Rejection of quasideterministic random processes when measuring phase difference in optical systems for atmospheric studies

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It is shown that when measuring the phase difference of oscillations under nonstationary conditions in systems for optical investigation of the atmosphere and ocean, separation of the useful signal from a mixture with a nonstationary fluctuating process having the $1/f$ type spectrum is required. Rejection algorithms for quasideterministic nonstationary current average processes with the $1/f$ type spectrum are proposed and examined. It is found that at rejection of fluctuating processes with the $1/f$ type spectrum under nonstationary conditions, detection of harmonic phase signals with an amplitude of 10^{-7} rad is possible under long-term relative instability of laser frequency up to 10^{-9} and short-term relative instability less than 10^{-24}.

Methods of optical investigation of the atmosphere and ocean employ measurements of phase difference between the measurement and reference oscillations.[1,2] Instability of oscillation frequencies leads to random phase changes giving rise to nonstationary fluctuation processes.[3] The frequency instability cannot be neglected in precision measurement systems, when the relative frequency instability is close to 10^{-9}, and the phase difference signal $\varphi_s(t)$ is less than 10^{-6} rad, as in laser detectors of gravitational waves. The problem of measurement of $\varphi_s(t)$ in detectors of gravitational waves is yet incompletely solved. This problem is very important and especially urgent when operating under real conditions of nonstationary fluctuations of the laser frequency.[4] The objective of this paper is to estimate parameters of $\varphi_s(t)$ under conditions of nonstationary fluctuations. Toward this end, we have to solve the problem of separating $\varphi_s(t)$ against the background of a slowly varying (ultra-low-frequency[5]) nonstationary random process of a fluctuation variation in time of the phase difference

$$\Phi(t) = \omega_m(t)t + \varphi_m - [\omega_{ref}(t)t + \varphi_{ref}] = \\ = \omega_{av}\, t + \Delta\omega(t)t + \varphi, \qquad (1)$$

where ω_{av} is the average value of the frequency difference between the measurement $\omega_m(t)$ and reference $\omega_{ref}(t)$ signals; $\Delta\omega(t)$ is the temporal variation of the frequency difference; $\varphi = \varphi_m - \varphi_{ref}$. In the practically important case of a highly stable frequency of sources of the measurement and reference signals, when $\Delta\omega(t)/\omega_{av} < 10^{-9}$, the random process $\Phi(t)$ has a spectral density of the form $1/f^{\alpha}$ (Ref. 3). Processes with the spectrum $1/f^{\alpha}$ have a mean value $M(t)$ varying with time:

$$\Phi(t) = M(t) + \varepsilon(t), \qquad (2)$$

where $\varepsilon(t)$ is a stationary random process with zero mean. Therefore, estimation of parameters of the phase difference signals, $\varphi_s(t)$, under conditions of nonstationary fluctuations can be reduced to separation of the useful signal from the additive mixture with the process having the spectrum of $1/f^{\alpha}$ type with a nonstationary mean value.

Effective means for analysis of nonstationary random processes of the form (2) are based on their quasideterministic representation with the use of physically and mathematically justified optimal basic functions. The optimal quasideterministic representation of $M(t)$ for the processes with the $1/f^{\alpha}$ spectrum has the form of polynomials with non-integer power Ψ_k of time and the random coefficients a_k (Ref. 5):

$$M(t) \approx \sum_{k=1}^{m} a_k\, t^{\Psi k}. \qquad (3)$$

The quasideterministic representation of nonstationary random processes allows them to be analyzed by the apparatus of random processes with stationary increments of the integer and fractional order, as well as the methods of generalized spectrum analysis based on the apparatus of fractional-order differentiation and integration[6] and tools for signal rejection according to its shape.[7] The algorithm for rejection of $M(t)$ can be realized based on the fractional-order differentiation operators Ψ_k or on the use of final stationary increments.[5,6] At $\max(\Psi_k,\ k = 1...m) \le n$, the stationarity of the nth increment can be achieved only approximately, and the error of approximation is estimated by the deviation of the nth derivative $M^{(n)}(t)$ or the stationary increment of the nth order

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$$\Delta_\tau^{(n)} M(t) = \sum_{e=0}^{n} (-1)^e C_n^e\, M(t - e\tau) \quad (4)$$

from a constant value, τ is the increment interval.

The *a priori* information about the $\varphi_s(t)$ shape in laser detectors of gravitational waves allows it to be represented in the form $\varphi_s(t) = f_0 \sin(2\pi t/T_0 + \varphi_0)$ with unknown f_0 and φ_0, but the *a priori* known period T_0 of a gravitational wave. The range τ for each stationary increment in this case should be taken equal or multiple to a half period T_0 to diminish distortions in the shape of the useful signal $\varphi_s(t)$. Performing current averaging procedures (low-frequency filtering) for signals after each increment gives a possibility to reduce the effect of the stationary random process $\varepsilon(t)$ in rejection of the nonstationary mean $M(t)$.

It should be emphasized that an important advantage of the rejection is a suppression of the quasideterministic random process $M(t)$ of the form (3) at arbitrary coefficients a_k, $k = 1, 2, \ldots, n$ (Ref. 8). Application of this approach to rejection of nonstationary quasideterministic processes requires a duration no shorter than $(n/2 + 1 + K)\, T_0$, where K is the minimal number of signal periods needed for estimation of φ_0 against the background of a stationary random process with a needed level of reliability.

In this paper, we estimate experimentally the capabilities of the proposed rejection of quasideterministic processes. Estimation was performed for harmonic signals of the phase variation $\varphi_s(t)$ in laser detectors of gravitational waves with the amplitude $f_0 \approx 10^{-7}$ rad. It was assumed that along with the quasideterministic process $M(t)$ of the form (3), random and oscillating noise signals are present. Fluctuations of the beat frequency of the reference and measurement signals in simulation were formed as sums of quasideterministic drifts of $\omega_m(t)$ described by Eq. (3) at $\Psi_k = k$, $k = 1 \ldots n$ (in the experiment $n \le 5$), periodic oscillations $\omega_{osc}(t) = \Delta\omega_{osc} \sin(2\pi t/T_{osc} + \varphi_{osc})$, random variations $\omega_{ran}(t) = Q_\omega\, \mathrm{rnd}(1)$, and random phase variations $\varphi_{ran}(t) = Q_\varphi\, \mathrm{rnd}(1)$. A full model of the nonstationary process $\Phi(t)$ had the form

$$\Phi(t) = \{\omega_m(t) + \omega_{osc}(t) + \omega_{ran}(t)\}\, t + \varphi_{ran}(t). \quad (5)$$

Simulation of $\omega_{ran}(t)$ and $\varphi_{ran}(t)$ was performed in the MathCad environment with random number generators rnd(1) having the uniform distribution law in the interval from 0 to 1. Time diagrams illustrating separation of the signal $\varphi_s(t) = f_0 \sin(2\pi t/T_0 + \varphi_0)$ from the additive mixture with $\Phi(t)$ of the form (5) are depicted in Figs. 1–3. Figure 1 shows, as an example, one version of the nonstationary phase difference signal $[\Phi(t) + \varphi_s(t)]$.

Signals obtained as a result of application of the second and third increments of the form (4) to $[\Phi(t) + \varphi_s(t)]$ are plotted in Fig. 2. The increment interval τ is equal to 500 s.

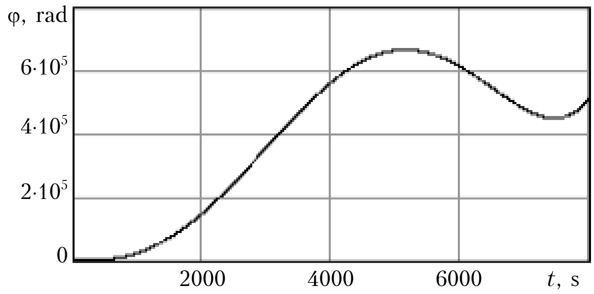

Fig. 1. Time diagram of the signal $[\Phi(t) + \varphi_s(t)]$.

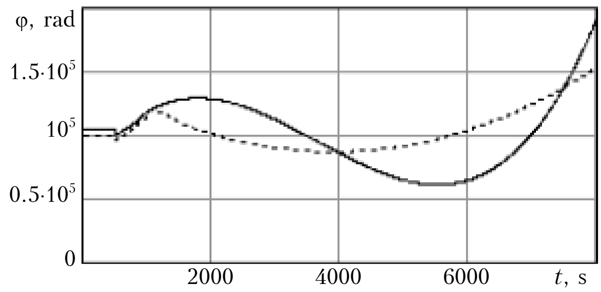

Fig. 2. Signals obtained as a result of application of the second (solid curve) and third (dashed curve) increments.

Figure 3 depicts the diagram of the signal obtained as a result of application of the sixth stationary increment to $[\Phi(t) + \varphi_s(t)]$ and, for comparison, $\varphi_s(t)$ with $f_0 = 10^{-7}$ rad.

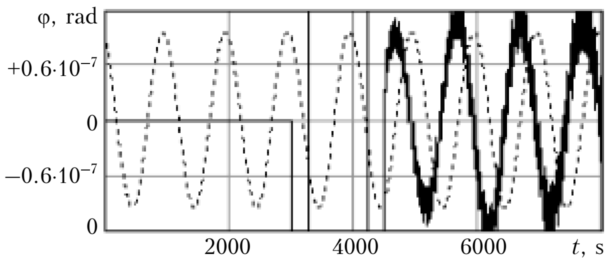

Fig. 3. Time diagrams obtained as a result of the sixth stationary increment (solid curve) and of $\varphi_s(t)$ (dashed curve).

For convenience, the signal obtained as a result of application of the sixth stationary increment is shown depressed 16 times. It can be seen that the sequential increase of the increment order allows a suppression of the nonstationary current average signal in $[\Phi(t) + \varphi_s(t)]$ and amplification of the useful signal $\varphi_s(t)$.

The results of experimental estimation of the acceptable values of $\Delta\omega_{osc}$, T_{osc}, Q_ω, and Q_φ when separating $\varphi_s(t)$ with $f_0 \approx 10^{-7}$ rad are tabulated below.

It should be noted that the acceptable values of $\Delta\omega_{osc}$, T_{osc}, Q_ω, and Q_φ can be increased by increasing the minimal number of signal periods K needed for estimation of f_0 with the required level of reliability. In our case $K \approx 3$.

Acceptable parameters of fluctuations of the beat frequency of nonstationary signals

$\Delta\omega_{osc} T_0$	T_{osc}/T_0	Q_ω	Q_φ
$\leq 3 \cdot 10^{-2}$	$\leq 10^4$	$\leq 10^{-10}$	$\leq 10^{-7}$

The absolute long-term instability of the beat frequency (comparable with the duration of analysis T) described by $M(t)$ of the form (3) can reach, depending on the coefficients a_k, from 10^3 to 10^5 Hz. At the same time, the short-term absolute instability of the beat frequency (at the frequency $\gg 2\pi/T_0$) should not exceed 10^{-10} Hz, and the mean value of these short-term instabilities should be close to zero.[4]

The obtained estimates allow us to determine the requirements to instability of frequencies of the measurement and reference signals. At the frequency of the measurement optical signal $\omega_m(t) \approx 10^{14}$ Hz, the acceptable relative long-term instability of the laser radiation frequency should not exceed 10^{-9}, while the relative short-term instability should be less than 10^{-24}. Fulfillment of the last requirement is a very complicated technical problem. A possible way of making the requirements to the relative short-term frequency instability less rigid may be quasideterministic description of its low-frequency part. The quasideterministic representation of this component of the frequency instability will allow an application of the considered methods of the signal rejection according to its shape to decrease the effect of short-term instabilities of the laser radiation frequency.

Based on the above-said, it is possible to draw the following conclusions:

1. Measurement of the phase difference of oscillations under conditions of nonstationary fluctuations of the laser frequency requires a separation of the useful signal from the additive mixture with a nonstationary random process having the spectrum of a $1/f^\alpha$ type with a nonstationary mean.

2. Quasideterministic representation of nonstationary random processes with the spectrum of the $1/f^\alpha$ type allows them to be analyzed by the apparatus of random processes with stationary increments, as well as the methods of generalized spectrum analysis based on signal rejection according to its shape.

3. When using rejection of nonstationary fluctuation signals, parameters of harmonic signals of phase difference variation with the amplitude $f_0 \approx 10^{-7}$ rad can be estimated, if the long-term relative instability of the laser radiation frequency is less than 10^{-9} and the short-term relative instability of the radiation frequency does not exceed 10^{-24} Hz.

References

1. E.D. Hinkley, ed., *Laser Monitoring of the Atmosphere* (Springer Verlag, New York, 1976).
2. V.E. Zuev, ed., *Laser Sensing of the Atmosphere and the Underlying Surface* (Nauka, Novosibirsk, 1991), 147 pp.
3. A.N. Malakhov, *Fluctuations in Self-Oscillating Systems* (Nauka, Moscow, 1968), 660 pp.
4. V.V. Afanasiev and G.I. Il'in, in: *Proc. of the IX Joint International Symposium on Atmospheric and Ocean Optics. Atmospheric Physics* (Tomsk, 2002), p. 114.
5. V.V. Afanas'ev, Izv. Vyssh. Uchebn. Zaved., Ser. Radioelektron. **27**, No. 11, 38–42 (1984).
6. V.V. Afanas'ev, Izv. Vyssh. Uchebn. Zaved., Ser. Radioelektron. **29**, No. 1, 95–97 (1986).
7. V.V. Afanas'ev and V.S. Knyazev, "*Rejection filter*," USSR Author's Certificate No. 723759, Bull. Izobret. No. 11, 1980.
8. V.V. Afanas'ev and G.I. Il'in, in: *Electronic Industry*. Collection of papers. Supplement to Vestn. Kazan. Gos. Tekhn. Un-ta, Issue 4(25), 80–88 (2002).