Spatial prediction of atmospheric parameters in the mesoscale region based on the dynamic-stochastic approach

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An original methodology and algorithms for spatial prediction of atmospheric parameters in the mesoscale region based on the use of Kalman filtering and the generalized dynamic-stochastic model constructed based on the two-dimensional equation of mesoscale diffusion are considered. The results of statistical assessment of the quality of the algorithms proposed are discussed as applied to the problem of spatial prediction of mesoscale temperature and wind fields.

Introduction

Among numerous problems of the modern mesometeorology, an important place is occupied by the problem of assessment of parameters of the atmosphere over the territory not covered by the data of aerological measurements in neighboring regions. Essentially, it is the procedure of spatial prediction (extrapolation) of meteorological fields in mesoscale regions. The results of such a prediction are used, in particular, for estimation of the spatial spread of technogenic pollutants to short (100–200 km) distances, as well as for meteorological provision of army during local warfare.

As known, for a long time the problem of spatial prediction was solved within the framework of objective analysis of meteorological fields carried out based on the method of optimal interpolation.[1,2] But in recent years, in connection with the increasing amount of meteorological information, the traditional procedure of objective analysis has been replaced by the data assimilation procedure. The data assimilation procedure is usually understood as taking into account both the measurements and the prediction by the chosen model of the atmosphere. As a model of the atmosphere, the hydrodynamic model being a set of fluid dynamics equations is usually used. The data assimilation procedure is based on the dynamic-stochastic approach with the use of the Kalman filtering theory.[3–9] However, application of this model to spatial prediction of mesometeorological fields faces significant difficulties. These are:

– impossibility of specifying the initial fields of meteorological parameters over the territory not covered by observations;

– difficulty of specifying the boundary conditions at the open side boundaries for hydrodynamic simulation of mesoscale processes;

– difficulty of correct solution (under the conditions of zonal-mean west-east transport) of the problem of extrapolation of the meteorological field to the territory lying to the west from the region covered by aerological information.

In addition, the use of the Kalman filter in the data assimilation procedure including modern fluid-dynamics models faces difficulties in its practical implementation because of the high order of covariance matrices of estimation and prediction errors involved in the calculations.[5,7] This is caused by the fact that the state vector includes the entire set of the weather parameters to be estimated on all standard isobaric surfaces and at all nodes of the specified regular grid. The number of parameters in the state vector may achieve several hundreds.[4] The high order of the state vector and the covariance matrix of estimation errors leads to difficulties in specifying their initial values, which, in its turn, lowers the quality of the filtering algorithm.

This paper is the extended version of our previous publication (Ref. 10). Taking into account the above difficulties, we propose a simpler approach to the spatial extrapolation. This approach is also based on the Kalman filtering algorithm, but as a mathematical model we took the system of first-order stochastic equations, describing the dynamics of the atmospheric state parameters in the simplified form.

It is assumed that each point, to which the results of observation are extrapolated, can be considered independently. This is caused by the fact that the information about the values of a weather parameter at each of the extrapolation points results from assimilation of the same measurement results, and the information about relations between the extrapolation points gives no new data to the assimilation system. Therefore, the Kalman filter of a higher dimensionality, whose state vector includes the values of a weather parameter at all measurement and extrapolation points, can be replaced by a set of filters of smaller dimension. For every extrapolation point, its own filter is constructed.

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The state vector for each filter should include the values of a weather parameter at the measurement points and one value at the extrapolation point, individual and corresponding to only this filter. The proposed approach allows us to considerably decrease (by several orders of magnitude) the dimension of the state vector and covariance matrices of estimation errors, facilitates realization of the algorithm, and improves its stability.

1. Formulation of the problem and the method for its solution

The problem of spatial extrapolation of the field of some atmospheric parameter ξ consists in estimation of its value at the point n with the Cartesian coordinates (x_n, y_n, z_n) from measurements at the points with the coordinates (x_i, y_i, z_i) $(i = 1, 2, 3, ..., n-1)$ and some mathematical model describing variations of the field of ξ in space and time. In this case, we use the dynamic-stochastic model based on first-order stochastic differential equations. As to estimation of the field of ξ, this problem is solved for an arbitrary altitude, ignoring the relations between neighboring levels. The incorrectness of the model arising in such a case is compensated for by introducing noise of the states.

For derivation of the equations of the mathematical model, we used the well-known equation of diffusion.[11] Note that we do not state in this paper that the equation of diffusion is an ideal model of evolution of any atmospheric field, pretending to be highly complete and realistic. However, any model of the fluid-dynamics type describes the processes of the ordered transport and diffusion of corresponding substances. Therefore, at this stage of investigations, we would like to reveal whether it is possible to obtain some constructive results based on only diffusion effects in the atmosphere.

Consider a low-dimensional (space and time) field of ξ and assume that its evolution in the mesoscale region is described by a two-dimensional equation of mesoscale diffusion[11]:

$$\frac{\partial \xi}{\partial t} = a^2 \nabla^2 \xi \, , \tag{1}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the two-dimensional Laplace operator; a^2 is the diffusion coefficient.

For Eq. (1), we introduce the function of sensitivity to unit distortion (Green's function) at the point with the coordinates (x_0, y_0), which has the form[12]:

$$G(x,y,t;x_0,y_0) = \left(\frac{1}{\sqrt{4\pi a^2 t}}\right)^2 \exp\left[-\frac{(x-x_0)^2 + (y-y_0)^2}{4a^2 t}\right]. \tag{2}$$

Designating $r^2 = (x - x_0)^2 + (y - y_0)^2$, we obtain

$$G(r,t) = \left(\frac{1}{\sqrt{4\pi a^2 t}}\right)^2 \exp(-r^2/4a^2 t) \, . \tag{3}$$

By differentiating Eq. (3) with respect to r, we get

$$\frac{\partial G(r,t)}{\partial r} = -\frac{1}{2a^2 t} r \left(\frac{1}{\sqrt{4\pi a^2 t}}\right)^2 \exp(-r^2/4a^2 t) =$$

$$= -\frac{1}{2a^2 t} r G(r,t). \tag{4}$$

After linearizing of the right-hand side of Eq. (4) near the point $r_0 = \sqrt{4a^2 t}$ and scale analysis, we obtain the equation

$$\frac{\partial G(\rho,t)}{\partial \rho} = -\beta G(\rho,t) \, , \tag{5}$$

where

$$\beta = \frac{r_0}{2a^2 t} = \frac{2}{r_0}, \ \rho = r - r_0.$$

Thus, if at the time $t = 0$ the filed of ξ was distorted at some point, then at any time the response to this distortion at a distance comparable with r_0 meets the following relationship:

$$\frac{\partial \xi}{\partial \rho} = -\beta \xi. \tag{6}$$

Represent now the function ξ as a complex Fourier integral over the spatial coordinates[13]:

$$\xi(x,y,t) =$$

$$= \int_{-k_{y\max}}^{k_{y\max}} \int_{-k_{x\max}}^{k_{x\max}} A(t,k_x,k_y) \exp[i(k_x x + k_y y)] \mathrm{d}k_x \mathrm{d}k_y, \tag{7}$$

where

$$\int_{-k_{y\max}}^{k_{y\max}} \int_{-k_{x\max}}^{k_{x\max}} A(t,k_x,k_y) \exp[i(k_x x + k_y y)] \mathrm{d}k_x \mathrm{d}k_y$$

is the principal value of the two-dimensional Fourier integral; k_x and k_y are, respectively, the x and y wave numbers.

It should be emphasized that in the integration we do not consider the entire spectrum of oscillations, but only its long-wave part corresponding to the wave numbers within the ranges $[-k_{x\,\max}; k_{x\,\max}]$ and $[-k_{y\,\max}; k_{y\,\max}]$. Thus, for example, we neglect the short-wave variations of the turbulence.

Differentiating the ξ twice with respect to x and y, according to the theorem of integral differentiation with respect to a parameter, we have

$$\nabla^2 \xi = \frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2} = \int_{-k_{y\max}}^{k_{y\max}} \int_{-k_{x\max}}^{k_{x\max}} -(k_x^2 + k_y^2) A(t,k_x,k_y) \times$$

$$\times \exp[i(k_x x + k_y y)] \mathrm{d}k_x \mathrm{d}k_y. \tag{8}$$

The function $-(k_x^2 + k_y^2)$ does not alternate the sign in the domain of integration; therefore, according to the theorem on the mean value of integral,[11] the right-hand side of Eq. (8) can be written as

$$\nabla^2\xi = -\overline{(k_x^2 + k_y^2)}\times$$

$$\times \int_{-k_{y\max}}^{k_{y\max}} \int_{-k_{x\max}}^{k_{x\max}} A(t,k_x,k_y)\exp[i(k_x x + k_y y)]\mathrm{d}k_x\mathrm{d}k_y =$$

$$= -k^2 \int_{-k_{y\max}}^{k_{y\max}} \int_{-k_{x\max}}^{k_{x\max}} A(t,k_x,k_y)\exp[i(k_x x + k_y y)]\mathrm{d}k_x\mathrm{d}k_y =$$

$$= -k^2\xi,$$

where k is the effective wave number.

Introducing the effective wavelength L (effective length of distortions in the field of ξ), we can write

$$k = \frac{2\pi}{L}, \quad k^2 = \frac{4\pi^2}{L^2}. \qquad (10)$$

Then Eq. (1) can be represented as

$$\frac{\partial\xi}{\partial t} = -\frac{4\pi^2}{L^2}a^2\xi \quad \text{or} \quad \frac{\partial\xi}{\partial t} = -\alpha\xi, \qquad (11)$$

where $\alpha = 4\pi^2a^2/L^2$.

Equation (11) is correct at all points, including the point r_0 (at $\rho = 0$), hence we have

$$\left.\frac{\partial\xi}{\partial t}\right|_{\rho=0} = -\alpha\xi. \qquad (12)$$

Thus, Eq. (6) can be used as a spatial interpolator (extrapolator) for the field of ξ, and Eq. (12) along with the condition $\xi(0,0) = \xi_0$ can be used for temporal prediction of the same field, including the point r_0.

Within the designations of the Kalman filtering,[14] introduce the vector of the space of states

$$\mathbf{X}(m)=|\mathbf{X}_1(m),\ \mathbf{X}_2(m),\ \mathbf{X}_3(m),\ \ldots,\ \mathbf{X}_{n-1}(m),\ \mathbf{X}_n(m)|^{\mathrm{T}},$$

with the components: $\mathbf{X}_i(m)$ is the value of the field of ξ at the points $i = 1, 2, 3, \ldots, (n-1)$ (measurement points); $\mathbf{X}_n(m)$ is the value of the field of ξ at the point n (point of prediction) lying in the area not covered by the meteorological information; T denotes transposition; m is the discrete time. Then, using the Euler method,[11] we can write Eqs. (6) and (12) in the difference form:

$$\frac{\mathbf{X}_i(m+1) - \mathbf{X}_n(m+1)}{\Delta\rho_{in}} = -\beta\mathbf{X}_n(m+1), \qquad (13)$$

$$\frac{\mathbf{X}_i(m+1) - \mathbf{X}_n(m)}{\Delta t} = -\alpha\mathbf{X}_n(m) \qquad (14)$$

or

$$\mathbf{X}_i(m+1) = (1 - \beta\Delta\rho_{in})\mathbf{X}_n(m+1), \qquad (15)$$

$$\mathbf{X}_n(m+1) = (1 - \alpha\Delta t)\mathbf{X}_n(m), \qquad (16)$$

where $\Delta\rho_{in}$ is the separation between the points i and n; Δt is the time interval between measurements.

Substituting the expression for $\mathbf{X}_n(m+1)$ from Eq. (16) to Eq. (15), we obtain the linear model for the evolution of the field $\mathbf{X}$:

$$\begin{aligned}\mathbf{X}_i(m+1) &= (1-\beta\Delta\rho_{in})(1-\alpha\Delta t)\mathbf{X}_n(m) + \omega_i(m),\\ \mathbf{X}_n(m+1) &= (1-\alpha\Delta t)\mathbf{X}_n(m) + \omega_n(m),\end{aligned} \qquad (17)$$

where $\omega_i(m)$ and $\omega_n(m)$ are random distortions accounting for the stochastic character of the model.

The system of equations (17) can be used as a model of the space of states when synthesizing the algorithm for estimation of the current values of the field $\mathbf{X}$ within the theory of Kalman filtering. A restriction on the use of Eq. (17) is an uncertain value of the parameters α and β. Indeed, under the conditions of occurrence of various mesoscale processes, these parameters depending on the coefficient of mesoscale diffusion exchange of the substance $\mathbf{X}$ should differ. Therefore, to lift this restriction, we should introduce additional variables $\mathbf{X}_{n+1} = \mathbf{X}_{n+1}(t) = \alpha$ and $\mathbf{X}_{n+2} = \mathbf{X}_{n+2}(t) = \beta$.

Assume that the evolution of $\mathbf{X}_{n+1}$ and $\mathbf{X}_{n+2}$ is described by the equations

$$\frac{\mathrm{d}\mathbf{X}_{n+1}}{\mathrm{d}t} = \omega'_{n+1}, \quad \frac{\mathrm{d}\mathbf{X}_{n+2}}{\mathrm{d}t} = \omega'_{n+2}, \qquad (18)$$

where ω'_{n+1}, ω'_{n+2} are random processes like white noise.

Then, using the method of spatiotemporal discretization (13)–(14), we can obtain the general equations for the equations of states:

$$\mathbf{X}_i(m+1)=\mathbf{X}_n(m)(1-\mathbf{X}_{n+2}(m)\Delta\rho_{in})(1-\mathbf{X}_{n+1}(m)\Delta t)+\omega_i(m),$$

$$\mathbf{X}_n(m+1) = \mathbf{X}_n(m)(1 - \mathbf{X}_{n+1}(m)\Delta t) + \omega_n(m),$$

$$\mathbf{X}_{n+1}(m+1) = \mathbf{X}_{n+1}(m) + \omega_{n+1},$$

$$\mathbf{X}_{n+2}(m+1) = \mathbf{X}_{n+2}(m) + \omega_{n+2}.$$

The equations of observations of the field $\mathbf{X}$ can be written as

$$\mathbf{Y}_i(m) = \mathbf{X}_i(m) + \varepsilon_i(m), \qquad (20)$$

where $\varepsilon_i(m)$ are measurement errors at the time m.

In this case, Eq. (20) uses, as $\mathbf{Y}_i(m)$, the centered value of the measurements obtained as

$$\mathbf{Y}_i(m) = \tilde{\mathbf{Y}}_i(m) - \bar{\mathbf{Y}}(m), \qquad (21)$$

where $\tilde{\mathbf{Y}}_i(m)$ is the actually measured value of a weather parameter at the ith observation point for the mth instant; $\bar{\mathbf{Y}}(m)$ is the spatially averaged value of the same weather parameter at the mth instant. The latter was calculated by the following equation:

$$\bar{\mathbf{Y}}(m) = \frac{1}{n-1}\sum_{i=1}^{n-1}\tilde{Y}_i(m), \qquad (22)$$

where $n - 1$ is the number of measuring stations at the considered territory.

Write Eqs. (19) and (20) in the matrix form

$$\mathbf{X}(m+1) = \mathbf{\Phi}[\mathbf{X}(m)] + \mathbf{\Omega}(m); \quad (23)$$

$$\mathbf{Y}(m) = \mathbf{H}\mathbf{X}(m) + \mathbf{E}(m), \quad (24)$$

where $\mathbf{\Phi}[\mathbf{X}(m)]$ is the transient vector-function of state; $\mathbf{\Omega}(m)$ is the $(n + 2)$ vector of state noise; $\mathbf{H}$ is the $(n - 1)\times(n + 2)$ matrix of observations; $\mathbf{E}(m)$ is the $(n - 1)$ vector of observation noise.

Equations (23) and (24) fully determine the structure of the estimation algorithm.[14] Since Eqs. (23) are nonlinear, the extended Kalman filter should be taken as a method for synthesis of the estimation algorithm. In this case, the equation for optimal estimation of the state vector $\mathbf{X}(m)$ has the form[14]:

$$\hat{\mathbf{X}}(m + 1| m + 1) = \hat{\mathbf{X}}(m + 1| m) + \mathbf{C}(\hat{\mathbf{X}}, m + 1) \times$$
$$\times [\mathbf{Y}(m + 1) - \mathbf{H}\cdot\hat{\mathbf{X}}(m + 1|m)], \quad (25)$$

where $\hat{\mathbf{X}}(m + 1| m + 1)$ is the estimate of the state vector $\mathbf{X}$ at the time $(m + 1)$; $\hat{\mathbf{X}}(m + 1| m)$ is the vector of estimates at the time $(m + 1)$ predictable from the data at the mth step, and

$$\hat{\mathbf{X}}(m + 1| m) = \mathbf{\Phi}[\hat{\mathbf{X}}(m)]; \ \mathbf{C}(\hat{\mathbf{X}}, m + 1)$$

is the $(n + 2)\times(n - 1)$ matrix of weighting coefficients.

The weighting coefficients in the extended Kalman filter are calculated by recursion matrix equations of the form[14]:

$$\mathbf{C}(\hat{\mathbf{X}}, m + 1) = \mathbf{P}(m + 1|m) \cdot \mathbf{H}^{\mathrm{T}} \cdot [\mathbf{H} \cdot \mathbf{P}(m + 1| m)\cdot\mathbf{H}^{\mathrm{T}} +$$
$$+ \mathbf{R}_{\mathbf{E}}(m + 1)]^{-1}; \quad (26)$$

$$\mathbf{P}(m + 1|m) = \mathbf{F}[\hat{\mathbf{X}}(m)] \cdot \mathbf{P}(m|m) \cdot \mathbf{F}^{\mathrm{T}}[\hat{\mathbf{X}}(m)] + \mathbf{R}_{\mathbf{\Omega}}(m); \quad (27)$$

$$\mathbf{P}(m + 1|m + 1) = [\mathbf{I} - \mathbf{C}(\hat{\mathbf{X}}, m + 1) \cdot \mathbf{H}] \cdot \mathbf{P}(m + 1| m), \quad (28)$$

where $\mathbf{P}(m + 1|m)$ is the $(n + 2)\times(n + 2)$ *a posteriori* covariance matrix of the prediction errors; $\mathbf{P}(m + 1|m + 1)$ is the $(n + 2)\times(n + 2)$ *a priori* covariance matrix of the estimation errors; $\mathbf{R}_{\mathbf{E}}(m + 1)$ is the $(n - 1)\times(n - 1)$ diagonal correlation matrix of the observation noise; $\mathbf{R}_{\mathbf{\Omega}}(m)$ is the $(n + 2)\times(n + 2)$ diagonal correlation matrix of the state noise; $\mathbf{F}[\hat{\mathbf{X}}(m)] = \dfrac{\partial\mathbf{\Phi}[\hat{\mathbf{X}}(m)]}{\partial\hat{\mathbf{X}}(m)}$ is the $(n + 2)\times(n + 2)$ Jacobi matrix of the transient vector-function; $\mathbf{I}$ is the $(n + 2)\times(n + 2)$ unit matrix.

For the filtering algorithm to start at the time $m = 0$ (initiation time), the following parameters should be specified: $\hat{\mathbf{X}}(0|0)$ – the initial estimation vector; $\mathbf{P}(0|0)$ – the initial correlation matrix of estimation errors, $\mathbf{R}_{\mathbf{E}}(0)$ – the correlation matrix of observation noise, and $\mathbf{R}_{\mathbf{\Omega}}(0)$ – the correlation matrix of the state noise.

In practice, the values of $\hat{\mathbf{X}}(0|0)$ and $\mathbf{P}(0|0)$ can be specified based on the minimum information about the real properties of the system,[14] and in the case of complete lack of the useful information it is set $\hat{\mathbf{X}}(0|0) = 0$ and $\mathbf{P}(0|0) = \mathbf{I}$. At the same time, the elements of the matrices $\mathbf{R}_{\mathbf{E}}(0)$ and $\mathbf{R}_{\mathbf{\Omega}}(0)$ can be determined from the errors of radiosonde observations.

2. Results of investigation of the Kalman filtering algorithm

Consider now the results of studying the Kalman filtering algorithm when it is used for spatial prediction of mesoscale temperature and wind fields. Since the spatial prediction in this paper is considered as applied to assessment of spread of a pollutant cloud, according to Ref. 15, we do not take wind and temperature measurements at some atmospheric levels, but their layer-mean values determined from the equation

$$< \xi >_{h_0,h} = \frac{1}{h - h_0}\int_{h_0}^{h}\xi(z)\mathrm{d}z, \quad (29)$$

where <·> denote vertical averaging over an atmospheric layer $\Delta h = h - h_0$ (here h_0 and h are the heights of the bottom and top boundaries of this layer, and $h_0 = 0$ corresponds to the ground); ξ is the value of a meteorological parameter.

It should be noted that to evaluate the quality of the Kalman filtering algorithm, we used many-year two-term (00:00 and 12:00 GMT) observations of five radiosonde stations: Warsaw (52°10′N, 20°58′E), Kaunas (54°53′N, 23°50′E), Brest (52°07′N, 23°41′E), Minsk (53°56′N, 27°38′E), and Lvov (49°49′N, 23°57′E) that form a typical mesometeorological network. The total volume of synchronous (for all the stations) measurements was 540 vertical profiles for winter and 560 vertical profiles for summer.

The number of stations taken determined the dimensionality of the state vector of the synthesized filter equal to $n + 2$ (here $n = 5$). As the initial conditions, we specified the values $\hat{\mathbf{X}}(0|0) = 0$ and $\mathbf{P}(0|0) = \mathbf{I}$, whereas the diagonal elements of the correlation matrices of observation noise $\mathbf{R}_{\mathbf{E}}(0)$ and state noise $\mathbf{R}_{\mathbf{\Omega}}(0)$ were taken based on the values of root-mean-square (rms) errors of radiosonde measurements (Ref. 16) equal to 1° for temperature and 2 m/s for wind velocity.

To assess the accuracy of the Kalman filtering algorithm, we took the stations Warsaw and Kaunas spaced by 185 and 277 km from the neighboring measurement stations as control points (extrapolation points).

The accuracy of the proposed algorithm was estimated using the rms error of the spatial extrapolation

$$\delta_\xi = \left[\frac{1}{N}\sum_{i=1}^{N}\left(\xi_i^* - \xi_i\right)^2\right]^{1/2}. \quad (30)$$

Here ξ_i and ξ_i^* are the measured and extrapolated values of a meteorological parameter; N is the number of realizations.

In addition, to assess the quality of the Kalman filtering algorithm, we used the procedure of comparing its errors with the errors of the traditional Gandin's method of optimal extrapolation.[1] This method has gained the wide recognition in Russia in the system for assimilation of meteorological information. The weighting coefficients involved in the equation of optimal extrapolation were obtained with the use of spatial correlation functions from Ref. 17.

Consider now the results of statistical assessment of the quality of the method proposed in the spatial prediction of mesometeorological fields using the data summarized in Tables 1 and 2.

Table 1. RMS (δ) and relative (θ, %) errors in prediction of the layer-mean values of temperature and zonal and meridional wind velocity components to the distance of 185 km as made based on the optimal extrapolation (1) and Kalman filter (2) algorithms

Layer, km	Winter				Summer			
	δ		θ		δ		θ	
	1	2	1	2	1	2	1	2
Temperature, °C								
0–0.2	1.9	1.7	41	37	1.8	1.6	42	37
0–0.8	2.0	1.7	47	40	1.8	1.6	45	40
0–1.6	2.1	1.6	51	39	1.9	1.5	53	42
0–2.0	2.2	1.6	52	38	2.0	1.4	57	40
0–4.0	2.9	1.6	60	33	2.7	1.1	82	33
0–6.0	3.3	1.5	66	30	3.0	1.0	86	28
0–8.0	3.5	1.4	73	29	3.2	0.9	89	25
Zonal wind velocity component, m/s								
0–0.2	3.2	2.2	80	55	2.8	1.8	70	45
0–0.8	3.3	2.7	62	51	2.7	2.0	52	38
0–1.6	3.1	2.7	53	46	2.7	2.0	51	38
0–2.0	3.0	2.6	51	44	2.6	1.9	49	36
0–4.0	2.8	2.4	42	36	2.6	1.8	46	32
0–6.0	3.2	2.8	42	36	2.6	1.9	44	32
0–8.0	3.6	3.1	40	35	2.7	2.3	42	36
Meridional wind velocity component, m/s								
0–0.2	2.7	2.0	71	53	3.0	1.6	86	46
0–0.8	2.9	2.6	62	55	3.1	1.7	70	39
0–1.6	3.0	2.7	56	50	3.0	1.6	71	38
0–2.0	3.0	2.7	54	48	2.9	1.8	67	42
0–4.0	3.0	2.7	44	40	2.9	2.0	67	46
0–6.0	3.5	3.2	42	38	3.0	2.2	64	47
0–8.0	3.8	3.5	39	36	3.2	2.5	62	48

Table 2. RMS (δ) and relative (θ, %) errors in prediction of the layer-mean values of temperature and zonal and meridional wind velocity components to the distance of 277 km as made based on the optimal extrapolation (1) and Kalman filter (2) algorithms

Layer, km	Winter				Summer			
	δ		θ		δ		θ	
	1	2	1	2	1	2	1	2
Temperature, °C								
0–0.2	2.0	1.7	49	41	2.2	1.7	54	41
0–0.8	2.2	1.8	58	47	2.2	1.7	58	45
0–1.6	2.5	1.6	64	41	2.2	1.6	63	46
0–2.0	2.7	1.5	66	37	2.3	1.5	68	44
0–4.0	3.2	1.4	70	30	2.9	1.4	88	42
0–6.0	3.4	1.4	72	30	3.1	1.3	94	39
0–8.0	3.6	1.4	80	31	3.4	1.3	97	37
Zonal wind velocity component, m/s								
0–0.2	2.9	2.2	76	58	3.0	2.0	81	54
0–0.8	3.0	2.7	68	61	3.1	2.3	74	55
0–1.6	3.3	2.8	60	51	3.0	2.5	65	54
0–2.0	3.5	2.9	60	50	3.0	2.5	62	52
0–4.0	4.0	3.1	59	46	3.0	2.6	60	52
0–6.0	4.2	3.2	55	42	3.2	2.6	54	44
0–8.0	4.8	3.3	56	39	3.4	2.6	51	39
Meridional wind velocity component, m/s								
0–0.2	3.2	2.0	89	55	3.5	1.9	97	53
0–0.8	3.3	2.3	80	56	3.5	2.3	88	57
0–1.6	3.4	2.6	61	46	3.4	2.5	77	57
0–2.0	3.5	2.7	57	44	3.4	2.6	69	53
0–4.0	3.8	2.8	49	36	3.4	2.8	59	48
0–6.0	4.3	2.8	47	31	3.5	3.0	54	46
0–8.0	4.8	3.0	44	29	3.5	3.0	48	41

Analysis of the data presented in Tables 1 and 2 shows the following.

– First, the proposed method based on the use of the Kalman filtering algorithm and the dynamic-stochastic model constructed based on the two-dimensional equation of mesoscale diffusion gives the results quite acceptable for practical needs, especially, in the case that spatial prediction is carried out up to the distance of 185 km. Indeed, at this distance regardless of the considered parameter, season, and atmospheric layer, the rms errors are about 25–55% of the rms deviations characterizing variability of these parameters.

– Second, this method gave the best results in spatial prediction up to the distance of 185 km for the layer-mean temperature values, for which the rms errors of prediction range within 1.4–1.7°C in winter and 0.9–1.6°C in summer. The summer values of δ for the layers with the top boundary $h > 3$ km are 0.9–1.1°C, that is, they are at the level of the acceptable error set by the WMO for the troposphere and equal to 1.0°C (Ref. 18).

– Third, as expected, the accuracy of spatial prediction of meteorological parameters decreases markedly with the distance. When predicting the layer-mean values of temperature to the distance of

277 km, only in winter in the free atmosphere the results obtained are somewhat better than those in the 185-km prediction, when it is carried out in the direction opposite to the zonal-mean west-east transport.

– Fourth, the algorithm proposed provides for better results of spatial prediction as compared to the method of optimal extrapolation, and the greatest gain in the accuracy (from 1.3 to 3 times) was obtained when predicting layer-average values of temperature (especially for the layers with the top boundary at the height $h > 3$ km).

Thus, it can be concluded that the proposed spatial prediction algorithm based on the extended Kalman filter and the dynamic-stochastic model taking into account only diffusion effects gives quite good results. Unlike the optimal extrapolation algorithm, it does not require *a priori* information about the statistical structure of the predicted fields of meteorological parameters. Therefore, this algorithm can be successfully applied in solving various practical problems, in particular, the problem of numerical estimation of mesoscale spread of the man-made pollutants.

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