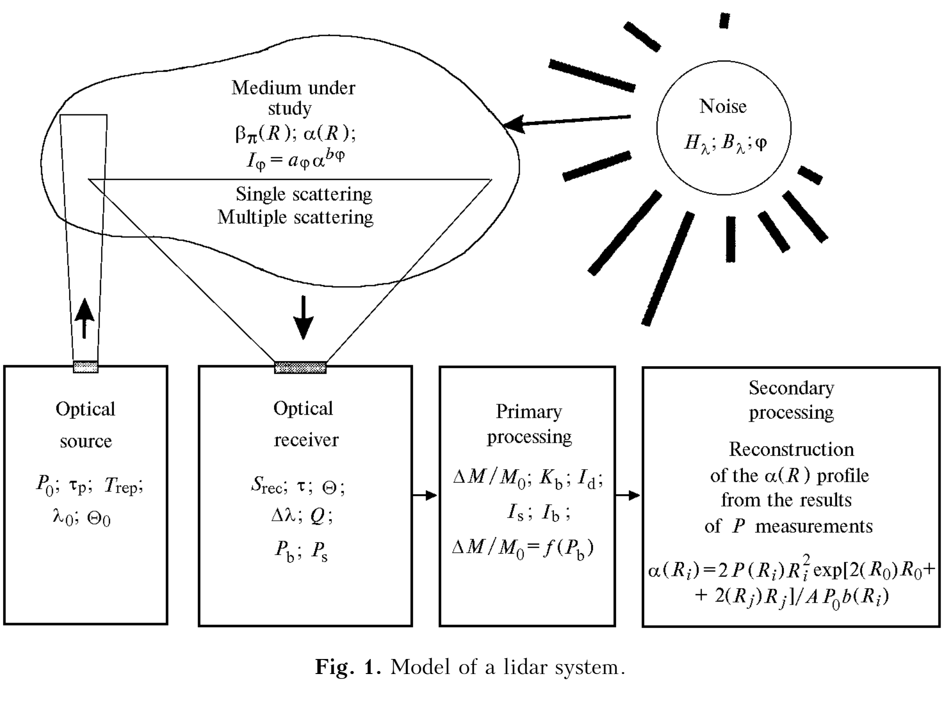

Fig. 1. Model of a lidar system.

0235-6880/99/07 610-06 $02.00

As a result of numerical simulation of the photodetector subject to the influence of background, the value of the change in the PMT gain coefficient is determined. Then this value is used to correct the level of the received signal and the background in order to increase the reliability of measurements and then to determine the profiles of the atmospheric extinction coefficient.

Simulation of a transceiver, medium, and background noise

The main parameters characterizing the transmitter are the emitted power (P_0), radiation wavelength (λ_0), angular width of the sounding beam (Θ_0), duration of the sounding pulse (τ_p), and pulse repetition period T_{rep}.

The return signals of a monostatic lidar are described by the lidar equation, which has the following form in the single-scattering approximation:

$$P(R) = AP_0\, \beta_\pi(R)\, R^{-2} \exp\left[-2\int\limits_0^R \alpha(R')\, dR'\right], \quad (1)$$

where P is the power of received signal; P_0 is the power of the emitted sounding pulse; A is the instrumental function; β_π is the volume backscattering coefficient; α is the volume extinction coefficient; R' is the current range.

It is often accepted that the background noise is described by the following equation[5]:

$$P_b = (\pi/4)\, B_\lambda\, \Theta^2\, S_{rec}\, T\tau, \quad (2)$$

where P_b is the power of the background radiation; B_λ is the spectral brightness of the sky background; Θ is the plane angle; S_{rec} is the area of the receiving optical system; T is the coefficient of signal extinction in the atmosphere; τ is the coefficient of signal extinction in the optical system.

Let us demonstrate now how parameters of the medium influence the intensity of the background noise. The scattering phase functions for different states of the atmosphere are measured and classified, in particular, by the coefficient of asymmetry of the scattered light flux. The analysis of the scattering phase functions[4] has shown that the correlation between the scattering phase function i_φ and the volume extinction coefficient α can be approximated by the functional dependence of the form

$$i_\varphi = a_\varphi\, \alpha^{b_\varphi}, \quad (3)$$

where a_φ and b_φ are the coefficients of approximation; φ is the angle between the lidar optical axis and the direction to the Sun; at $\varphi < 50°$ the coefficient $b_\varphi > 0$, and at $\varphi > 60°$ $b_\varphi < 0$.

The intensity of the background noise P_b is usually estimated at the known parameters of the receiver on some assumption on the value of the background illumination of a horizontal surface: $H_\lambda = B_\lambda/i_\varphi$, where the maximum value of B_λ varies from 1 to 300 W/(m^2·sr·μm) (Ref. 2).

Let us estimate the level of background radiation scattered by the atmosphere, which falls within the field of view of the receiver. On the assumption that in the optical wavelength region the volume extinction coefficient is equal to the volume scattering coefficient, the total power of the background radiation from the whole path from 0 to ∞ is equal to[2]:

$$P_b = H_\lambda Q \int\limits_0^\infty i_\varphi(R)\, \alpha(R) \exp\left(-\int\limits_0^R \alpha(R')\, dR'\right) dR, \quad (4)$$

where $Q = \Omega\Delta\lambda S_{rec}\tau$ depends on the parameters of the lidar receiving system; $\Omega = (\pi/4)\Theta^2$, Θ is the plane angle, which serves as a measure of the solid angle Ω for the conic field of view. For the actual values: $\Theta = 10^{-4}-10^{-3}$ rad, pass band of the optical filter $\Delta\lambda = 1-10$ nm, area of the receiving optical system $S_{rec} \approx 10^{-2}$ m^2, transmittance of the receiving optics $\tau \approx 0.5$, we have $Q = 3\cdot 10^{-20}-4\cdot 10^{-17}$ m^2·sr·μm.

With the allowance for approximation (3), we have the following estimate of the received background power:

$$P_b = H_\lambda\, Qa_\varphi \int\limits_0^\infty \alpha^{b_\varphi+1}(R) \exp\left(-\int\limits_0^R \alpha(R')\, dR'\right) dR. \quad (5)$$

According to Eq. (5), the calculated values were scaled to the maximum value. Then they were used for drawing the plot of the scaled power of the background illumination versus the optical density (Fig. 2).

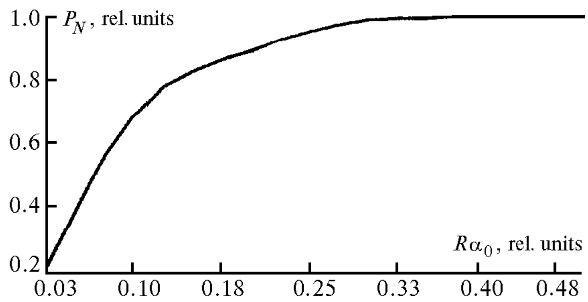

Fig. 2. Scaled power of background illumination vs. the optical density.

As seen from Fig. 2, the characteristic of the scaled power of background illumination is limited from above. This limit corresponds to the maximum value of the background power P_b at the photodetector input with the given profile $\alpha(R)$ and background illumination of the horizontal surface H_λ.

Simulation of the influence of background noise on the amplitude characteristics of a photodetector

According to several criteria, the PMT-based photodetectors have advantages over other photodetectors in the visible and near IR regions. The action of high-intensity noise on such a photodetector manifests itself, first, in an increase of the output signal at the constant signal illumination at the input. With the high-intensity background, the resulting output current may exceed the sum of output currents due to only signal and the background illumination. This fact can be explained by the change in the PMT gain coefficient under the effect of the background.[3,4]

Figure 3*a* shows the calculated dependence of the change in the PMT gain coefficient $\Delta M/M_0$ (ΔM is the absolute change in the PMT gain coefficient; M_0 is the PMT gain coefficient in the absence of background) on the background intensity characterized by the current through the photocathode $I_{\rm pc}$. This characteristic agrees well with the experimental dependence (Ref. 4, p. 51, Fig. 3.3) recorded for typical PMTs with louvered dynodes, which can endure high current load. The plots in Fig. 3 correspond to FEU–84 PMT at the initial current through the uniform voltage divider $I_{\rm d0} = 3.5$ mA, the supply voltage $E = 1500$ V, and $M_0 \approx 10^6$.

The relative change in the gain coefficient of the PMTs with louvered dynodes can be presented as

$$\Delta M/M_0 = (I_{\rm a}/I_{\rm pc} - M_0)/M_0, \qquad (6)$$

where $I_{\rm a}$ is the current through the anode; $I_{\rm pc}$ is the current through the photocathode; M_0 is the PMT's gain coefficient in the absence of a background.

The mathematical model of such a photodetector can be presented by the following system of equations[4]:

$$\begin{cases} i_1 = i_0\,(1 + kR\,(I_{\rm d} - i_0)), \\ \qquad \cdots\cdots \\ i_n = i_{n-1}\,(1 + kR\,(I_{\rm d} - i_{n-1})), \\ \qquad \cdots\cdots \\ i_N = i_{N-1}\,(1 + kR\,(I_{\rm d} - i_{N-1})\,K_{\rm t}(U_{n+1})), \\ \sum\,(I_{\rm d} - i_n) = E/R,\ n = 0 \ldots N, \end{cases} \qquad (7)$$

where N is the number of PMT stages; k is the constant factor, which depends on the material of dynodes; E is the applied voltage; i_n is the current through the nth dynode; I_0 is the current through the photocathode; $I_{\rm a} = i_N$ is the electric current through the anode; $U_{n+1} = R(I_{\rm d} - i_N)$ is the voltage applied to the 12th dynode–anode gap; $K_{\rm t}$ is the transmission coefficient of the final PMT's stage, which is described as[4]:

$$K_{\rm t} = k_0 + k_1 U_{n+1}, \quad \text{if} \quad U_{n+1} > U_{\rm op},$$

$$K_{\rm t} = (1 + k_2 U_{n+1})/(1 + kE/(N+1)),$$
$$\text{if } 0 < U_{n+1} < U_{\rm op}, \qquad (8)$$

where $U_{\rm op}$ depends on the PMT type and characterizes its operating mode; $K_{\rm t}$ is the function of voltage applied to the 12th, dynode–anode, gap, U_{n+1}. The change of U_{n+1} with the growing cathode current $I_{\rm pc}$ due to background causes the change in the transmission of the final PMT's stage, as shown in Figs. 3*b* and *c*. The change in the transmission coefficient of the final stage $K_{\rm t}$ (the fall off of the characteristic, see Fig. 3*c*), in its turn, leads to a fall off of the characteristic of the anode current.

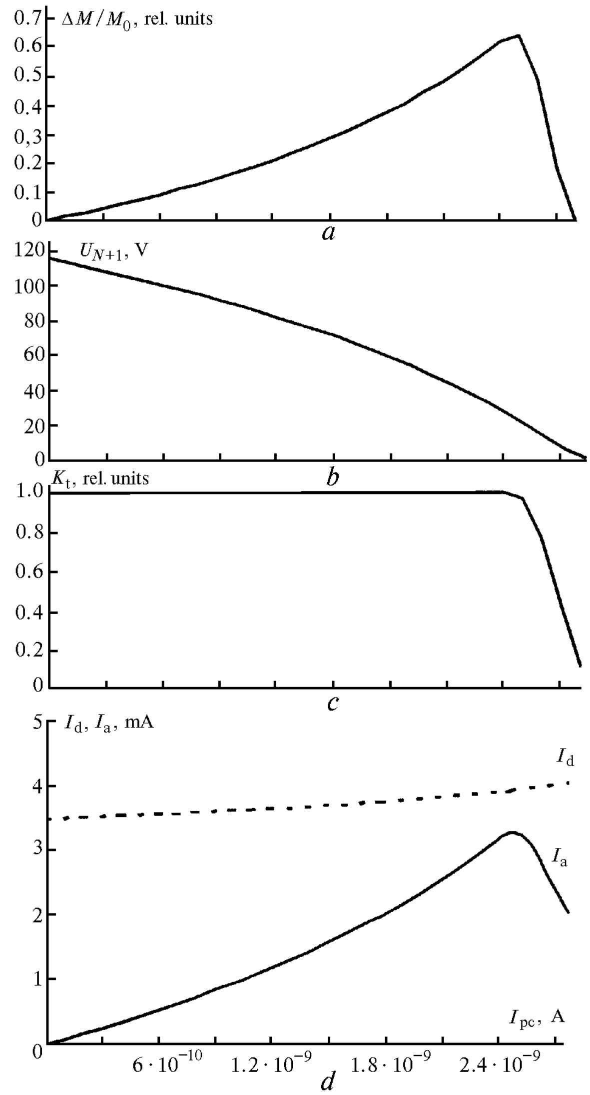

Fig. 3. Change of the PMT characteristics: relative gain coefficient $\Delta M/M_0$ (*a*); the voltage at last dynode–anode gap (*b*); transmission coefficient of the last PMT stage (*c*); current through the voltage divider and the background anode current (*d*) vs. the background current of the photocathode $I_{\rm pc}$.

Figure 3*d* shows the generalized plots drawn based on the above-considered changes in the parameters influencing the relative gain coefficient $\Delta M/M_0$. The plots present the behavior of the current through the voltage divider $I_{\rm d}$ and the PMT anode current $I_{\rm a}$ due to the background in response to current through the photocathode.

The results of simulation of the background characteristics of lidar photodetectors allow us to predict changes in their amplitude characteristics (Fig. 3*a* and *d*), from the measured current through the anode caused by the background. The amplitude characteristics can be generalized in the following form[4]:

$$M(I_b) = M_0\,[1 + \Delta M(I_b)/M_0]. \qquad (9)$$

This allows one to correct measured lidar signals.

Thus, in the following calculations on reconstruction of optical profiles of the volume extinction coefficient, we apply a correction for received components of the signal and the background noise according to Eq. (9) and the results depicted in Fig. 3.

Model experiments on reconstruction of the profile of volume extinction coefficient

Reconstruction of the profiles of volume extinction coefficient was numerically simulated by the following scheme: (1) reconstruction of the profile $\alpha(R)$ in the absence of the background at varying parameters of the medium, (2) reconstruction of the profile $\alpha(R)$ taking into account the influence of the background noise and changes in the characteristics of the photodetector, (3) influence of the photodetector noise on the results of reconstruction of the profiles.

At present there exist various methods for solution of the lidar equation. Below we present the results obtained with the use of the method of successive layers, which is based on the assumption of the constant scattering coefficient and scattering phase function inside an elementary layer. In a slightly turbid atmosphere, this method allows sufficiently accurate reconstruction and good description of both the fronts and plane peaks of a model profile.[1] To extend the domain of applicability of the method to the model of scattering, we take into account both single and multiple scattering by applying the correcting coefficients according to Ref. 1.

If the volume extinction coefficient in the initial layer is known, then for any other, *i*th layer, in the single-scattering approximation, it is accepted to write the signal in the following form[1]:

$$P_s(R_i) = AP_0\alpha(R_i)b(R_i)R^{-2} \times$$
$$\times \exp\left\{-2\left[\alpha(R_0)\,R_0 + \sum_{j=1}^{n}\alpha(R_j)\,\Delta R_j\right]\right\}. \qquad (10)$$

The formal solution to which is as follows:

$$\alpha(R_i) = 2P_s(R_i)\,R_i^2\exp\left[2\alpha(R_0)\,R_0 + \right.$$
$$\left. + 2\sum_{j=1}^{n}\alpha(R_j)\Delta R_j\right] \Big/ \left[(AP_0\,b(R_i)\right], \qquad (11)$$

where $P_s(R_i)$ is signal power received from the *i*th layer; P_0 is the power of the transmitter; A is the instrumental function characterized by the parameters of the transceiver, losses in the optical systems, etc.; b is the known constant factor for a certain class of media; ΔR_i is the thickness of the *i*th layer; n is the number of layers; α is the volume extinction coefficient.

1. First, the profile $\alpha(R)$ was reconstructed in the absence of the background at varying parameters of the medium in order to determine the conditions for best reconstruction and then to carry out calculations for them with the account of the background.

The initial parameters of simulation are the following: the model profile of the volume extinction coefficient $\alpha(R)$; the mean extinction coefficient $\alpha_0 = 0.03$, 0.1, and 1.0 corresponds to the clear sky, a light and a moderate haze.

The results calculated by the method of successive layers are shown in Fig. 4. The reliable reconstruction of the profile is limited by the conditions of haze and clear sky ($\alpha_0 = 0.3$ km^{-1} – $\alpha_0 = 0.03$ km^{-1}), what supports the results of analysis of the accuracy characteristics.[1]

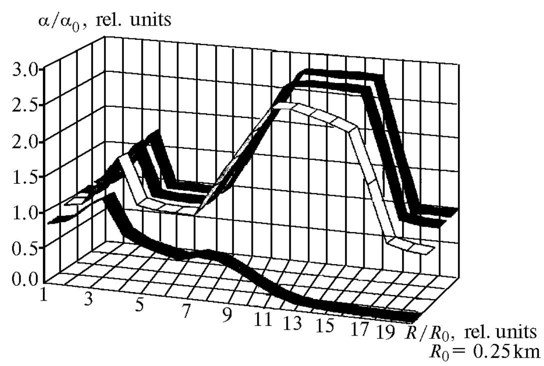

Fig. 4. Reconstruction of the scaled volume extinction coefficient α/α_0 by the method of successive layers in the absence of the background under different atmospheric conditions; from the top down: model profile; $\alpha_0 = 0.03$ km^{-1} (clear sky); $\alpha_0 = 0.1$ km^{-1} (a light haze); $\alpha_0 = 1.0$ km^{-1} (a moderate haze).

2. Then we consider the influence of the background noise on the results of reconstruction of the profile $\alpha(R)$ under clear sky conditions, under which the profile of the extinction coefficient is reconstructed most accurately neglecting the background. The relative background illumination of the horizontal surface, $h_{rel} = H_\lambda/H_{\lambda max}$, of 10^{-2}, 10^{-1}, and 10^0 corresponds to the background illumination of the horizontal surface $H_\lambda = 10^2$, 10^3, and 10^4 W/(m^2·sr·μm). For the "clearB atmosphere ($\alpha_0 = 0.03$ km^{-1}) we obtained the results shown in Fig. 5. These results were obtained using the method of successive layers under different background conditions with compensation by the "traditionalB method of simple subtraction of the background.

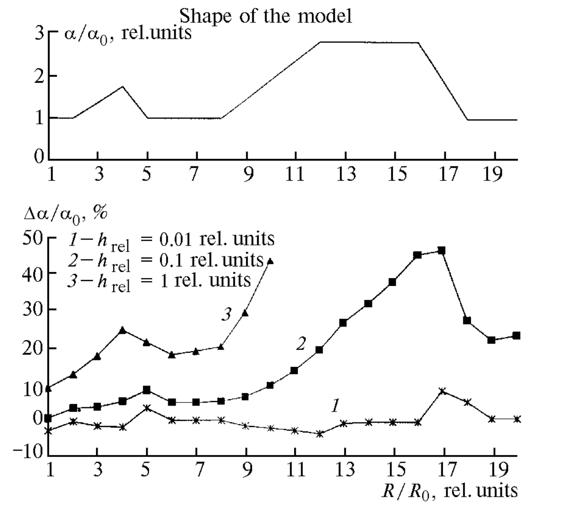

Fig. 5. Relative error of reconstruction of the volume extinction coefficient α by the method of successive layers with compensation for the background by the traditional method under different background conditions for $\alpha_0 = 0.03$ km^{-1}; $R_0 = 0.25$ km.

As seen from Fig. 5, the reconstruction is good at low level of the background ($h_{rel} = 0.01$); the relative error of reconstruction in this case does not exceed 10%. If the error of reconstruction is restricted to 10–20%, the relative range of reliable sounding within the preset accuracy at the relative background intensity $h_{rel} = 0.1$ proves to be 1.5 times less than the maximum range for $h_{rel} = 0.01$. As the background intensity further increases, the solution rapidly diverges, and at $h_{rel} = 1.0$ the preset error of reconstruction is achieved at the relative range R/R_0, which is five to eight times less than the maximum range corresponding to a weak background.

Influence of noise in the receiving system

Let us consider the reconstruction of the volume extinction coefficient profile by making use of the developed technique and taking into account the change in characteristics and the increasing influence of noise in the photodetector.

The noise in the photodetector increases significantly under the influence of high-intensity background. The preset signal-to-noise ratio at the output of the generalized system of direct photodetection with the internal current gain coefficient ρ determines the minimum signal current[7]:

$$I_{s\,min} = e\,\Delta f\,\rho\,\{1 + [1 + (I_b + I_d)/(e\,\Delta f\,\rho)]^2\}^{1/2}, \quad (12)$$

where e is the electron charge; $\Delta f = f_{max}$ is the frequency band; I_b is the photodetector anode current due to the background; I_d is the dark current.

Fluctuations of the signal current increase under the effect of the background. Therefore, the reconstructed profile can be thought lying inside some amplitude band corresponding to the noise band of the lidar signal. Reconstruction of the profile with compensation for the background noise using the proposed approach taking into account the increase in the noise under the effect of the background gives the result shown in Fig. 6. The outer boundaries of the band of the reconstructed profile correspond to the maximum intensity of the background for the mid-latitude conditions ($h_{rel} = 1$).

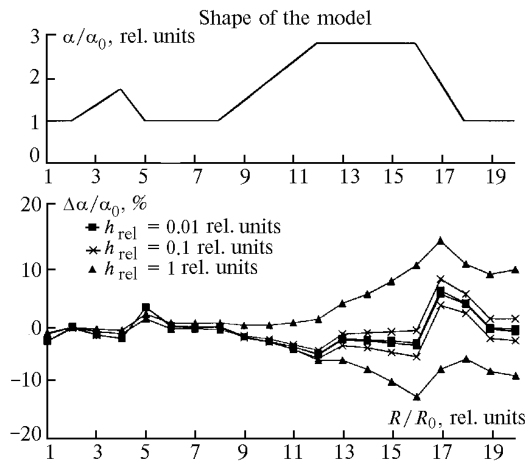

Fig. 6. Relative error of reconstruction of the volume extinction coefficient α by the method of successive layers based on compensation for the background with the allowance for the noise of the photodetector under different background conditions at $\alpha_0 = 0.03$ km^{-1}; $R_0 = 0.25$ km.

The maximum error in the reconstructed profile compared with the model profile does not exceed 15%, what is quite tolerable. The comparison of the results shown in Figs. 5 and 6 demonstrates high efficiency of the proposed approach in comparison with the traditional method.

Thus, this approach allows reconstruction of the profile of the volume extinction coefficient at the maximum background influence $h_{rel} = 1$ with the accuracy of the traditional method at the level of the background $h_{rel} = 0.01$.

Conclusion

The developed mathematical model of the system “transmitter – medium under study – receiver – device for secondary processingB adequately describes the process of measurements under actual noise situation and correction of the received lidar signal for the effect of the background. The proposed approach provides for higher accuracy of measurements and reconstruction of the optical profiles of $\alpha(R)$ under severe noise conditions. Thus, at the maximum value of the spectral brightness of the sky background in the mid-latitudes $B_\lambda = 300$ W/(m^2·sr·μm), what corresponds to $h_{rel} = 1$, the relative error of measurements, in accordance with the proposed technique, does not exceed 15%. With the use of the traditional approach, this can be achieved only at a 100 times lower background. The range of reliable sounding under the same background conditions increases up to ten times with the use of the described approach in comparison with the traditional methods of processing lidar returns.

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