Effect of turbo-engine jet on laser radiation. Part 2. Random wandering of disturbed beam

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The variance of centroid displacements has been experimentally measured for a laser beam intersecting a turbo-engine jet at different angles (φ = 90, 45, 10°). The obtained values range from 60 to 180 μrad for λ = 1.06 μm and from 110 to 300 μrad for λ = 0.53 μm. A difference has been found between the variances of beam wandering in the horizontal and vertical (orthogonally to the jet axis) directions. An analytical model developed for calculating the variance of laser beam wandering agrees with experimental data. The experimental results have allowed us to assess the high-frequency component of refraction index fluctuations in the jet, which supplements the von Karman spectrum for a turbulent medium.

Random wandering of a laser beam as a whole at radiation propagation along natural atmospheric paths is studied rather thoroughly,[1] because it is important for many practical applications using laser radiation for precise measurements. In many cases, experimental measurements of the variance of beam centroid displacements and the correlation characteristics of wandering allow refining characteristics of a turbulent medium, for example, parameters of the spatial spectrum of refractive index fluctuations. In this connection, it is of undoubted interest to study this effect under the conditions that the atmospheric turbulence is caused by technogenic factors. In such a situation, the level of turbulence in a medium several times exceeds the natural atmospheric turbulence and other peculiarities caused by the nature of a source of medium perturbations are possible as well. Because of these peculiarities, the methods of theoretical analysis that have already been developed cannot be directly applied to analysis of random beam wandering.

In this paper, we present measurement results on the variance of centroid wandering for laser beams disturbed by a high-speed turbo-engine jet and interpret the obtained results based on the comparison between the experimental and calculated data.

The measurements were conducted within the same cycle of experimental studies, as in Ref. 2, which describes in detail the experimental technique and the optical arrangement of the experiment. Let us note briefly most important features of our experiment. The jet was generated by an R-25-300 turbo-engine with the nozzle diameter of 55 cm. The jet temperature at the exit from the nozzle was ≈ 380°C, and the jet speed at the axis 1 m far from the nozzle was about 600 km/h. The axes of the jet and the laser beam in the experiments lied in the horizontal plane at the altitude of 150 cm above the ground, and the beam entered the jet at the distance of half a meter from the nozzle and passed through its central part. The studies were conducted as the laser beam propagated both crosswise (intersection angle $\varphi_1 = 90°$ and $\varphi_2 = 45°$) and almost along ($\varphi_3 = 10°$) the jet axis; the length of the beam path in the turbulent zone was L_t = 0.8, 1.4, and about 60 m, respectively.

A Nd:YAG laser was operated in a repetitively pulsed mode at the wavelengths λ = 1.06 and 0.53 μm simultaneously. The duration of laser pulses with the energy of 3–5 mJ did not exceed 30 ns, and the pulse repetition rate was 12.5 Hz. The diameter of the beam with almost homogeneous intensity distribution was determined by the diaphragm (10 or 30 mm) installed 2 to 3 m far from the jet boundary.

The distance from the laser to the receiving system was about 80 m. The far-field intensity distribution in each pulse was recorded simultaneously at two wavelengths λ = 0.53 and 1.06 μm by CCD cameras installed in the focal plane of an objective. The receiving objective had the diameter of 300 mm and the focal length of 265 cm. Practically instantaneous images of the beam cross section were recorded in the digital form and stored in the memory of a computer by entering signals from the CCD cameras. The number of images recorded in every cycle of measurements, i.e., the size of the statistical array for the following processing of data, was no less than 1500 frames.

It should be emphasized that our experimental technique allowed recording instantaneous images of laser beams having passed through the same inhomogeneities in the high-speed turbulent jet.

The expected angular displacement of the beam was very small – fractions of milliradian. Therefore, a considerable attention was paid to vibroprotection of all elements of the optical assembly and the laser

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located several meters far from the engine. Besides, the beam may experience jitter under the effect of natural turbulence along the atmospheric path more than 80 m long. In this connection, control experiments were conducted to assess the effect of the operating vibrational background of the engine on the characteristics of laser radiation. In these experiments, the vibrational scenario and the parameters of the optical path were kept unchanged, but the laser beam did not pass through the jet of the engine operating in its standard mode.

Table 1 gives the variance of centroid displacements and the angular halfwidth of laser beams for the horizontal (σ_x and θ_x – along the jet) and vertical σ_y and θ_y cross sections. The values were measured in the control experiments and at the turned-off engine and then averaged over the ensemble of realizations. The values obtained at the engine off characterize the effect of the atmospheric path on the laser radiation.

It can be seen that laser beams are not subject to the effect of vibrations excited by the engine, and the noticed small jitter of the image is caused by natural atmospheric turbulence.

Table 2 presents the measured results averaged over the ensemble of more than 1500 frames of root-mean-square random angular displacements of the centroid of a laser beam intersecting the engine jet at different angles to its axis.

As could be expected, the random wandering of a narrow 10-mm-diameter beam exceeds the amplitude of wandering of a wider 30-mm-diameter beam and increases with the decrease of the intersection angle. The data demonstrate also the azimuth asymmetry of the angular displacements of the beam: the variance of displacements in the direction *OY* orthogonal to the jet axis exceeds that of the displacements in the direction *OX*. This difference in the amplitude of wandering is especially marked for the beams intersecting the jet across (φ = 90 and 45°). It should be noted here that this observation agrees with the results obtained for the angular spectrum of laser beams disturbed by the jet in the same experimental cycle.[2] The results showed similar asymmetry in the angular intensity distributions. This asymmetry can be explained theoretically assuming the anisotropy of the spectrum of inhomogeneities in the jet in the region of the outer scale of turbulence.

The most interesting result among those given in Table 2 is the strong wavelength-dependence of the variance of random displacements of the beam centroid. To interpret this effect, let us use one of the main results obtained in Ref. 2 from analysis of the angular spectra of disturbed beams: the spatial spectrum of fluctuations of the refractive index in the engine jet should include, along with the von Karman spectral function, an additional function that describes the increased contribution of the high-frequency components.

In Ref. 2 it was shown that the laser beam with the wavelength λ = 1.06 μm is almost insensitive to the effect of this high-frequency component of the spectrum of inhomogeneities, whereas its contribution to widening of the angular intensity distribution is decisive for the beam with λ = 0.53 μm. Using these arguments, let us analyze the results obtained.

Table 1. Variance of centroid displacements for a laser beam (σ_x, σ_y, in μrad) and the averaged angular halfwidth at the level 1/e I_{max} (θ_x, θ_y, in μrad) in the horizontal and vertical directions

Experiment	Beam diameter, mm	σ_x	σ_y	θ_x	θ_y
	λ = 1.06 μm				
Engine off	30	11	11	26	26
	10	9	11	31	42
Engine on	30	7	10	24	26
	10	7	8	29	39
	λ = 0.53 μm				
Engine off	30	10	23	22	22
	10	12	13	39	46
Engine on	30	16	25	26	29
	10	11	12	36	39

Table 2. RMS angular displacements of the beam centroid in the horizontal and vertical directions: σ_x and σ_y, μrad

Beam parameters		λ = 1.06 μm		λ = 0.53 μm	
		Diameter, mm			
		30	10	30	10
φ = 90°	σ_x	50 ± 5	85 ± 10	115 ± 5	175 ± 20
	σ_y	80 ± 10	130 ± 15	135 ± 5	195 ± 20
φ = 45°	σ_x	85 ± 10	130 ± 15	175 ± 20	245 ± 25
	σ_y	115 ± 10	160 ± 15	220 ± 20	295 ± 30
φ = 10°	σ_x	130 ± 15	175 ± 20	245 ± 25	–
	σ_y	145 ± 15	190 ± 20	275 ± 30	–

Let us use the known approximate equation for the variance of angular tilts of the wave passed through a turbulent medium (the tilts are observed in the focal plane of the objective)[3]:

$$\sigma^2 = \frac{1}{\pi k^2 a^2} \int_0^{2a} \left[D''(\rho) + \frac{D'(\rho)}{\rho} \right] \times$$

$$\times \left[\arccos\left(\frac{\rho}{2a}\right) - \left(\frac{\rho}{2a}\right)\sqrt{1 - \frac{\rho^2}{4a^2}} \right] \rho d\rho, \qquad (1)$$

where a is the objective radius; D'' and D' are the second and first derivatives of the structure phase function of the plane wave; $k = 2\pi/\lambda$ is the wave number.

Equation (1) can be used for simulating experimental situations, when the laser beam crosses the jet, because its cross size in this case is markedly different at the exit from the jet. As is seen from Table 1, additional angular displacements of the beam on the atmospheric path behind the jet are negligibly small. Therefore, in the focal plane of the receiving objective at the complete interception of radiation, the beam wandering caused just by the turbulent layer is analyzed. Under these conditions, Eq. (1), if the parameter a in it is believed to be the beam radius, allows us to estimate the variance of displacements of the beam centroid.

In Ref. 2, the following analytical equation was derived for the structure phase function of the wave disturbed by the turbulent layer (this equation accounts for the anisotropy of the spectrum of inhomogeneities of the medium in the region of the outer scale of turbulence):

$$D(x, y) = 2.92\, k^2 L_t C_n^2 \left\{ (x^2 + y^2)^{5/6} - 0.2 \times \right.$$

$$\left. \times \left[\frac{3\left[\left(\frac{L_{0x}}{L_{0y}}\right)^2 x^2 + \left(\frac{L_{0y}}{L_{0x}}\right)^2 y^2\right] + x^2 + y^2}{(L_{0x} L_{0y})^{1/6}} \right] \right\}. \qquad (2)$$

Here the parameters L_{0x} and L_{0y} characterize the outer scale of turbulence in the directions OX and OY, respectively.

Using Eqs. (1) and (2), we can easily derive the equations for the components of the variance of the beam centroid displacements that account for the anisotropy of the turbulent layer:

$$\sigma_x^2(a) = 2.84 C_n^2 L_t (2a)^{-1/3} \times$$

$$\times \left\{ 1 - 0.1 \left(\frac{4a^2}{L_{0x} L_{0y}} \right)^{1/6} \left[3\left(\frac{L_{0x}}{L_{0y}}\right)^2 + 1 \right] \right\}, \qquad (3)$$

$$\sigma_y^2(a) = 2.84 C_n^2 L_t (2a)^{-1/3} \times$$

$$\times \left\{ 1 - 0.1 \left(\frac{4a^2}{L_{0x} L_{0y}} \right)^{1/6} \left[3\left(\frac{L_{0y}}{L_{0x}}\right)^2 + 1 \right] \right\}. \qquad (4)$$

Because of the approximations accepted in deriving the Eq. (2), these equations are valid only for narrow beams ($a \ll L_{0x}$, L_{0y}) and not very strong anisotropy.

The structure function (2) corresponds to the von Karman spectrum of refractive index fluctuations in the approximation allowing, in the first order, for the finiteness of the outer scale of turbulence. As was mentioned above, this form of the spectrum, according to Ref. 2, rather adequately simulates the angular intensity distribution of the laser beam ($\lambda = 1.06$ μm) disturbed by the engine jet. It could be expected that wandering of the 1.06-μm beam is also described by Eqs. (3) and (4). In Ref. 2, it was shown that to simulate the experimentally observed angular spectrum of laser radiation with the wavelength $\lambda = 0.53$ μm, disturbed by the turbulent jet, the von Karman spectrum of inhomogeneities should be supplemented with the function increasing the contribution of high-frequency components.

It was also noted that the angular distribution of 1.06-μm beam is practically insensitive to this modification of the spectrum. This is connected both with the peculiarities in the manifestation of small-scale inhomogeneities on the beam path in the angular spectrum of radiation at different wavelengths and with the peculiarities of the technique of recording the beam images in the experiment. The angles of light scattering at inhomogeneities are proportional to the radiation wavelength. Therefore, in the angular intensity distribution of the 1.06-μm beam, the high-frequency components of the spectrum of refractive index fluctuations contribute only to the far wings of the distribution, whereas in the angular spectrum of the 0.53-μm radiation these components widen the core of the intensity distribution.

Besides, when recording fluctuating images of the beam, by selecting the proper attenuators, the CCD camera is tuned in such a way that the brightest speckles of images are within its dynamic range (256 grades). As a result, the brightness of the low-intensity wings decreases down to the level of background and camera noise that is subtracted at processing of the readouts. Consequently, the wings are practically not recorded in the case of radiation at $\lambda = 1.06$ μm. In this connection, the contribution of the high-frequency portion of the spectrum to the beam wandering is only noticeable in the case of radiation with $\lambda = 0.53$ μm.

In Ref. 2, the agreement between the experimental and theoretically calculated angular spectra of radiation at different wavelengths was achieved with the use of both the multiscale high-frequency spectral function and the function of the single-scale type. Let us write the corresponding structure phase functions for both of these versions:

$$D_1(r) = 2.77\, k^2 C_{n1}^2 L_t \times$$

$$\times \left[\frac{\pi L_{0s}^{5/3}}{\Gamma(1/6)} - (r L_{0s}/2)^{5/6} K_{-5/6}(r/L_s) \right], \quad (5)$$

$$D_2(r) = 2\sqrt{\pi}\, k^2 \langle n_1^2 \rangle L_t L_{0s} \{1 - \exp[-(r/L_s)^2]\}, \quad (6)$$

where L_s is the outer scale of inhomogeneities for the multiscale spectrum or the characteristic size of the inhomogeneities for the single-scale spectrum; n_1 is the amplitude of inhomogeneities of the refractive index.

From Eq. (2) using Eq. (5) or (6), we can easily calculate the corresponding variances of the beam wandering σ_1^2 and σ_2^2 caused by the contribution of the additional spectral function of some or other form. To calculate the variance of the beam wandering σ_t^2 under the effect of the whole spectrum of inhomogeneities in the jet given by σ^2 and σ_1^2 (or σ_2^2), we used statistical averaging in deriving the Eq. (2). Therefore, it is necessary to reveal how strong is the correlation between the angular displacements caused by the main (von Karman spectrum) and additional high-frequency components of the spectrum of the inhomogeneities. The experimental technique used here allows us to easily determine the correlation of angular displacements between the beams with $\lambda = 1.06$ and 0.53 μm from the displacements measured within every laser pulse. Table 3 gives the coefficients of correlation between the components of the vector of angular displacements σ_x and σ_y calculated from the experimental data.

It is seen that for the beams having different wavelengths, the correlation of angular displacements of the beam centroid $K_\sigma(\lambda_1, \lambda_2)$ is close to 100%. For a comparison, we present also the calculated correlation coefficients for the orthogonal components of the vector of angular displacements $K_\lambda(\sigma_x, \sigma_y)$, which demonstrate almost full absence of the correlation. Since measured displacements of the 1.06-μm beam are caused only by the von Karman spectrum and those of the 0.53-μm beam are caused by both components, the contributions from them to the spectrum of inhomogeneities correlate as well. Consequently, the vector of rms angular displacements of the 0.53-μm beam should be determined from the equation: $\sigma_t = \sigma + \sigma_{1(2)}$.

The agreement between the experimentally measured components of the vector of rms angular displacements of a narrow laser beam (10 mm in diameter) and those calculated based on the equations derived for the experimental situations analyzed here (φ = 90 and 45°) is demonstrated in Fig. 1.

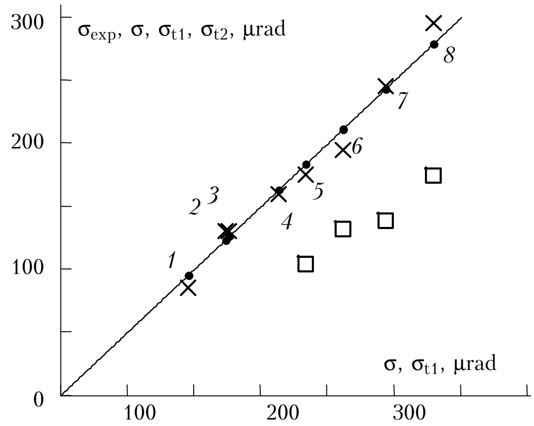

Fig. 1. Calculated and experimental (×) values of the components σ_x and σ_y of the vector of rms angular displacements of the centroid of the disturbed laser beam: experiment σ_{exp} (×), calculation (●) of σ (*1–4*) and σ_{t1} (*5–8*), calculation (□) of σ_{t2} (*5–8*); *x*- and *y*-components, $\lambda = 1.06$ μm, $\varphi = 90°$ (*1* and *2*); *x*- and *y*-components, $\lambda = 1.06$ μm, $\varphi = 45°$ (*3* and *4*); *x*- and *y*-components, $\lambda = 0.53$ μm, $\varphi = 90°$ (*5* and *6*); *x*- and *y*-components, $\lambda = 0.53$ μm, $\varphi = 45°$ (*7* and *8*).

The value of the angular displacement calculated with the use of the von Karman spectrum for $\lambda = 1.06$ μm (σ) and its combination with the multiscale high-frequency spectral function for $\lambda = 0.53$ μm ($\sigma_{t1} = \sigma + \sigma_1$) is plotted along the abscissa. The ordinate gives the experimental σ_{exp} and calculated $\sigma_{t1} = \sigma + \sigma_1$ and $\sigma_{t2} = \sigma + \sigma_2$ values for $\lambda = 0.53$ μm with the use of the combination of the von Karman spectrum and the single-scale high-frequency component. In calculations, we used the following values of the parameters of the turbulent layer $C_n^2 = 1.5\cdot10^{-9}$ m$^{-2/3}$, $L_{0y} = 0.7$ m, $C_{n_1}^2 = 1.2\cdot10^{-8}$, $\langle n_1^2 \rangle = 2\cdot10^{-11}$, and $L_s = 1$ mm, which were determined in Ref. 2 from the comparison of the calculated and experimental angular spectra of the disturbed beams. The value $L_{0x} = 0.32$ m chosen when calculating wandering from the condition of closeness to the experiment differs somewhat from $L_{0x} = 0.35$ m obtained in Ref. 2.

Table 3. Coefficients of correlation between the components of the vector of angular displacements of the beam centroid at the wavelengths λ_1 =1.06 μm and λ_2 =0.53 μm

Correlation coefficient	Intersection angle φ, deg					
	90		45		10	
	Diameter, mm					
	30	10	30	10	30	10
$K_{\sigma x}(\lambda_1, \lambda_2)$	1	0.96	0.93	1	1	1
$K_{\sigma y}(\lambda_1, \lambda_2)$	1	0.85	0.97	1	0.96	1
$K_{\lambda 1}(\sigma_x, \sigma_y)$	−0.2	−0.01	−0.03	0.05	−0.06	−0.09
$K_{\lambda 2}(\sigma_x, \sigma_y)$	−0.23	0.003	−0.05	0.001	0.09	0.16

It is seen from Fig. 1 that calculated σ_{t1} agrees well with the experimental results, whereas the values of σ_{t2} are strongly underestimated. In other words, the amplitude of wandering of the 0.53-μm beam strongly depends on the shape of the high-frequency spectral component and, consequently, analysis of the beam wandering allows this shape to be specified. It is very important, because in Ref. 2 it was shown that the angular spectrum of the laser beams disturbed by the jet is practically independent of the high-frequency spectral function.

Thus, we have measured and studied experimentally random wandering of laser beams intersecting a turbo-engine jet at different angles to its axis: $\varphi = 90$, 45, and 10°. The random displacements of the beam of 10 mm in diameter exceed the amplitude of the displacements of a 30-mm beam at $\lambda = 1.06$ and 0.53 and increase as the intersection angle decreases. The strong wavelength-dependence of the amplitude of the beam wandering was revealed. The rms angular centroid displacements of the beam with the wavelength $\lambda = 0.53$ μm are roughly twice as large as those of the beam with $\lambda = 1.06$ μm under the same experimental conditions. It was also found that the azimuth asymmetry exists in the angular displacements of the beam: the variance of the displacements in the direction OY orthogonal to the jet axis exceeds that in the orthogonal direction OX.

The applied technique of simultaneous recording of the images of the beams with different wavelengths in every pulse allowed us to determine the correlation between the components of the vector of angular displacements of the beams with $\lambda = 0.53$ and 1.06 μm. It was shown that the correlation of the same components of the vector of angular displacements of the beam centroid is close to 100% for the beams with different wavelengths. At the same time, no correlation is practically observed for the orthogonal components of the vector of angular displacements of the same beam.

The equations have been derived for the variance of the random angular displacements of a narrow laser beam having passed through a thin anisotropic turbulent layer characterized by the von Karman spectrum of the refractive index fluctuations. The analysis has shown that to explain the spectral dependence of wandering of the beam disturbed by the jet, the von Karman spectrum should be supplemented with an additional high-frequency spectral function and the peculiarities of recording the fluctuating beam images should be taken into account. With the allowance for the determined coefficient of correlation between angular displacements of the beams with different wavelengths, the derived equations allow the experimental results to be simulated for the case of a narrow beam.

It follows from calculations that the amplitude of angular displacements of the beam with the wavelength $\lambda = 0.53$ μm strongly depends on the shape of the additional high-frequency component of the spectrum of refractive index fluctuations – the experimental data correspond to the values calculated based on the function of the multiscale type, whereas the single-scale function strongly underestimates the angular displacements of the beam.

Thus, the analysis of wandering of the beams disturbed by the turbo-engine jet, unlike the analysis of their averaged angular intensity distributions,[2] allows us to specify the shape of the spatial spectrum of inhomogeneities in the jet, what is necessary for the adequate simulation of the process of radiation propagation.

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