How to reach likeness of evolutions in behavior models of two ring optical systems?

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The concept of isodynamism (equivalence) of the influence of parameters on the evolution of a dynamic system is introduced. The method of isodynamism detection is demonstrated for the case of a nonlinear ring interferometer, which is used in atmospheric adaptive optics. The concept of isodynamism allows constructing an algorithm providing for likeness of the evolutions of two systems. This approach, for example, suggests a method for compensation for optical vortex influence on the processes in a nonlinear ring interferometer.

Introduction

Nonlinear ring optical systems are of interest in connection with the development of methods and devices for adaptive beaming and imaging in the atmosphere.[1] A version of ring systems is a two-channel laser system (for example, bichromatic emitters) used in spectroscopy and remote sensing of the atmosphere.[2]

Starting from the early 1990s, nonlinear ring optical systems have been considered as prototype of information processing devices. The investigations by Ikeda and then by Akhmanov and Vorontsov, as well as other authors showed that such systems can serve generators of regular optical structures and optical turbulence in the laser beam cross section.[3–5] In a ring interferometer including an element with nonlinear-optical properties, not only temporal and spatiotemporal chaos,[3–6] but also spatial one[7] are possible.

Because of the complexity of processes in nonlinear ring optical systems, an important way to theoretically study them is, naturally, computer simulation. However the large number of parameters $\mathbf{p}(\mathbf{r}, t)$ [and, often, dynamic variables $\mathbf{U}(\mathbf{r}, t)$] significantly complicates revealing the regularities of the influence of $\mathbf{p}(\mathbf{r}, t)$ – all together and separately – on the evolution of the variables. It is an urgent question how to reach likeness of evolution of two optical systems with similar structures, but different values of the parameters? The trial-and-error method realized on a computer proves to be inefficient and does not guarantee obtaining the sought regularities.

Consequently, an analytical approach should be developed. In this paper, it is proposed to develop one of its versions starting from analysis of two model dynamic systems together with the so-called relation of isodynamism (equivalence) of their evolution. Here the relation of isodynamism of evolutions is some operation of comparing the evolutions of two systems.[8]

Concept of isodynamism of evolutions of two systems

The idea of equivalence can be illustrated if we look at the Ohm's law. If some particular values of current i_i and resistance r_i satisfy the relations $i_1r_1 = i_2r_2$, $i_1 \neq i_2$, $r_1 \neq r_2$, then the values (i_1, r_1) and (r_2, i_2) are equivalent from the standpoint of the effect on the voltage u across each of the two different conductors, because $u = i_1r_1 = i_2r_2$. Complementing the traditional sign of equality with a subscript, we get the designation $=_u$, which will mean "equivalent from the viewpoint of the effect on u". Then in these designations we have $(i_1, r_1) =_u (r_2, i_2)$. It can be easily seen that the following is valid:

$$(i_1, r_1) =_u (r_2, i_2) \Leftrightarrow i_1r_1 = i_2r_2,$$

or, in a more general form,

$$(i_1, r_1) =_F (r_2, i_2) \Leftrightarrow F(i_1, r_1, i_2, r_2) = 0,$$

where

$$F(i_1, r_1, i_2, r_2) = i_1r_1 - i_2r_2.$$

It is obvious that in this example for any fixed value (i_1, r_1) we can find arbitrarily many different values (r_2, i_2) such that $(i_1, r_1) =_F (r_2, i_2)$. That is, equivalent (in the above meaning) values of current i and resistance r form some set $M(\text{const}_i) = \{i, r : ir = \text{const}_i\}$. The sets $M(\text{const}_k)$ and $M(\text{const}_j)$ obviously do not overlap, if $\text{const}_k \neq \text{const}_j$, and at $\text{const}_k = \text{const}_j$ they coincide: $M(\text{const}_k) = M(\text{const}_j)$.

Now let us generalize the above-said in application to the equivalence of evolution of two systems with similar structure and dynamic variables $U_{i,j}(\mathbf{r}, t) \in \mathbf{U}_i(\mathbf{r}, t) \equiv \{U_{i,j}(\mathbf{r}, t)\}$ and parameters $\mathbf{p}_i(\mathbf{r}, t) \equiv \{p_{i,k}\}$, where $i \in \{1; 2\}$ enumerates the systems, $j \in \{1; ...; m_i\}$, $k \in \{1; ...; N_i\}$, m_i and N_i are the numbers of dynamic variables and parameters for the ith system.

The evolutions of two systems will be called *equivalent* in the meaning $\mathbf{F}$, if the relation of equivalence of the evolutions is fulfilled:

$$\mathbf{F}[\mathbf{r}, t, \mathbf{U}_1(\mathbf{r}, t), \mathbf{U}_1(\mathbf{r}, 0), \mathbf{U}_2(\mathbf{r}, t), \mathbf{U}_2(\mathbf{r}, 0), \mathbf{p}_1(\mathbf{r}, t), \mathbf{p}_2(\mathbf{r}, t)] \approx 0, \quad (1)$$

where $\mathbf{F}[...] \equiv \{F_1[...]; ...; F_N[...]\}$ is some vector function; $\mathbf{U}_1(\mathbf{r}, 0)$ and $\mathbf{U}_2(\mathbf{r}, 0)$ are the initial conditions. In the general case, the arguments in Eq. (1)

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$[\mathbf{U}_1(\mathbf{r}, t), \mathbf{U}_1(\mathbf{r}, 0), \mathbf{U}_2(\mathbf{r}, t), \mathbf{U}_2(\mathbf{r}, 0), \mathbf{p}_1(\mathbf{r}, t), \mathbf{p}_2(\mathbf{r}, t)]$

should be understood as full spatiotemporal realizations of the functions, rather than their individual values $\mathbf{U}_1$, $\mathbf{U}_2$, $\mathbf{p}_1$, $\mathbf{p}_2$ at a point $(\mathbf{r}, t)$. Relation (1) in a particular case can take the meaning of the relation of identity of the evolutions: $\mathbf{U}_1(\mathbf{r}, t) - \mathbf{U}_2(\mathbf{r}, t) = 0$.

We shall consider the values $[\mathbf{U}_1(\mathbf{r}, 0), \mathbf{p}_1(\mathbf{r}, t)]$ and $[\mathbf{U}_2(\mathbf{r}, 0), \mathbf{p}_2(\mathbf{r}, t)]$ equivalent in the sense of the relation $\mathbf{F}[...] \approx 0$ chosen (expressing this fact symbolically: $[\mathbf{U}_1(\mathbf{r}, 0), \mathbf{p}_1(\mathbf{r}, t)] =_F [\mathbf{U}_2(\mathbf{r}, 0), \mathbf{p}_2(\mathbf{r}, t)]$), if the condition (1) is fulfilled and $\mathbf{p}_1(\mathbf{r}, t) \neq \mathbf{p}_2(\mathbf{r}, t)$.

The operation of comparison (1) of the evolutions of two systems is constructed having in mind the problem of reaching the likeness of their evolutions, that is, similar structures of the phase space and/or its separate basins, in particular, similar modes of system functioning.

The above-mentioned analysis of two models and the relation of equivalence of the evolutions consist in their interpretation as a system of equations. Now the unknown values are the values of (a) the vector $\mathbf{p}_1(\mathbf{r}, t)$ of the parameters under study and (b) the initial conditions $\mathbf{U}_1(\mathbf{r}, 0)$ for the first system. The values of $\mathbf{p}_1(\mathbf{r}, t)$ and $\mathbf{U}_1(\mathbf{r}, 0)$ can be expressed as functions of the parameters $\mathbf{p}_2(\mathbf{r}, t)$ and the initial conditions $\mathbf{U}_2(\mathbf{r}, 0)$ of the second system, that is, the solution is presented by the following dependences:

$$\mathbf{p}_1(\mathbf{r}', t') = \mathbf{f}_{p1}[\mathbf{r}, t, \mathbf{U}_2(\mathbf{r}, 0), \mathbf{p}_2(\mathbf{r}, t)];$$

$$\mathbf{U}_1(\mathbf{r}', 0) = \mathbf{f}_{u1}[\mathbf{r}, t, \mathbf{U}_2(\mathbf{r}, 0), \mathbf{p}_2(\mathbf{r}, t)] \qquad (2)$$

or

$$\mathbf{p}_2(\mathbf{r}', t') = \mathbf{f}_{p2}[\mathbf{r}, t, \mathbf{U}_1(\mathbf{r}, 0), \mathbf{p}_1(\mathbf{r}, t)];$$

$$\mathbf{U}_2(\mathbf{r}', 0) = \mathbf{f}_{u2}[\mathbf{r}, t, \mathbf{U}_1(\mathbf{r}, 0), \mathbf{p}_1(\mathbf{r}, t)].$$

It would be logical to call Eqs. (2) the relations of equivalence of the parameters of two systems (in the sense of equivalence of the evolutions (1)). Here the initial conditions can be understood as some parameter of the dynamic system, whose variation though being able to affect the system evolution, cannot give rise to bifurcations in it.

Below the procedure of revealing the equivalence properties is demonstrated as applied to the model of structure-forming processes in a nonlinear ring interferometer (NRI).

Nonlinear ring interferometer and model of the processes in it

The optical layout of a ring interferometer is shown in Fig. 1*a*, where *NM* stands for the nonlinear medium with the length *l*, *G* is a linear element that performs large-scale transformation of the field (for example, rotation in the plane *xOy*, beam compression, and so on); the mirrors M_1 and M_2 have the intensity reflection coefficients *R*, while the mirrors M_3 and M_4 have the unit reflection coefficient.

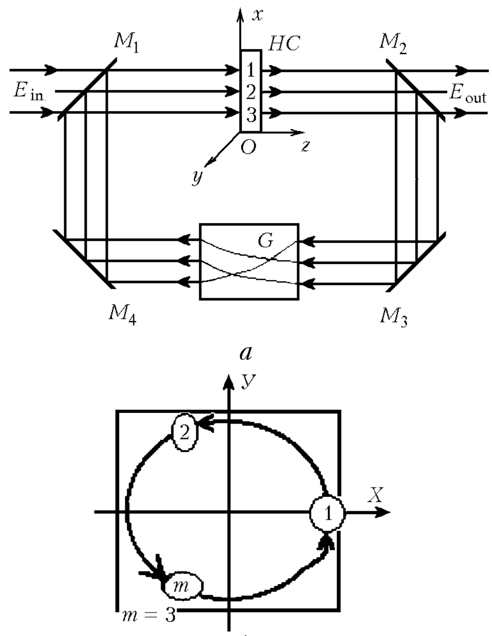


Fig. 1. Schematic layout of a nonlinear ring interferometer (*a*). The ray path (*b*) is depicted for the case of beam turn (by the element *G*) in the plane *xOy* by 120°.

Let the field at the entrance into the NRI consist of two components with the circular polarization (Fig. 2):

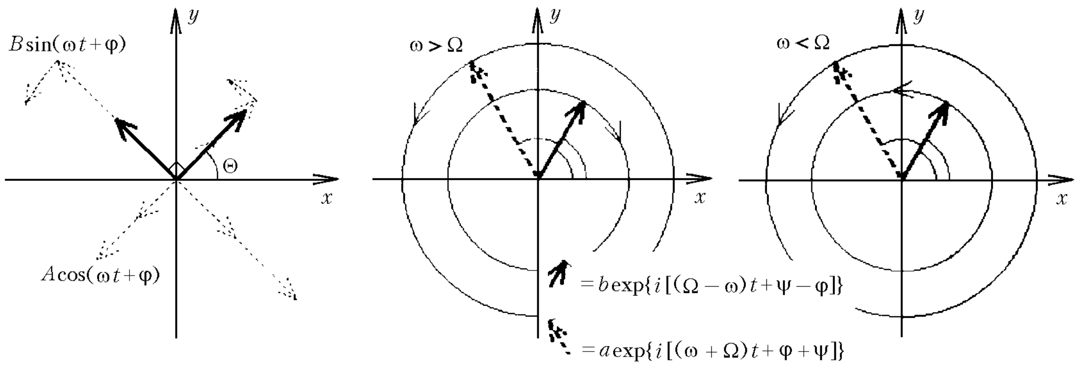
$$\mathbf{E}(\mathbf{r}, t) = \mathbf{e}[\Theta(\mathbf{r}, t)]\, A(\mathbf{r}, t) \cos[\omega t + \varphi(\mathbf{r}, t)] +$$
$$+ \mathbf{e}[\Theta(\mathbf{r}, t) + \pi/2]\, B(\mathbf{r}, t) \sin[\omega t + \varphi(\mathbf{r}, t)], \qquad (3)$$

where ω is the basic frequency of the light field; $\Theta(\mathbf{r}, t) = \psi(\mathbf{r}, t) + \Omega t$ is the angle between the vector $\mathbf{e}(\Theta)$ specifying the polarization direction and the axis Ox lying in the plane (xOy) of the beam cross section (Ω may be comparable with ω); Ω is the frequency of synchronous rotation of polarization vectors $\mathbf{e}$ lying in the plane referred to here as the polarization plane; $A(\mathbf{r}, t)$, $B(\mathbf{r}, t)$, $\varphi(\mathbf{r}, t)$, $\psi(\mathbf{r}, t)$ are the amplitude, phase, and position of the polarization plane of the light field that vary only slightly for the time $T = 2\pi/\omega$. The sign of Ω characterizes the direction of rotation of the polarization vectors $\mathbf{e}$.

If we introduce the following designations: $a(\mathbf{r}, t) \equiv [A(\mathbf{r}, t) + B(\mathbf{r}, t)]/2$, $b(\mathbf{r}, t) \equiv [A(\mathbf{r}, t) - B(\mathbf{r}, t)]/2$, then Eq. (3) can be expressed through projections of $\mathbf{E}(\mathbf{r}, t)$ as

$$E_x(\mathbf{r}, t) = a(\mathbf{r}, t) \cos[(\omega + \Omega)t + \varphi(\mathbf{r}, t) + \psi(\mathbf{r}, t)] +$$
$$+ b(\mathbf{r}, t)\cos[(\omega - \Omega)t + \varphi(\mathbf{r}, t) - \psi(\mathbf{r}, t)],$$

$$E_y(\mathbf{r}, t) = a(\mathbf{r}, t)\sin[(\omega + \Omega)t + \varphi(\mathbf{r}, t) + \psi(\mathbf{r}, t)] -$$
$$- b(\mathbf{r}, t)\sin[(\omega - \Omega)t + \varphi(\mathbf{r}, t) - \psi(\mathbf{r}, t)].$$



a b c

Fig. 2. Structure of a bichromatic optical radiation $E(r, t)$: bold lines correspond to the instantaneous state of the strength vectors, dashed lines show the possible states of these vectors at arbitrary $\omega t + \varphi(r, t)$.

Thus, at the interferometer's entrance we have a sum of two quasi-monochromatic fields with the amplitudes $a(\mathbf{r}, t)$ and $b(\mathbf{r}, t)$ and frequencies $\omega \pm \Omega$ of the circular polarization with different (at $\omega > \Omega$) or identical (at $\omega < \Omega$) directions of rotation (Figs. 2*b*, *c*). Here ω (or Ω at $\omega < \Omega$) has the meaning of a mean frequency, and 2Ω (2ω at $\omega < \Omega$) is the frequency interval between the field components. To reflect the specificity of the optical field under consideration, we deal with the parameter of bichromaticity (nonmonochromaticity according to Ref. 9) $q \equiv \Omega/\omega$.

Then the model of the dynamics of nonlinear phase change $U(\mathbf{r}, t)$ in the cross section of the laser beam in NRI in the large loss or single passage approximation can be represented by the following equations[9]:

$$\tau_n(\mathbf{r})\mathrm{d}U(\mathbf{r}, t)/\mathrm{d}t = D_e(\mathbf{r})\Delta U(\mathbf{r}, t) - U(\mathbf{r}, t) + f(\mathbf{r}, t);$$

$$f(\mathbf{r}, t) = n_2(\mathbf{r})lk\ an\langle \mathbf{E}^2_{\mathrm{nm}}(\mathbf{r}, t)\rangle_T =$$

$$= an\ n_2(\mathbf{r})lk[a^2_{\mathrm{nm}}(\mathbf{r}, t) + b^2_{\mathrm{nm}}(\mathbf{r}, t)] =$$

$$= K_{ab}(\mathbf{r}, t, \mathbf{r}) + pK_{ab}(\mathbf{r}', t - \tau, \mathbf{r}) + [\gamma(\mathbf{r}', t)/\sigma] \times$$

$$\times \{ K_a(\mathbf{r}, t, \mathbf{r}', t - \tau) \cos[(1 + q)\omega\tau + \varphi(\mathbf{r}, t) -$$

$$- \varphi(\mathbf{r}', t - \tau) + \psi(\mathbf{r}, t) - \psi(\mathbf{r}', t - \tau)] +$$

$$+ K_b(\mathbf{r}, t, \mathbf{r}', t - \tau) \cos[(1 - q)\omega\tau + \varphi(\mathbf{r}, t) - \varphi(\mathbf{r}', t - \tau) -$$

$$- \psi(\mathbf{r}, t) + \psi(\mathbf{r}', t - \tau)] \}. \quad (4)$$

Here $k = \omega/c$; $U(\mathbf{r}, t) \equiv \omega t_u(\mathbf{r}, t)$ is the nonlinear phase change;

$$\tau \equiv \tau(\mathbf{r}', t) = t_e(\mathbf{r}', t) + U(\mathbf{r}', t - t_e(\mathbf{r}', t))/\omega;$$

$$\gamma(\mathbf{r}', t) \equiv 2R\kappa(\mathbf{r}', t)C_n(\mathbf{r}'); \ p = 0$$

in the large loss approximation, but $p = [\gamma(\mathbf{r}', t)/\sigma/2]^2$ in the single passage approximation; the "mixed" (K_{ab}) and "partial" (K_a, K_b) nonlinearity parameters are

$$K_{ab}(\mathbf{r}, t, \mathbf{r}_n) \equiv$$

$$\equiv (1 - R)\ an\ n_2(\mathbf{r}_n)lk\ [a^2(\mathbf{r}, t) + b^2(\mathbf{r}, t)],$$

$$K_a(\mathbf{r}, t, \mathbf{r}', t - \tau) \equiv$$

$$\equiv (1 - R)\ an\ n_2(\mathbf{r})lk\ a(\mathbf{r}, t)a(\mathbf{r}', t - \tau),$$

$$K_b(\mathbf{r}, t, \mathbf{r}', t - \tau) \equiv$$

$$\equiv (1 - R)\ an\ n_2(\mathbf{r})lk\ b(\mathbf{r}, t)b(\mathbf{r}', t - \tau),$$

$an = 1$ or $an = 2$; $\Omega = 0$; $\psi = \mathrm{const}$.

The simulation practice shows[10] that variations of some parameters of NRI and radiation significantly affect the dynamics of the processes in NRI, that is, formation of regular structures $U(\mathbf{r}, t)$ or appearance of the deterministic chaos conditions. These parameters form the set

$$P \equiv P_0 = \{\Delta,\ q,\ \varphi(\mathbf{r}, t),\ \psi(\mathbf{r}, t),\ \nu,\ \omega t_e(\mathbf{r}, t),$$

$$K_{ab}(\mathbf{r}, t, \mathbf{r}_n),\ K_a(\mathbf{r}, t, \mathbf{r}', t'),\ K_b(\mathbf{r}, t, \mathbf{r}', t')\}.$$

Equivalence of the effect of parameters φ, ψ, ωt_e on the evolution of nonlinear phase change

In model (4) the parameters φ, ψ, t_e additively enter the arguments of the *cos* functions (the last one as a component of the delay time τ). In this connection it is logical to put the question: whether or not there are such sets of the values of φ_1, ψ_1, t_{e1} and φ_2, ψ_2, t_{e2}, non equal to each other, for which the evolution of the nonlinear phase change in the NRI nonlinear medium is identical? It is obvious that at transition from φ_1, ψ_1, t_{e1} to φ_2, ψ_2, t_{e2} the dynamics of $U(\mathbf{r}, t)$ keeps unchanged, if the values $\gamma(\mathbf{r}', t)/\sigma$, $K_a(\mathbf{r}, t, \mathbf{r}', t - \tau)$, $K_b(\mathbf{r}, t, \mathbf{r}', t - \tau)$, $K_{ab}(\mathbf{r}, t, \mathbf{r})$, $K_{ab}(\mathbf{r}', t - \tau, \mathbf{r})$ and the values of *cos* arguments in Eq. (4) do not change, that is,

$$\gamma_1(\mathbf{r}', t)/\sigma_1 = \gamma_2(\mathbf{r}', t)/\sigma_2,$$

$$K_{ab1}(\mathbf{r}, t, \mathbf{r}) = K_{ab2}(\mathbf{r}, t, \mathbf{r}),$$

$$K_{ab1}(\mathbf{r}', t - \tau_1, \mathbf{r}) = K_{ab2}(\mathbf{r}', t - \tau_2, \mathbf{r}),$$

$$K_{a1}(\mathbf{r}, t, \mathbf{r}', t - \tau_1) = K_{a2}(\mathbf{r}, t, \mathbf{r}', t - \tau_2),$$

$$K_{b1}(\mathbf{r}, t, \mathbf{r}', t - \tau_1) = K_{b2}(\mathbf{r}, t, \mathbf{r}', t - \tau_2),$$
$$(1 + q)\omega\tau_1 + \varphi_1(\mathbf{r}, t) - \varphi_1(\mathbf{r}', t - \tau_1) +$$
$$+ \psi_1(\mathbf{r}, t) - \psi_1(\mathbf{r}', t - \tau_1) =$$
$$= (1 + q)\omega\tau_2 + \varphi_2(\mathbf{r}, t) - \varphi_2(\mathbf{r}', t - \tau_2) +$$
$$+ \psi_2(\mathbf{r}, t) - \psi_2(\mathbf{r}', t - \tau_2) - 2\pi i,$$
$$(1 - q)\omega\tau_1 + \varphi_1(\mathbf{r}, t) - \varphi_1(\mathbf{r}', t - \tau_1) -$$
$$- \psi_1(\mathbf{r}, t) + \psi_1(\mathbf{r}', t - \tau_1) =$$
$$= (1 - q)\omega\tau_2 + \varphi_2(\mathbf{r}, t) - \varphi_2(\mathbf{r}', t - \tau_2) -$$
$$- \psi_2(\mathbf{r}, t) + \psi_2(\mathbf{r}', t - \tau_2) - 2\pi j, \quad (5)$$

where i, j are integer numbers.

Introduce now the following designations :

$$\delta t_e(\mathbf{r}', t) \equiv t_{e2}(\mathbf{r}', t) - t_{e1}(\mathbf{r}', t) = \tau_2 - \tau_1,$$
$$\delta\varphi(\mathbf{r}, t) \equiv \varphi_2(\mathbf{r}, t) - \varphi_1(\mathbf{r}, t),$$
$$\delta\psi(\mathbf{r}, t) \equiv \psi_2(\mathbf{r}, t) - \psi_1(\mathbf{r}, t).$$

Since

$$K_{ab}(\mathbf{r}, t, \mathbf{r}_n) \equiv (1 - R)\, an\, n_2(\mathbf{r}_n) lk\, [a^2(\mathbf{r}, t) + b^2(\mathbf{r}, t)],$$

taking into account the second equality in Eq. (5), it is reasonable to require that amplitudes of the fields are equal:

$$a_1(\mathbf{r}, t) = a_2(\mathbf{r}, t),\ b_1(\mathbf{r}, t) = b_2(\mathbf{r}, t).$$

Assume that $|\delta t_e(\mathbf{r}', t)| \le \delta t_{e\,\max}$, and $\delta t_{e\,\max}$ is so small that

$$a(\mathbf{r}, t) \approx a(\mathbf{r}, t + \delta t_{e\,\max}),\ b(\mathbf{r}, t) \approx b_2(\mathbf{r}, t + \delta t_{e\,\max}),$$
$$\varphi(\mathbf{r}, t) \approx \varphi(\mathbf{r}, t + \delta t_{e\,\max}),\ \psi(\mathbf{r}, t) \approx \psi(\mathbf{r}, t + \delta t_{e\,\max}).$$

For example, $\delta t_{e\,\max} = \pi l/\omega$. At small l this assumption is not a strong restriction, because in construction of the model (4) the approximation of slowly varying characteristics of the optical field was used.

The equalities 3–5 in Eqs. (5) are not correct in the general case. However, being smoothed down to approximate equalities and because

$$a(\mathbf{r}, t) \approx a(\mathbf{r}, t + \delta t_{e\,\max}),\ b(\mathbf{r}, t) \approx b_2(\mathbf{r}, t + \delta t_{e\,\max}),$$

they become valid. As

$$\varphi(\mathbf{r}, t) \approx \varphi(\mathbf{r}, t + \delta t_{e\,\max}),\ \psi(\mathbf{r}, t) \approx \psi(\mathbf{r}, t + \delta t_{e\,\max}),$$

two last equalities in Eq. (5) can be written in a more compact form:

$$\omega\delta t_e(\mathbf{r}', t) + \delta\varphi(\mathbf{r}, t) - \delta\varphi(\mathbf{r}', t - \tau_1) \approx \pi(i + j),$$
$$q\omega\delta t_e(\mathbf{r}', t) + \delta\psi(\mathbf{r}, t) - \delta\psi(\mathbf{r}', t - \tau_1) \approx \pi(i - j). \quad (6)$$

The equivalence of the effect of φ, ψ, t_e on the evolution of nonlinear phase change $U(\mathbf{r}, t)$ in the NRI is just expressed by the equalities (6) similar to the relations of equivalence of the parameters (2). So, if $\delta t_e(\mathbf{r}', t) \ne 0$, then to compensate for different values of $t_{e1}(\mathbf{r}', t)$ and $t_{e2}(\mathbf{r}', t)$ it is sufficient to modulate the input field in NRI (in addition to the space–time modulation of $\varphi_1(\mathbf{r}, t)$ and $\psi_1(\mathbf{r}, t)$) by the law

$$\delta\varphi(\mathbf{r}, t) = \pi(i + j) + \delta\varphi(\mathbf{r}', t - \tau_1) - \omega\delta t_e(\mathbf{r}', t),$$
$$\delta\psi(\mathbf{r}, t) = \pi(i - j) + \delta\psi(\mathbf{r}', t - \tau_1) - q\omega\delta t_e(\mathbf{r}', t)$$

or, what is the same, to provide for fulfillment of the equalities

$$\varphi_2(\mathbf{r}, t) = \pi(i + j) + \varphi_1(\mathbf{r}, t) +$$
$$+ \delta\varphi(\mathbf{r}', t - \tau_1) - \omega\delta t_e(\mathbf{r}', t),$$
$$\psi_2(\mathbf{r}, t) = \pi(i - j) + \psi_1(\mathbf{r}, t) +$$
$$+ \delta\psi(\mathbf{r}', t - \tau_1) - q\omega\delta t_e(\mathbf{r}', t). \quad (7)$$

Similarly, if $\delta\varphi(\mathbf{r}, t) \ne 0$, then for compensation it is sufficient to meet the conditions

$$\omega\delta t_e(\mathbf{r}', t) = \pi(i + j) + \delta\varphi(\mathbf{r}', t - \tau_1) - \delta\varphi(\mathbf{r}, t),$$
$$\delta\psi(\mathbf{r}, t) = \pi[(1 - q)i - (1 + q)j] + \delta\psi(\mathbf{r}', t -$$
$$- \tau_1) + q[\delta\varphi(\mathbf{r}, t) - \delta\varphi(\mathbf{r}', t - \tau_1)]. \quad (8)$$

When $\delta\varphi(\mathbf{r}, t) \ne 0$, one of the important cases is the presence of an optical eddy (screw-type dislocation) in the structure of the phase front of a monochromatic light field ($\psi_k(\mathbf{r}, t) = 0$ and $q = 0$) at the NRI entrance. Then, using the property of equivalence, we can identify the order of the screw-type dislocation of the optical field and compensate for the effect of an optical eddy on the dynamics of nonlinear phase change in the laser beam cross section in the NRI.[11]

Analyzing the form of the second equality in Eq. (8), it is logical to assume that

$$\delta\psi(\mathbf{r}, t) = q\delta\varphi(\mathbf{r}, t) + \text{const}. \quad (9)$$

Substituting Eq. (9) into Eq. (8), we obtain the condition $\pi[(1 - q)i - (1 + q)j] = 0$ relating the parameter of bichromaticity q to the numbers i and j. The condition is fulfilled for any value q (that is, any frequency interval 2Ω of the components of the spectrum of bichromatic radiation at the NRI entrance) at $i = 0$, $j = 0$ or for $q = (i - j)/(i + j)$ at $i \ne 0$, $j \ne 0$. The latter alternative can be written in the form $q = 2i/(i + j) - 1$. In this case, for any pair of the numbers i, j we can find the pair n and l by the rule $i = nN$, $j = N(l - n)$, where N is an arbitrary integer, and *vice versa*.

In other words, the assumption (9) on the relation between the functions $\psi(\mathbf{r}, t)$ and $\varphi(\mathbf{r}, t)$ in Eq. (8) is valid either for $i = 0$ and $j = 0$ or when the bichromaticity parameter q is determined by the equality $q = 2n/l - 1$. It can be shown that the equality $q = 2n/l - 1$ provides for repetition (periodicity) of the properties of a ring optical system as $\omega t_e(\mathbf{r}', t)$ changes by the value multiple of πl.

Substituting $(i + j) = Nl$, $i - j = N(2n - l)$, and Eq. (9) into Eq. (6), we obtain, in addition to Eq. (9), the following equation:

$$\omega\delta t_e(\mathbf{r}', t) \approx \pi l N + \delta\varphi(\mathbf{r}', t - \tau_1) - \delta\varphi(\mathbf{r}, t). \quad (10)$$

Turning to the version that $i = 0$ and $j = 0$, we can check that the substitution $i = 0$, $j = 0$ and Eq. (9) into Eq. (6) gives the equation that can be derived from Eq. (10) at $N = 0$.

It can readily be seen that for any bichromaticity parameter q at $i = 0$ and $j = 0$ $(N = 0)$ or for $q = 2n/l - 1$ at $i \neq 0$, $j \neq 0$ fulfillment of the equalities (10) and (9) guarantees the validity of Eqs. (6) and Eqs. (7) and (8) obtained from them. However, the form of Eqs. (9) and (10) is much simpler than that of Eqs. (6)–(8) because the function $\delta\psi(\mathbf{r}, t)$ in Eq. (9) depends only on $\delta\varphi(\mathbf{r}, t)$ rather than on three functions $\delta\psi(\mathbf{r}', t - \tau_1)$, $\delta\varphi(\mathbf{r}, t)$ and $\delta\varphi(\mathbf{r}', t - \tau_1)$. In addition, in Eqs. (9) and (10) there is no relation, through i, j, between the pairs of equalities in Eqs. (6)–(8).

Thus, if the equalities (6) or (9) and (10) are valid, then the evolution of the nonlinear phase change $U(\mathbf{r}, t)$ in ring optical systems with the parameters φ_1, ψ_1, t_{e1} or φ_2, ψ_2, t_{e2} is the same. That is, *the property of equivalence* exists on the subset of the parameters $\{\varphi, \psi, \omega t_e\}$ for any initial conditions and any values of the parameters. It is only necessary to remember that $\delta t_{e\,\max}$ is small, which is difficult to express in the statement on the property of the equivalence.

If $\delta\varphi(\mathbf{r}, t)$ is fixed in Eqs. (9) and (10), then a sole value of $\delta\psi(\mathbf{r}, t)$, but more than one values of $\omega\delta t_e(\mathbf{r}', t)$ correspond to it. The set of the values of $\omega\delta t_e(\mathbf{r}', t)$ forms an equidistant series. And this suggests that the property of periodicity is a particular case of equivalence of the values of some parameter (for example, ωt_e).

Practicing the approach described here, we can prove that the property of equivalence exists at the subset $\{q, \omega t_e, K_{ab}, K_a, K_b\}$ for any initial conditions and any values of the parameters from the set P. The property is formulated as

$$q_2 = 1/q_1,\ K_{a2}(\mathbf{r}, t, \mathbf{r}', t') = q_1 K_{a1}(\mathbf{r}, t, \mathbf{r}', t'),$$

$$K_{b2}(\mathbf{r}, t, \mathbf{r}', t') = q_1 K_{b1}(\mathbf{r}, t, \mathbf{r}', t'),$$

$$K_{ab2}(\mathbf{r}, t, \mathbf{r}_n) = q_1 K_{ab1}(\mathbf{r}, t, \mathbf{r}_n),$$

$$\omega_2 t_{e2}(\mathbf{r}', t) = q_1 \omega_1 t_{e1}(\mathbf{r}', t),$$

providing the equivalence of the evolutions in the sense of the relation $U_2(\mathbf{r}, t) = q_1 U_1(\mathbf{r}, t)$. (Therefore, at $q = \Omega/\omega > 1$, when the role of optical frequency passes from ω to Ω, the measure of bichromaticity is $1/q$, rather than the parameter q.) We can demonstrate the equivalence of the evolutions by drawing the bifurcation diagram of static states based on the model (4) for the NRI (Fig. 3).

Conclusion

A new concept of equivalence of the parameters of a dynamic system in the sense of the relation $\mathbf{F}[\ldots] \approx 0$ chosen for the equivalence of the evolutions is used.

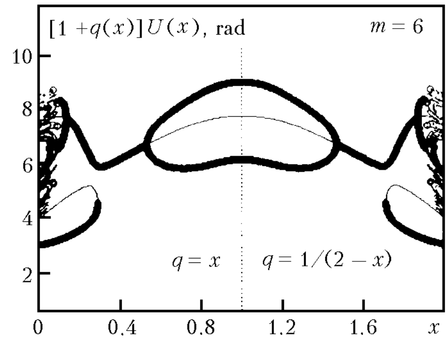


Fig. 3. Demonstration of the property of equivalence: diagram branches are symmetric about the straight line $x = 1$ (dashed line). $K_a = K_b = K_0/(1 + q)$, $K_{ab} = 2K_0/(1 + q)$, $\omega t_e = \omega t_{e0}/(1 + q)$, $K_0 = 3$, $\omega t_{e0} = 0$, $q(x) = x$ at $x \in [0; 1]$ and $q(x) = 1/(2 - x)$ at $x \in [1; 2)$.

Thanks to the property of equivalence, it is possible to manipulate with some parameters having unchanged characteristics of evolution of a nonlinear system that were laid in the condition $\mathbf{F}[\ldots] \approx 0$ in constructing it. It is also possible to provide for the identical evolutions (for example, periodic behavior) of two similar optical systems.

The property of equivalence in the sense of identical evolution of the systems allows the determined dependences of the system dynamics on some parameters to be extended to those on other parameters. Thus, we can, for example, save computer resources in drawing maps of dynamic modes of an optical system showing the conditions for the regular and chaotic behavior of the system.

The efficiency of the method for revealing equivalence is demonstrated from the model (4) of the processes in a nonlinear ring interferometer used in atmospheric adaptive optics.[1] Dealing with the concept of equivalence allows us:

(a) to control the law of space–time variation of any two parameters: phase (for example, the order of a screw-type dislocation of the optical field with an eddy), position of the polarization plane of the optical field at the entrance to an NRI, and delay time in the NRI, to identify the law of variation of the third parameter and compensate for or simulate its effect on the dynamics of the processes in the NRI;

(b) to determine the conditions for periodic repetition of the NRI properties at variation of the phase change in the feedback loop;

(c) to find that increasing the bichromaticity parameter it is possible to decrease the nonlinearity parameters and the phase lag in the NRI, providing for the preset character of the evolution.

Development of the formalism for description of optical devices proposed in Ref. 7 in combination with the results obtained permits developing the principles of synthesis of nonlinear systems based on changing the structure of their phase space.

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