Use of laser reference stars for ground-based astronomical telescopes

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In this paper we study the problems associated with one of the most promising tendencies in development of current ground-based adaptive telescopes, in particular, its equipping with additional optical system for formation of a laser reference star. The calculated results are presented for such a scheme of formation of a laser reference star, which allows one to obtain arbitrary correlation between random angular shifts of the scatterer image due to fluctuations at the forward and backward paths. The expressions for the monostatic and bistatic schemes of formation of the laser reference star can be derived as limiting cases.

Introduction

The efficiency of modern opto-electronic systems operating through the atmosphere is restricted by the perturbing effect introduced by the atmosphere. Adaptive optical systems are among most promising tools decreasing this effect. Development and creation of adaptive systems require more thorough theoretical study of the optical radiation propagation in conditions of the radiation interaction with the atmosphere under adaptive phase control.

From wide variability of adaptive optical systems, we consider the coherent adaptive optical systems. Operation of these systems is characterized by the presence of reference radiation that provides information about fluctuations in the propagation channel.

Even the first papers on adaptive optics by Linnik[1] and Hardy[2] and others mentioned the "reference source." It was assumed that the following objects could serve as a reference source:

(a) Natural source;
(b) Specialized artificial source;
(c) Radiation backscattered from an object; and
(d) Radiation backscattered (or re-emitted) from atmospheric inhomogeneities.

Correlation of random shifts of beams and images

Various problems arising in the use of specialized reference sources, as well as natural sources (stars) were studied in our works in 1977–1983 (Refs. 3–10).

Thus, in 1979–1980 we solved the problem of stabilization of the laser beam direction in the turbulent atmosphere.[4,5] In particular, in Ref. 4 it is shown that fairly efficient stabilization of the axis of the laser beam directional pattern can be achieved by tracing the random position of an image formed by the reference plane wave. In 1980 these results were generalized for the cases of formation in the turbulent atmosphere of laser beams with arbitrary parameters and for different scenarios of formation of the reference radiation.[5]

Just in that work the idea to use a signal backscattered from atmospheric aerosol for adaptive correction was put forward for the first time.

Thus, the first investigations of laser reference stars have been started as early as in 1978–1980 when studying the problem of stabilization of the laser beam propagation direction in the turbulent atmosphere. Those studies considered the measurements of shift of the image of reference source (including a natural star image) in the focal plane of a telescope. In particular, in Refs. 4 and 5 the correlation function $\langle \rho_{\rm l.b}\, \rho_F \rangle$ was calculated for the vector $\rho_{\rm l.b}$, characterizing the random shift of the energy centroid of the optical beam propagating through the turbulent medium, and the vector ρ_F determining the image centroid for a star or any other reference source formed by the same optical system. It was assumed that it might be the image of a reference source (beacon) or optical beam reflected from some object. The particular cases can be images of a natural star, image of a specularly reflected laser beam, or a point-like reference source.

In Ref. 4 the correlation function for the random shift of the Gaussian beam centroid and the image centroid of an unbounded plane wave was calculated. The beam and the plane wave propagated along the same optical path. Random shifts of the beam centroid were described by the vector[11]:

$$\rho_{\rm l.b} = \frac{1}{P_0} \int\limits_0^X {\rm d}\xi\, (X-\xi) \iint {\rm d}^2 R\, I(\xi, \mathbf{R})\, \nabla_R\, n_1(\xi, \mathbf{R}),$$

$$P_0 = \iint {\rm d}^2 R\, I(0, \mathbf{P}), \qquad (1)$$

where $n_1(\xi, \mathbf{R})$ are the fluctuations of the refractive index at the point $(\xi, \mathbf{R})$; $I(\xi, \mathbf{R})$ is the field intensity at the point $(\xi, \mathbf{R})$ from the laser source situated at the origin of coordinates in the initial plane (for $\xi = 0$); X is the thickness of the atmospheric layer. Random shifts of image of reference plane wave in the focal plane of the optical system (telescope or equivalent "thin" lens with focal length F and area $\Sigma = \pi R_0^2$) are described by the equation[12]:

$$\rho_F = -\frac{F}{k\Sigma}\iint_\Sigma \nabla_\rho S(x,\rho)\mathrm{d}^2\rho , \qquad (2)$$

where k is the radiation wave number; $S(x, \rho)$ are phase fluctuations in the optical wave at the aperture of the optical system (in the plane $\xi = X$) at the point ρ. The correlation between the random vectors $\rho_{1.\mathrm{b}}$ and ρ_F can be determined as

$$K = \langle\rho_{1.\mathrm{b}}\, \rho_F\rangle / [\langle\rho_{1.\mathrm{b}}^2\rangle\, \langle\rho_F^2\rangle]^{1/2}. \qquad (3)$$

From there on $\langle\ldots\rangle$ denotes averaging over an ensemble of realizations of a random function $n_1(\xi, \mathbf{R})$. Assume that the distribution of the average intensity $\langle I(\xi, \mathbf{R})\rangle$ and the spectrum of atmospheric turbulence $\Phi_n(\xi, \kappa)$ are isotropic functions, and the average intensity $\langle I(\xi, \mathbf{R})\rangle$ for the Gaussian beam is described by the equation[11]:

$$\langle I(\xi,\mathbf{R})\rangle = \frac{a^2}{a_{\mathrm{eff}}^2(\xi)}\exp(-R^2 / a_{\mathrm{eff}}^2(\xi)) , \qquad (4)$$

where

$$a_{\mathrm{eff}}^2(\xi) = a^2\left[\left(1-\frac{X}{f}\xi\right)^2 + \Omega^{-2} + \Omega^{-2}\left(\frac{1}{2}D_s(2a)\right)^{6/5}\right];$$

$\Omega = \dfrac{ka^2}{X\xi}$; a and f are the initial parameters of the Gaussian beam; $D_s(2a)$ is the structural phase function. As a result we obtain[4]:

$$K = \int_0^1 \mathrm{d}\xi\,(1-\xi) \int_0^\infty \mathrm{d}\kappa\,\kappa^3\,\Phi_n(\kappa)\exp\left(-\frac{\kappa^2\,(R_0^2 + a_{\mathrm{eff}}^2)}{4}\right)\times$$

$$\times\cos\left(\frac{\kappa^2\,x(1-\xi)}{2k}\right)\left[\int_0^1 \mathrm{d}\xi\,(1-\xi)^2 \int_0^\infty \mathrm{d}\kappa\,\kappa^3\,\Phi_n(\kappa)\times\right.$$

$$\left.\times\exp\left(-\frac{\kappa^2 a_{\mathrm{eff}}^2(\xi)}{2}\right)\right]^{-1/2}\left[\int_0^1 \mathrm{d}\xi \int_0^\infty \mathrm{d}\kappa\,\kappa^3\,\Phi_n(\kappa)\times\right.$$

$$\left.\times\exp\left(-\frac{\kappa^2 R_0^2}{2}\right)\cos^2\left(\frac{\kappa^2\,x(1-\xi)}{2k}\right)\right]^{-1/2}. \qquad (5)$$

The calculations used the spectrum of turbulence in the form:

$$\Phi_n(\xi, \kappa) = 0.033\, C_n^2(\xi)\,(\kappa^2 + \kappa_0^2)^{-11/6}, \qquad (6)$$

which took into account the deviation from the exponential function in the region of the turbulence outer scale $L_0 = 2\pi\kappa_0^{-1}$; $C_n^2(\xi)$ is the structure parameter of the turbulent atmosphere.

The estimates were done for the homogeneous path (initial diameter of the laser beam was equal to the diameter of the telescope entrance aperture) for the following parameters:

$$\kappa_0^{-1} \gg (R_0,\ a_{\mathrm{eff}}\sqrt{x/k}),\ kR_0^2 \gg x;$$

$$\Omega^{-2}\left(\frac{1}{2}D_s(2a)\right)^{6/5} \ll 1.$$

As a result, we have obtained (for the focused beam $f = X$) the value $K = 0.84$.

Thus, as early as in 1979 the strong positive correlation between the shifts of the Gaussian beam and the shift of the plane wave image centroid was demonstrated providing the laser beam and the plane wave propagated along the same path and in the same direction.[4]

In 1980 those results were generalized to the case when the beam and the reference wave propagated towards each other.[5] In that case it was assumed that the reference image was formed in the focal plane of the telescope for the following scenarios: plane wave, spherical wave, and random Gaussian beam reflected from a plane mirror. For the plane wave at the homogeneous path it was obtained: $K = -0.87$ (for collimated beam), $K = -0.82$ (for focused beam).

Experiments on the use of reflected waves for adaptive correction

In about 1978–1983 a series of experiments was conducted. Thus, on the initiative of V.G. Vygon the experiments were performed on recording the returns from high altitudes using a ground-based pulsed laser source. "esides, first experiments were conducted on using the photoelectric meter of jitter of an image formed from real stars. Estimates and the first experiments conducted at that time have shown that technical capabilities of tracing systems allowed real-time observation of sources not weaker than +4 starbrightness units. Measurements of natural bright stars jitter were conducted (1981) nearby the 6-meter "TA telescope with the use of a tracing system based on an image dissector. The obtained results were reported in Refs. 13–15.

Similar investigations associated with stabilization of direction of the optical radiation propagation were performed as applied to operation of opto-electronic systems along horizontal paths (for laser sources).[6,9] Naturally, in that case there were no problems with reception of an optical signal. Some attempts to apply light amplifiers (Strelets and Kozerog systems) were undertaken as well.[15] From the literature we also know about the experience on construction of the ground-based adaptive telescope GRAF–1.[15]

All these investigations were the subject of discussion at the first meeting (within the USSR) on the problem "Atmospheric Instability and Adaptive Telescope" held in 1986 in the Crimean Astrophysical Observatory.

First investigations on creation of laser reference stars in the USA

In the USA the works on creation of laser reference stars started in 1982 within the framework of the Strategic Defense Initiative. The history of these researches can be found in Ref. 16. Signals from laser stars at different altitudes were measured for the first time in the USA at the Lincoln National Laboratory (under the direction of Dr. D. Greenwood) and the Philips Laboratory of Air Forces of the USA (under the direction of Dr. R. Fugate) in 1984. However, those results were first reported only in 1993 (at the Summer School of NATO in France); they are published in Ref. 17.

Performance of an adaptive telescope utilizing laser reference stars

Our paper on limiting capabilities of the systems, forming images of extended objects with the use of reference sources, was published in 1983.[10] It presented the first calculated results on the efficiency of correcting formation of an image of some extended object through the turbulent atmosphere with the help of an adaptive telescope using the reference star. That paper also presented rather simple quantitative estimates of the isoplanatism angle of the entire atmospheric depth and integral resolution of an adaptive optical system.

In 1993 at the conference in Munnich I presented the report[18] on calculation of ultimate capabilities of adaptive telescopes.

In 1995 we calculated[19] the behavior of the Strehl parameter and the integral resolution of a ground-based adaptive telescope for two types of reference stars: the Rayleigh and sodium ones. However, it should be noted that in Ref. 19 it is assumed that the reference star is fixed, that is, the information on instantaneous position of a star is obtained from some auxiliary measurements.

The use of laser reference stars for the problems of correction of images of extraterrestrial objects (astronomy, photography of near-space objects) faces some fundamental problems.[20,21] One of them is impossibility of "direct" correction of the wavefront total tilt when using the signal from some laser reference star.

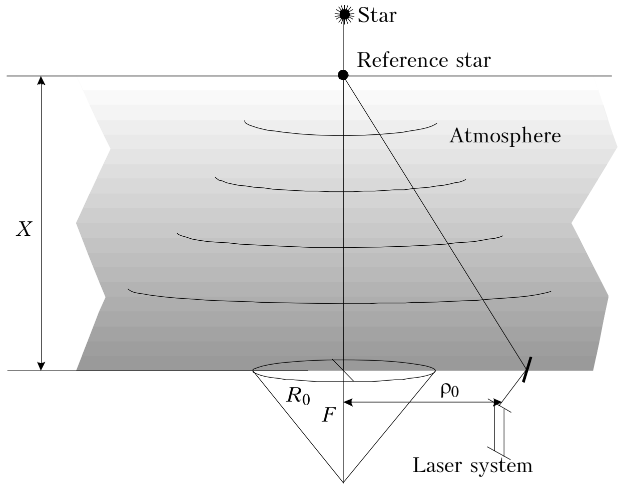

Fig. 1. Scheme of formation of a laser reference star: R_0 is the size of aperture of the main telescope; X is the altitude of formation of the laser reference star (entrance aperture of the telescope is in the plane $x = 0$); a_0 is the size of the aperture of the auxiliary telescope forming the laser reference star; $\boldsymbol{\rho}_0$ is the vector of the auxiliary telescope center shift relative to optical axis of the main telescope.

Some authors proposed other schemes of formation of reference stars, which helped to overcome that problem.[22–25] Figure 1 shows the scheme proposed by Ragazzoni.[22,23] It employs the reference star formed by an additional laser source. "ased on this scheme, in our recent works we have calculated the correlation between the jitter of images of the laser star and the natural star.[26–28]

Peculiarities of reflected waves fluctuations

Image jitter was the subject of many papers of Soviet authors who studied the backscattered radiation.[29] In the early 70s researchers in the fields of optical vision and formation of optical beams in the atmosphere have realized the importance of taking into account fluctuations of reflected waves. In contrast to the transmitting systems, in sounding systems a signal always twice passes the atmospheric path. Researchers of those systems have introduced such terms as *effective scattering volume, monostatic optical scheme, bistatic scheme of laser sounding*, and some others. In my opinion, such terms as *effective scattering volume* and *laser reference star* are the scientific synonyms.

In the period 1975–1983 in the framework of those studies the fluctuations of image centroid were considered for the problem of sounding the atmospheric inhomogeneities with a focused laser beam.[29] Monostatic and bistatic sounding schemes were treated. The equations were derived[30] for the variance of fluctuations of centroid of the optical image shift in the plane of a photodetector for the case of sounding the surfaces with random scattering properties.[30] For the case of strongly scattering surface (in the approximation of Lambert scattering) and the bistatic sounding scheme, it was obtained that the variance of a linear shift of the image centroid $\rho_{\rm im}$ at reflection can be described by the following equation[30]:

$$\langle\rho_{\rm im}^2\rangle = \frac{F^2}{x^2}\langle\rho_{\rm l.b}^2\rangle + F^2\langle(\varphi_F^{\rm ss})^2\rangle, \qquad (7)$$

where $\langle\rho_{\rm l.b}^2\rangle$ is the variance of a random shift of the laser beam centroid in the sounding plane (upward propagation), and $\langle(\varphi_F^{\rm ss})\rangle$ is the variance of a random angular shift of the image of the fixed "secondary" source (downward propagation); F is the telescope focal length; X is the distance between the laser source and the scattering volume.

Thus, it was shown that for the bistatic scheme (the limiting bistatic scheme was assumed without correlation between fluctuations on forward and backward paths) the variance of the image angular shift was formed by the variance of angular shifts of the sounding beam and that of angular shifts of image of the fixed secondary source. For strongly scattering medium and focused beam, the secondary source is in fact a point-like source.

The importance of works[31,32] devoted to the study of jitter of extended sources images should be particularly emphasized. In Ref. 31 the equation was derived for the jitter variance of an extended source image (luminous thin thread). Reference 32 deals with the study of correlation between jitter of image centroids of two randomly oriented laser beams.

Problems in the use of a laser reference star

The use of a laser reference star (LRS) in telescopes faces some serious problems mentioned by Fugate in Ref. 21, namely, influence of focal non-isoplanatism and incapability to practically separate (for the monostatic scheme) the contributions to LRS image jitter from beam propagation in upward and downward directions.

The effect of focal non-isoplanatism is attributed to the fact that the natural star forms a plane wavefront, while the laser reference star always forms a spherical wavefront.

In my opinion, the sufficiently complete review of the current works (unfortunately, except for the works done in the USSR and then in Russia) on the main stages in development of systems of laser reference stars formation is presented by Ragazzoni.[22]

Correlation between shifts of the laser beam and the image of a natural star for the bistatic scheme of LRS formation

Following Ragazzoni,[22,23] let us consider the scheme of a laser reference star formation (see Fig. 1). LRS is created with the use of a laser device having a separate transmitter.

Assume that the main telescope operates under conditions of adaptive correction using the radiation from LRS, which is formed by the laser device on the optical axis of the main telescope at a distance (altitude) u from the entrance aperture. According to Fig. 1 the main telescope is directed strictly to zenith, and both a weak natural star and the laser reference star are observed simultaneously along its optical axis. The zenith angle of the laser beam direction (under the condition $|\rho_0| \ll X$) is $|\rho_0|/X$. The natural star is spaced at almost infinite distance and forms the plane wavefront. The vector characterizing a random tilt of this wavefront due to atmospheric turbulence can be written as follows[12]:

$$\varphi_F^{\rm pl} = -\frac{1}{\Sigma}\iint\limits_{\Sigma} d^2\rho\nabla_\rho\, S^{\rm pl.w}(0, \rho), \qquad (8)$$

where

$$S^{\rm pl.w}(0, \rho) = k\int\limits_0^\infty d\xi \iint d^2\, n(\kappa, X-\xi)\exp(i\kappa\rho)$$

are the phase fluctuations of the plane wavefront at the entrance aperture; Σ is the area of the entrance aperture; k is the radiation wave number. In Eq. (8) it is taken into account that the optical wave from a real star propagates in the atmosphere from top to bottom, and the fluctuations of the refractive index of the atmosphere are presented as:

$$n_1(\xi, \rho) = \iint d^2\, n(\kappa, \xi)\exp(i\kappa\rho).$$

Random angular shifts of the LRS centroid formed by laser device at the altitude X can be written[11] with the help of Eq. (1) upon substitution of

$$I = I(\xi, \mathbf{R} + \rho_0(1 - \xi/X)) \qquad (9)$$

in it.

Thus, in the last expression it is taken into account that the optical axis of the laser source is shifted by the vector ρ_0 and inclined to zenith at an angle $|\rho_0|/X$.

Let us calculate the function of correlation between random angular shifts of the natural star image (function (8)) formed by the telescope and the shift of laser beam centroid formed by the laser source (function (1)). This correlation has been calculated repeatedly in many papers (Refs. 4, 5, 10, and 33). In some calculations the model of the turbulence spectrum was used, taking into account the deviation from the exponential function at large scales[33–44]:

$$\Phi_n(\kappa, \xi) = 0.033\, C_n^2(\xi)\, \kappa^{-11/3}\{1 - \exp(-\kappa^2/\kappa_0^2)\}, \qquad (10)$$

where $C_n^2(\xi)$ is the intensity of turbulence along the propagation path; $\kappa_0^{-1}(\xi)$ is the outer scale of turbulence. Accounting for the results of Refs. 4, 5, 33, and 45, this correlation function can be presented as

$$\langle \varphi_{\rm l.b}(\rho_0)\, \varphi_F^{\rm pl.w.} \rangle = \left(-2\pi^2\, 0.033\Gamma\left(\frac{1}{6}\right)\right) 2^{1/3}\, R_0^{-1/3} \times$$
$$\times \int_0^X d\xi\; C_n^2(\xi)\, (1 - \xi/X) \left\{ [1 + b^2(1 - \xi/X)^2]^{-1/6} \times \right.$$
$$\times\, {}_1F_1\left(\frac{1}{6}, 1; -\frac{d^2\,(1 - \xi/X)^2}{(1 + b^2\,(1 - \xi/X)^2)}\right) -$$
$$- [1 + b^2\,(1 - \xi/X)^2 + 4c^2]^{-1/6} \times$$
$$\left. \times\, {}_1F_1\left(\frac{1}{6}, 1; -\frac{d^2\,(1 - \xi/X)^2}{(1 + b^2\,(1 - \xi/X)^2 + 4c^2)}\right)\right\}, \quad (11)$$

where $b = a_0/R_0$; $d = |\rho_0|/R_0$; $c = \kappa_0^{-1}\, R_0^{-1}$; a_0 is the initial size of the focused laser beam; ${}_1F_1(\ldots)$ is the degenerate hypergeometric Gaussian function. It is seen from Eq. (11) that the second term in braces is attributed to the influence of the outer scale of turbulence. For the infinite outer scale ($c \to \infty$) the second term in Eq. (11) is negligible, then the correlation function assumes the following form:

$$\langle \varphi_{\rm l.b}(\rho_0)\, \varphi_F^{\rm pl.w.} \rangle = \left(-2\pi^2\, 0.033\Gamma\left(\frac{1}{6}\right)\right) 2^{1/3}\, R_0^{-1/3} \times$$
$$\times \int_0^X d\xi\; C_n^2(\xi)\, (1 - \xi/X)\, [1 + b^2(1 - \xi/X)^2]^{-1/6} \times$$
$$\times\, {}_1F_1\left(\frac{1}{6}, 1; -\frac{d^2\,(1 - \xi/X)^2}{(1 + b^2\,(1 - \xi/X)^2)}\right). \quad (12)$$

The case $d = 0$ corresponds to the monostatic scheme of formation of the laser reference star. In the opposite situation (for the bistatic scheme) the condition $d \gg 1$ corresponds to the asymptotic case for the hypergeometric function ${}_1F_1(\ldots)$, then

$$\langle \varphi_{\rm l.b}(\rho_0)\, \varphi_F^{\rm pl.w.} \rangle = \left(-2\pi^2\, 0.033\Gamma\left(\frac{1}{6}\right)\right) 2^{1/3}\, R_0^{-1/3} \times$$
$$\times\, \Gamma^{-1}\left(\frac{5}{6}\right) d^{-1/3} \int_0^X d\xi\; C_n^2(\xi)\, (1 - \xi/X)^{2/3}. \quad (13)$$

Analysis of the last equation allows the conclusion that the correlation between the plane wave and beam decreases to about 0.1 if $d \geq 10^3$. This actually corresponds to the "limiting" bistatic scheme of formation of the laser reference star.

A lot of experimental data[33–44,48] demonstrate that the outer scale of turbulence $\kappa_0^{-1}(\xi)$ in the atmosphere has a finite value. Our numerical estimates based on different models of the altitude distributions of $C_n^2(\xi)$ and $\kappa_0^{-1}(\xi)$ show[46] that under conditions of optical radiation propagation through the entire atmospheric depth *the effective outer scale of turbulence* can be introduced for the atmosphere as a whole (similarly to Ref. 44). It turned out[46] that under medium conditions of vision[47] the value of such *effective outer scale* is from 5 to 60 m. Then for a telescope with $R_0 = 4$ m the parameter $c = \kappa_0^{-1}\, R_0^{-1} = 10$.

The asymptotic analysis[45] (assuming that the outer scale of turbulence has a finite value) shows that for $c < 5$, if the axes of the main and auxiliary telescopes are separated by $\rho_0 \geq 2\,\kappa_0^{-1}$, we obtain practically the limiting bistatic scheme.

To confirm the conclusions of our asymptotic analysis, let us calculate numerically the correlation coefficient

$$K(d, b, c, X) = \frac{\langle \varphi_{\rm l.b}(\rho_0)\, \varphi_F^{\rm pl.w.} \rangle}{\sqrt{\langle \varphi_{\rm l.b}(\rho_0)^2 \rangle\, \langle (\varphi_F^{\rm pl.w.})^2 \rangle}}, \quad (14)$$

which can be expressed in terms of the correlation function (11) and the corresponding variances as:

$$\langle (\varphi_F^{\rm pl.w.})^2 \rangle = \left(2\pi^2\, 0.033\Gamma\left(\frac{1}{6}\right)\right) 2^{1/6}\, R_0^{-1/3} \times$$
$$\times \int_0^\infty d\xi\; C_n^2(\xi)\, [1 - [1 + 4c^2]^{-1/6}], \quad (15)$$

$$\langle (\varphi_{\rm l.b}(\rho_0))^2 \rangle = \left(2\pi^2\, 0.033\Gamma\left(\frac{1}{6}\right)\right) 2^{1/6}\, R_0^{-1/3} \int_0^X d\xi\, C_n^2(\xi) \times$$
$$\times \{(b^2(1 - \xi/X)^2)^{-1/6} - (b^2(1 - \xi/X)^2 + 4c^2)^{-1/6}\}. \quad (16)$$

The calculations were performed for the model of $C_n^2(\xi)$ (Ref. 47) corresponding to the medium condition of vision and for two most characteristic altitudes $u = 10$ and 100 km, which approximately correspond to the positions of the Rayleigh and sodium laser reference stars. The values of the parameter b were taken equal to 0.1, 0.3, 0.7, 1.0, 3.0, and 5.0. The variable d characterizing the separation between the axes of the main and auxiliary telescopes was used as an argument of the function.

It should be noted that the use of the model of altitude variation of the outer scale and, especially, just the value of this parameter for vertical optical paths continuously provokes the skepticism. It has been shown for homogeneous ground paths, that the outer scale of turbulence is a finite value comparable with the altitude above the underlying surface.[48] At the same time, some scientists recommend that the outer scale of turbulence equal to hundreds meters and even several kilometers be used for astronomical observations. However, there are enough observations in which the outer scale does not exceed one meter.[14,35,42,43]

Certainly, the outer scale of atmospheric turbulence varies widely with the altitude both in the ground layer and at large altitudes. Therefore, it cannot be considered as a fixed value for the atmosphere as a whole. We propose to consider the possible versions of the altitude behavior of the outer scale:

A) $\kappa_0^{-1}(h) \approx 0.4\, h$,

") $\kappa_0^{-1}(h) \approx \begin{cases} 0.4h & h \leq 25 \text{ m} \\ 2\sqrt{h} & h > 25 \text{ m} \end{cases}$,

$$\text{C) } \kappa_0^{-1}(h) \approx \begin{cases} 0.4h & h \le 25 \text{ m} \\ 2\sqrt{h} & 25 \text{ m} < h \le 2000 \text{ m} \\ 88.4 \text{ m} & h > 2000 \text{ m} \end{cases},$$

$$\text{D) } \kappa_0^{-1}(h) = 4/[1 + ((h - 8500)/2500)^2],$$

$$\text{E) } \kappa_0^{-1}(h) = 5/[1 + ((h - 7500)/2000)^2].$$

The model A was recommended in Ref. 12 to be used at low altitudes; the model ” was proposed by Fried[34]; the model C is the generalization of the two above models; the models D and E were obtained from the data of direct measurements in the USA, France, and Chili[34,40,41,50] (it is interesting that very close values for that parameter were obtained at the Mauna Kea Observatory (Hawaii)[41]). These models are not beyond doubts,[34,35] but accepting wide altitude variations of the outer scale is more justified than assuming some definite value for it at an inhomogeneous optical path.

The calculated results are shown in Figs. 2 and 3. Each figure consists of five fragments: A, ” , C, D, and E, which correspond to various models of the altitude behavior of the turbulence outer scale.

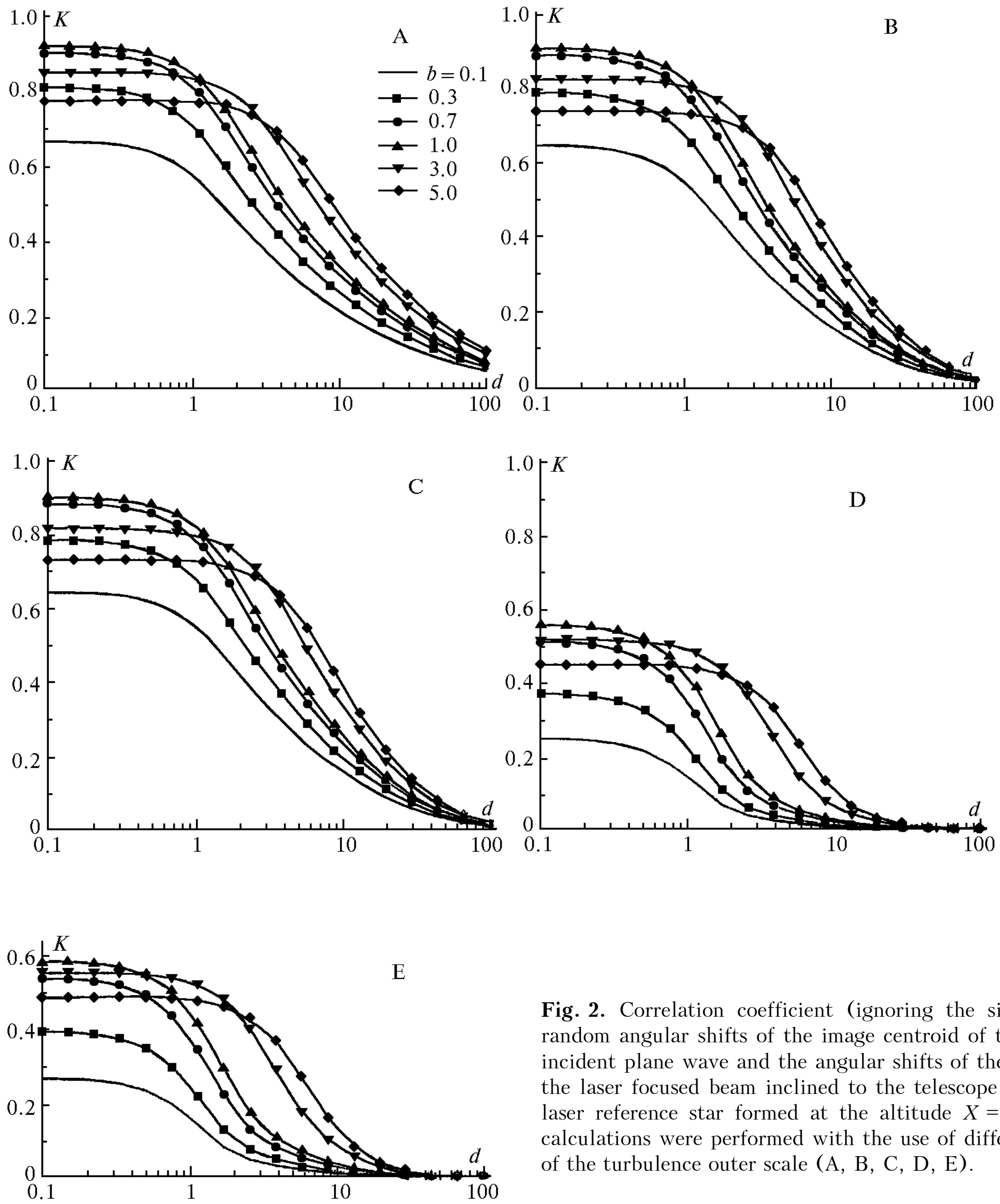

Fig. 2. Correlation coefficient (ignoring the sign) for the random angular shifts of the image centroid of the normally incident plane wave and the angular shifts of the centroid of the laser focused beam inclined to the telescope axis for the laser reference star formed at the altitude X = 10 km. The calculations were performed with the use of different models of the turbulence outer scale (A, B, C, D, E).

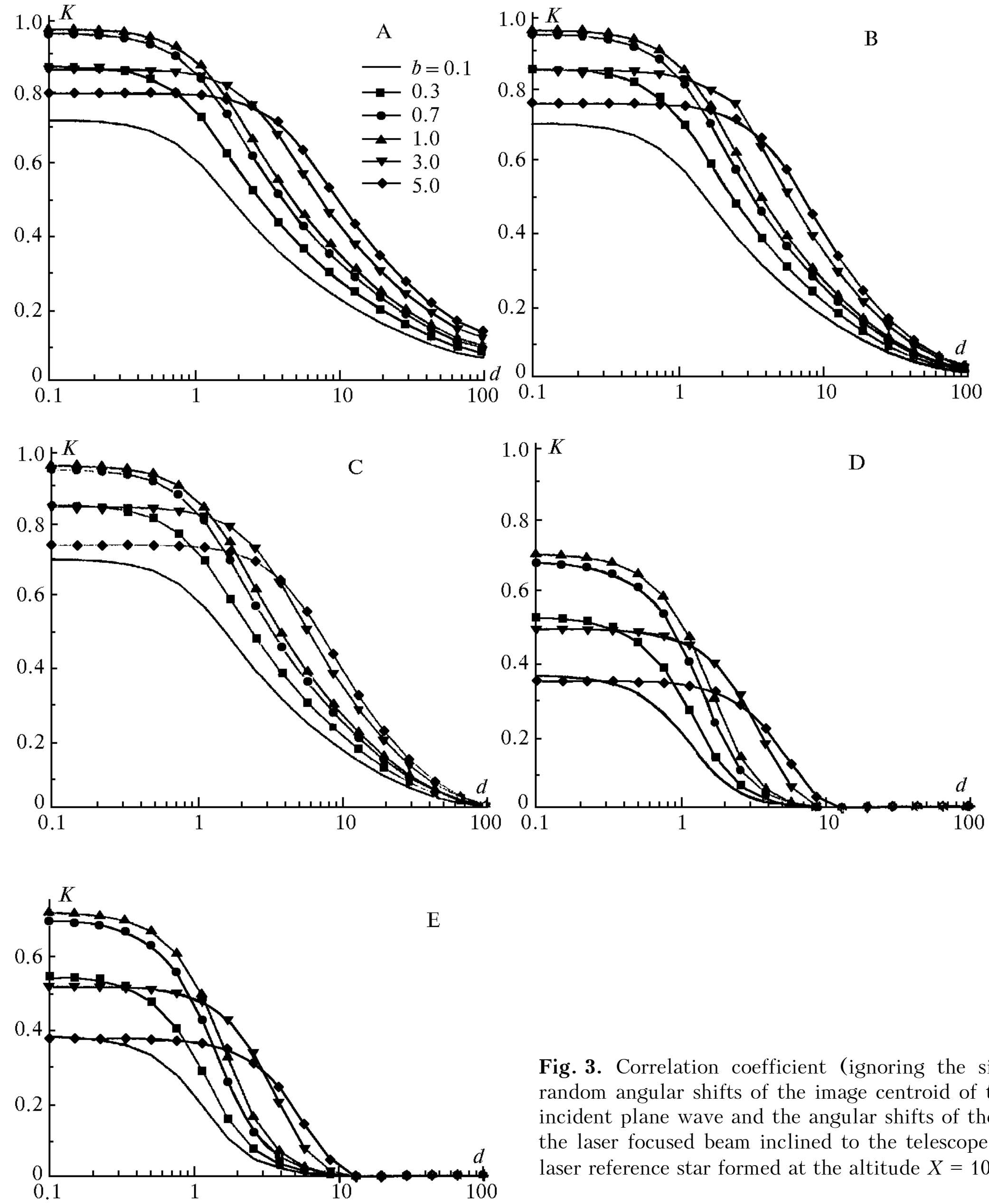

Fig. 3. Correlation coefficient (ignoring the sign) for the random angular shifts of the image centroid of the normally incident plane wave and the angular shifts of the centroid of the laser focused beam inclined to the telescope axis for the laser reference star formed at the altitude X = 100 km.

In particular, the calculated results allow the following conclusions:

1. For the models A, ", and C of the outer scale altitude behavior, the transition from the monostatic scheme (d = 0) to the limiting bistatic one occurs when the separation between axes of the main and auxiliary telescopes is from 100 to $200R_0$.

2. At small values of the outer scale (models D and E) this transition occurs already at the separation about two to five times larger than the size of the telescope aperture.

3. It should be noted here that our calculations differ from those presented in Refs. 25 and 49, because the subject of those works was the angular dependence of correlation between two plane waves coming from the infinity at different angles. In our case, in particular, the correlation coefficient K for the argument $d = 0$ (see Eqs. (11)–(13)) is not identically equal to minus one. Only with increasing X the value of K (for $d = 0$) asymptotically tends to unity (with the minus sign). I have made those calculations for the first time as early as in 1980 (Ref. 5).

4. It is of interest to note the peculiarity in the behavior of the correlation coefficient K for small characteristic values of the outer scale (models D and E) and at large values of the parameter b (b = 3, 5). The case takes place when a big telescope forms a reference star for a small telescope. The results show that in this case the correlation for $0.1 < d < 5$ remains practically constant (from 0.5 to 0.55).

5. As the characteristic values of the outer scale increase, the correlation function K becomes more large-scale. It should be noted that a similar effect was mentioned in Ref. 25: the radius of angular correlation increases as a function of the outer scale.

6. Such phenomenon as alternation of the sign of the correlation K (to see this, we should come back and compare Eqs. (11) and (12)), which was predicted in Ref. 45 based on the asymptotic analysis, is connected with manifestation of the outer scale finiteness. For small values of the turbulence outer scale (models D and E) and $d > 20–30$, the correlation K changes its sign. For large values of the outer scale this phenomenon is not observed. For the infinite outer scale (see Eq. (12)) the correlation coefficient K does not change its sign.

Algorithm of the "optimum" correction of the total wavefront tilt

It is known that the use of the laser reference star increases the stable operation zone of an adaptive system. However, because the laser star is formed at a finite distance and its position is accidental, the necessity arises to correct the data of optical measurements from the laser star to ensure the effective correction of distortions for real astronomical objects. I believe that it is important to study the possibility of better correction of atmospheric distortions with the use of the atmospheric models.[33,34,48]

In Refs. 26–28 it is proposed to construct the algorithm for correction of the star image jitter in the form

$$\varphi_F^{\text{pl.w.}} - A\varphi_{\text{m}}, \tag{17}$$

thus providing, due to selection of the coefficient A, the minimum variance of residual distortions:

$$\langle\beta^2\rangle = \langle(\varphi_F^{\text{pl.w.}} - A\varphi_{\text{m}})^2\rangle = \langle(\varphi_F^{\text{pl.w.}})^2\rangle + A^2\,\langle(\varphi_{\text{m}})^2\rangle - 2A\,\langle\varphi_F^{\text{pl.w.}}\,\varphi_{\text{m}}\rangle. \tag{18}$$

Once the minimum variance in the form (18) is found, we have

$$\langle\beta^2\rangle_{\min} = \langle(\varphi_F^{\text{pl.w.}})^2\rangle - \langle\varphi_F^{\text{pl.w.}}\,\varphi_{\text{m}}\rangle^2/\langle(\varphi_{\text{m}})^2\rangle, \tag{19}$$

where the correcting coefficient A is expressed only through the deterministic functions as follows:

$$A = \langle\varphi_F^{\text{pl.w.}}\,\varphi_{\text{m}}\rangle/\langle\varphi_{\text{m}}^2\rangle. \tag{20}$$

From the form of Eq. (20) for the correcting coefficient A it follows that this coefficient can be found directly in the process of optical experiment from the data of direct measurements. Another alternative approach is a calculation of the correcting coefficient using the models of the turbulent atmosphere by Eqs. (20), (9), (12), and (13).

It should be noted that the traditional correction algorithm (17), which employs $A = -1$, naturally, does not give the minimum variance (18). To confirm this, let us compare the values of the residual variance for the optimal and non-optimal algorithms of correction.

In the actual experiment, we have, as a rule, only the measured values of φ_{m}. The reason is that we cannot measure the vector $\varphi_F^{\text{pl.w.}}$ characterizing the angular jitter of the actual star, whose image is to be corrected. This is because the actual star gives a little light, not sufficient to be measured with a wavefront sensor. In this case the correcting coefficient A can be estimated by Eq. (20) using the model description of altitude distribution of the turbulence.[26–28]

In our designations, minimum possible variance of residual fluctuations of angular shifts of the laser star image (under assumption that the reference star is point-like and cannot be resolved by the aperture of the initial telescope) for the scheme shown in Fig. 1 can be estimated by the following equation:

$$\langle\beta^2\rangle_{\min} = \langle(\varphi_F^{\text{pl.w.}})^2\rangle \times \left\{1 - \frac{2^{1/3} f(X, b, d, C_n^2)}{\left[1 + b^{-1/3} - 2^{7/6}(1+b^2)^{-1/6}\,{}_1F_1\left(\frac{1}{6}, 1; -\frac{d^2}{(1+b^2)}\right)\right]}\right\}, \tag{21}$$

where the function

$$f(X, b, d, C_n^2) = \left(\int_0^X \mathrm{d}\xi\; C_n^2(\xi)\;(1 - \xi/X) \times \left\{[1 + (1 - \xi/X)^2]^{-1/6} - [1 + b^2(1 - \xi/X)^2]^{-1/6} \times {}_1F_1\left(\frac{1}{6}, 1; -\frac{d^2\,(1 - \xi/X)^2}{1 + b^2\,(1 - \xi/X)^2}\right)\right\}\right)^2 \times \left[\left(\int_0^X \mathrm{d}\xi\; C_n^2(\xi)\,(1 - \xi/X)^{5/3} \int_0^\infty \mathrm{d}\xi\; C_n^2(\xi)\right)\right]^{-1} \tag{22}$$

depends on both the parameters of the optical experiment and the atmospheric model used (these equations are written under the assumption of the infinite outer scale). As the numerical analysis of the last equations shows,[6–28,45,51] the optimum correction allows a decrease in the value of residual angular distortions as compared to the traditional scheme.

The effect of optimum algorithm application is demonstrated by the Table 1, which presents the residual angular distortions of images formed by the telescope employing the bistatic scheme of the reference star. The reference star in this case is assumed a point-like source. For the optimum and traditional (non-optimum) correction algorithms, the third and fourth columns of the table give the values of normalized variance of residual angular distortions

$\langle e^2\rangle = \langle\beta^2\rangle_{\min}/\langle(\varphi_F^{\text{pl.w.}})^2\rangle$. The fifth column gives the values of the coefficient A calculated for the model of turbulence from Ref. 47. It is seen from the table that the optimum correction using the optimizing coefficient obtained from the models of the turbulent atmosphere reduces the value of residual distortions by more than half, whereas the traditional algorithm (that is, when $A = -1$) in some cases even increases the value of residual distortions.

Table 1. Comparison of the algorithms of optimum and non-optimum (traditional) correction of the wavefront random tilts for the limiting bistatic scheme of the laser reference star formation

X, km	b	Residual angular distortions		A
		optimum algorithm	non-optimum algorithm	
8	0.3	0.640	1.291	− 0.427
	0.5	0.603	1.105	− 0.471
	0.7	0.578	0.999	− 0.500
	1	0.552	0.899	− 0.532
	2	0.500	0.736	− 0.593
	3	0.471	0.656	− 0.628
	5	0.434	0.570	− 0.671
20	0.3	0.612	1.354	− 0.420
	0.5	0.572	1.148	− 0.463
	0.7	0.545	1.030	− 0.492
	1	0.516	0.918	− 0.523
	2	0.461	0.736	− 0.583
	3	0.429	0.647	− 0.618
	5	0.330	0.551	− 0.660
40	0.3	0.602	1.406	− 0.413
	0.5	0.561	1.187	− 0.456
	0.7	0.533	1.062	− 0.484
	1	0.504	0.944	− 0.515
	2	0.447	0.751	− 0.574
	3	0.414	0.657	− 0.608
	5	0.374	0.556	− 0.650
80	0.3	0.600	1.446	− 0.407
	0.5	0.558	1.22	− 0.450
	0.7	0.531	1.091	− 0.478
	1	0.501	0.969	− 0.508
	2	0.443	0.769	− 0.567
	3	0.410	0.672	− 0.600
	5	0.370	0.567	− 0.641
100	0.3	0.599	1.455	− 0.406
	0.5	0.588	1.227	− 0.448
	0.7	0.530	1.097	− 0.477
	1	0.501	0.974	− 0.507
	2	0.443	0.774	− 0.565
	3	0.410	0.676	− 0.598
	5	0.370	0.570	− 0.639

Thus, it is proved that the optimum correction based on the use of information on vertical profiles of turbulence has some efficiency. However, it should be noted that the level of residual distortions is still too large for this correction can be recommended for actual experiments.

In this connection, we can conclude that the optimum way to achieve high quality in correction for total tilt of the wavefront is the use of different hybrid schemes,[23,25,51] including simultaneous use of the natural reference star (to determine the total tilt of the wavefront) and the laser one (to measure the higher aberrations of the wavefront).

It should be noted that because the visible image of a bistatic reference star is an extended, rather than point-like object, some optimistic assumptions have been put forward[22–25] that the allowance for the size of the laser reference star can improve the estimate of the total wavefront tilt. However, the estimates show[45,52] that in practice (at actual schemes of formation of the laser star) the additional gain can be only about 20–25%.

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