Acoustic levitation of aerosols

V.G. Chernyak and N.S. Kiseleva

Ural State University, Ekaterinburg

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Physical mechanisms of acoustic suspension of aerosol particles in the gravity field under the effect of the traveling and standing sound waves are analyzed. The equations of particle equilibrium are obtained taking into account these mechanisms. These equations establish the relation between the properties of the sound wave, aerosol particle, and gas, so that the particle levitation averaged over the period of oscillations realizes. The parameters of equilibrium are calculated numerically for a water drop suspended in air, depending on its size. Based on numerical calculations, the contributions of various mechanisms to the particle suspension conditions are revealed for the traveling and standing sound waves at different Knudsen numbers.

Introduction

The success of experimental investigations into the aerosol photochemistry significantly depends on the possibility of suspending particles without a mechanical contact. This problem is directly related to the investigation of the processes of drop evaporation and condensation, phoretic phenomena (photophoresis, thermophoresis, diffusiophoresis), aerosol coagulation, etc. Electrostatic, electromagnetic, cryogenic, and acoustic suspension are quite often used now. Each of these methods has its own advantages and disadvantages, as well as the domain of applicability. The suspension of particles in the field of a sound wave is used in acoustic levitators – special facilities intended for contactless installation and fixation of liquid and solid bodies in certain positions.[1]

The conditions of acoustic levitation of particles are calculated now based on consideration of only the acoustic pressure force.[1] This force is known to be significant only for particles larger than 700 μm in diameter in the traveling wave and larger than 25 μm in diameter in the standing wave.[2,3] The size of atmospheric aerosol particles ranges within 0.1–10 μm, where the sound pressure is very low. Therefore, it seems urgent to estimate the contribution of other possible mechanisms, providing for the phenomenon of levitation of fine aerosols. In this case, the dependence of the parameters characterizing the interaction of a particle with a sonicated gas on the Knudsen number should be taken into account.

This work is aimed at the theoretical investigation of the possibility of suspending an aerosol particle in the field of the traveling and standing waves, establishing the relation between the characteristics of the sound wave, on the one hand, and the properties of the gas and the suspended particle, on the other hand. To estimate the contributions of one or another mechanisms of particle levitation under various conditions, it is necessary to calculate numerically the parameters of the equilibrium equations in the cases of the traveling and standing waves as functions of the Knudsen number.

Formulation of the problem

Consider a spherical particle of radius r in a gas, whose state is disturbed by the traveling or standing sound waves. Let ω be the cyclic frequency, $\lambda = 2\pi c_g/\omega$ be the wavelength, and c_g be the speed of sound in the gas.

Restrict our consideration to the case when the mean free path of gas molecules l is much shorter than the sound wavelength λ. This means that gas is a continuous medium with respect to the sound propagation. On the other hand, assume that the mean free path of gas molecules can have any value with respect to the radius of the aerosol particle (arbitrary Knudsen number $\mathrm{Kn} = l/r$). This means that the gas is rarefied with respect to the aerosol particle. Let the particle radius be shorter than the sound wavelength. Thus, we have

$$l \ll \lambda,\ r \forall l,\ r \ll \lambda.$$

The aerosol particle can be suspended, if the gravity force of the particle is balanced by the resulting force, acting on the particle from the sound wave.

Consider the following mechanisms of sound action on the particle.

1. Acoustic radiation pressure on a particle

The pulse given to the particle by the incident wave is greater than the pulse scattered by the particle in the direction of propagation of the incident wave. An excessive force arises, which, when averaged over the period of oscillations, has the following form for a nonviscous gas[2,3]:

$$F_R = \frac{11}{9}\pi\left(\frac{\omega}{c_g}\right)^4 r^6 \mu_g^2 \frac{J}{c_g}, \tag{1}$$

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where μ_g is the particle flow coefficient; J is the sound wave intensity, in W/m^2.

In the field of the standing wave, we have to take into account the pulses of both the direct and the backward waves, whose values depend on the position of the particle in the sound wave. Finally, the following equation has been obtained for the force of the radiation pressure on a particle in the standing sound wave[2,3]:

$$F_R = \frac{8}{3}\pi\left(\frac{\omega}{c_g}\right) r^3 \mu_g^2 \frac{J}{c_g}\sin(2kx), \quad (2)$$

where x is the distance from a node of the standing wave.

The force F_R, directed along the direction of propagation of the traveling wave, is equal to zero at nodes and antinodes of the standing wave.

2. Periodic alternation of medium viscosity

Adiabatic compressions and rarefactions of the medium in the sound field result in the periodic rise and drop of its temperature. The change of the gas temperature causes the corresponding change of its viscosity. The difference of the medium viscosity in the compression and rarefaction phases causes a certain difference in the force, with which the gas acts on the suspended particle at the forward and backward motions. This phenomenon leads to formation of the force, acting on the particle in the direction of the sound source in the traveling wave or a node in the standing wave. The equation for this force on the average for a period, obtained in the hydrodynamic approximation, is presented in Ref. 3. As in the case that the Kn number is not small, correction for the factor of gas rarefaction f should be introduced. Then the equation for the force will have the following form:

$$F_\eta = 3\pi(\gamma - 3)\frac{\nu}{c_g} r \mu_g^2 \frac{J}{c_g} f, \quad (3)$$

where γ is the adiabatic exponent; ν is the coefficient of kinematic viscosity of the gas, in m^2/s. The equation for the factor f, obtained in Ref. 4, has the form

$$f = \frac{0.619}{\mathrm{Kn} + 0.619}\left(1 + \frac{0.310\mathrm{Kn}}{\mathrm{Kn}^2 + 1.152\mathrm{Kn} + 0.785}\right). \quad (4)$$

For the standing wave, Eq. (3) should be multiplied by $\sin(2kx)$.

3. Distortion of the shape of the sound wave

For anharmonic sound oscillations of the gas, the particles suspended in it carry out non-sinusoidal oscillatory motions. As this takes place, some force arises, which shifts the equilibrium position, with respect to which the particle suspended in the gas oscillates. The mechanism of appearance of the drift force for the saw-tooth waveform of the sound wave is discussed in detail in Ref. 3. In Ref. 2, the sound wave is represented as a superposition of harmonic oscillations and the force acting on the particle is calculated in the approximation of second harmonic

$$F_h = -6h_2 \sin\psi r^2 \mu_g^2 J / c_g, \quad (5)$$

where h_2 is the amplitude ratio of the second harmonic of the sound wave to the fundamental harmonic; ψ is the phase shift of the second harmonic. It can be seen that the direction of this force coincides with the direction of propagation of the sound wave, if the phase shift ψ is negative. If $\psi > 0$, then the force is directed backward the sound wave.

Note that Eq. (5) was obtained from the Oseen correction to the Stokes law applied to a sphere flowed by a viscous liquid and therefore it is valid only at small Kn numbers. The rarefaction factor f (4) was obtained using linear relative to the Reynolds number approximation and cannot be used for correction of Eq. (5). However, numerical estimates show that the force F_h decreases quickly with the increase of the Kn number. For example, at Kn ≈ 1 the force F_h is four orders of magnitude smaller than F_η and its contribution to the resulting force is negligibly small. The force F_h should be taken into account only at Kn ≤ 0.1, that is, in that range of the Kn numbers, where Eq. (5) is a good approximation.

As was noted in Ref. 3, the calculation of the force F_h in the standing wave is to difficult because of the fact that the waveform changes with time at finite amplitudes of oscillations.

4. Asymmetry of oscillatory motion in the standing wave

In the standing sound wave, the amplitudes of the shift and velocity of the gas increase sinusoidally with the distance from the node of oscillations. This leads to the accelerated motion of the gas as it moves toward an antinode and the decelerated motion on the way back. The particle suspended in the gas, having certain inertia, lags behind the gas at the forward shift and passes ahead of the gas at the backward shift. As a result, the asymmetry of the oscillatory motion of the gas gives rise to the force acting on the aerosol particle[5]:

$$F_a = \frac{\pi}{3} k r^3 \mu_p \left[\frac{9}{2}\left(b^2 + b\right)\mu_g - \left(3 + \frac{9}{2}b\right)\mu_p\right]\frac{J}{c_g}\sin(2kx), \quad (6)$$

where

$$b = \sqrt{2\nu/\omega}/r; \quad \mu_p = \sqrt{1 - \mu_g^2}$$

is the dragging factor, determined by the ratio of oscillation amplitudes of the gas and the particle; k is the wave number.

The condition of particle suspension is determined by the zero resultant force acting on the particle, including the force of gravity, $m\mathbf{g}$.

For the traveling wave

$$m\mathbf{g} + \mathbf{F}_R + \mathbf{F}_\eta + \mathbf{F}_h = 0, \tag{7}$$

and for the standing wave

$$m\mathbf{g} + \mathbf{F}_R + \mathbf{F}_\eta + \mathbf{F}_a = 0. \tag{8}$$

Note that the particle oscillates about the equilibrium position. The amplitude and the phase shift of the particle oscillations with respect to the gas oscillations depend on the particle size and density, as well as on the gas properties (pressure, temperature, viscosity) and on the sound frequency. The corresponding calculations can be found in Ref. 6.

The numerical estimates show that, under normal conditions for the cyclic frequency of $62.8 \cdot 10^3\ \mathrm{s}^{-1}$, the radiation force F_R is significant only for particles with radius $r > 700\ \mu$m in the traveling wave and $r > 25\ \mu$m in the standing wave. The motion of such particles is well described by the hydrodynamic theory. We are interested in particles of the radius of 10 μm and smaller, which corresponds to the atmospheric aerosol. In this case, the contribution of F_R to the resultant force is negligible. Therefore, from here on the radiation force is omitted in Eqs. (7) and (8).

Results and discussion

The equilibrium equations (7) and (8) allow us to establish such a relation between the characteristics of the sound wave, on the one hand, and the particle and the gas, on the other hand, at which the suspension of the aerosol particle takes place. From Eq. (7), neglecting the radiation pressure force, we obtain

$$\frac{\mu_g^2 J}{\rho_p} = \frac{4}{3}\frac{\pi g r^2 c_g^2 \rho_g}{6 h_2 \rho_g c_g \sin(\psi) r + 3\pi(3-\gamma)\eta f}. \tag{9}$$

Usually, $\psi > 0$ [Ref. 3] and $\gamma < 3$. Then the forces $\mathbf{F}_\eta$ and $\mathbf{F}_h$ are directed toward the emitter, that is, against the direction of propagation of the traveling wave. Therefore, small particles ($r \le 10\ \mu$m) can be suspended, only if the sound force propagates in the direction of the force of gravity.

The right-hand side of Eq. (9) includes only characteristics of the gas and the particle radius. The results of its numerical calculation in the case of acoustic suspension of the particle in the air depending on the Kn number are presented in Table 1. The following values of the parameters were used in the calculation:

$\gamma = 1.4$; $c_g = 331.2$ m/s; $\eta = 1.85 \cdot 10^{-5}$ Pa·s;

$\rho_g = 1.29$ kg/m^3; $p = 10^5$ Pa; $h_2 = 0.5$; $\psi = \pi/2$.

From Eq. (8), neglecting the radiation pressure force, for the standing wave we obtain

$$\frac{\mu_g^2 J}{\rho_p}\sin(2kx) = a_1\left(1 + \frac{a_2}{\sqrt{\omega}} + \frac{a_3}{\omega} - \frac{a_4}{\omega\sqrt{\omega}}\right)^{-1}, \tag{10}$$

where

$$a_1 = \frac{4}{9}\frac{c_g^2 g r^2}{(3-\gamma)\nu f}, \quad a_2 = \frac{9\sqrt{2\nu}\,\rho_g}{4(3-\gamma) r \rho_p},$$
$$a_3 = \frac{3\rho_g\left(3\nu - r^2/\tau\right)}{2\rho_p(3-\gamma) r^2}, \quad a_4 = \frac{9\rho_g\sqrt{2\nu}}{4\rho_p(3-\gamma)\tau r}. \tag{11}$$

Table 1. Calculation of Eq. (9), determining suspension of a particle in the field of the traveling sound wave

r, μm	Kn	f	$\mu_g^2 J/\rho_p$, m^3/s^3
10	0.01	0.991	$4.4 \cdot 10^{-2}$
5	0.02	0.977	$2.2 \cdot 10^{-2}$
2	0.05	0.941	$8.0 \cdot 10^{-3}$
1	0.1	0.889	$4.0 \cdot 10^{-3}$
0.5	0.2	0.800	$2.0 \cdot 10^{-3}$
0.2	0.5	0.606	$5.0 \cdot 10^{-4}$
0.1	1	0.422	$2.0 \cdot 10^{-4}$
0.05	2	0.257	$1.0 \cdot 10^{-4}$
0.02	5	0.116	$4.0 \cdot 10^{-5}$
0.01	10	0.060	$2.0 \cdot 10^{-5}$

At the nodes ($x = 0$, $\lambda/2$, λ) and at antinodes ($x = \lambda/4$, $3\lambda/4$) of oscillations, the balancing forces are zero. These forces are maximum at the center of sections between nodes and antinodes of oscillations, where $\sin 2kx = 1$. Thus, the equilibrium positions of particles suspended in the standing wave are the points $x = \lambda/8$, $3\lambda/8$, $5\lambda/8$, and $7\lambda/8$.

The coefficients a_2, a_3, and a_4 are proportional to the density ratio of the gas and the particle. If $\rho_g/\rho_p \ll 1$ and the frequency ω is high enough, then the right-hand side of Eq. (10) is mostly determined by the value of a_1. Its dependence on the Kn number is taken into account through the gas rarefaction factor f. From Eq. (4) it follows that $f \to 1$ at Kn $\to 0$ (hydrodynamic limit) and $f \to 0$ (Kn^{-1}) at Kn $\to \infty$ (free-molecule mode). Thus, the coefficient a_1 decreases monotonically at the increase of the Kn number. This means that, at the fixed values of the gas pressure and the sound frequency, for suspension of large particles the intensity of the sound wave should be higher, while for small particles it should be lower in accordance with Eq. (10). This result seems obvious in the qualitative respect. The practical significance of Eq. (10) is that it establishes such quantitative relations between the characteristics of the wave, particle, and gas, which provide for the levitation of particles. The coefficient a_2 is independent of the Kn number. The dependence of the coefficients a_3 and a_4 on Kn is determined through the particle relaxation time τ, which was calculated in Ref. 6 and is presented in Table 2. The results of numerical calculation of the coefficients a_i by Eqs. (11) for water drops ($\rho_p = 10^3$ kg/m^3) at the same values of the parameters, which were used in the case of the traveling wave, are presented in Table 2 as well.

Table 2. Calculated parameters (11) for water drops at different values of the radius and the Knudsen number

r, μm	Kn	τ, s	f	a_1, m^3/s^3	a_2, s$^{-1/2}$	a_3, s^{-1}	a_4, s$^{-3/2}$
10	0.01	$1.22 \cdot 10^{-3}$	0.991	2.12	0.971	$5.20 \cdot 10^2$	$0.795 \cdot 10^3$
5	0.02	$3.08 \cdot 10^{-4}$	0.977	0.537	1.94	$20.8 \cdot 10^2$	$6.30 \cdot 10^3$
2	0.05	$5.10 \cdot 10^{-5}$	0.941	$8.90 \cdot 10^{-2}$	4.85	$1.30 \cdot 10^4$	$95.2 \cdot 10^3$
1	0.1	$1.35 \cdot 10^{-5}$	0.889	$2.40 \cdot 10^{-2}$	9.70	$5.20 \cdot 10^4$	$71.9 \cdot 10^4$
0.5	0.2	$3.75 \cdot 10^{-6}$	0.800	$6.0 \cdot 10^{-3}$	19.4	$20.8 \cdot 10^4$	$51.9 \cdot 10^5$
0.2	0.5	$7.93 \cdot 10^{-7}$	0.606	$1.0 \cdot 10^{-3}$	48.5	$13.0 \cdot 10^5$	$61.1 \cdot 10^6$
0.1	1	$2.84 \cdot 10^{-7}$	0.422	$5.0 \cdot 10^{-4}$	97.0	$52.0 \cdot 10^5$	$34.2 \cdot 10^7$
0.05	2	$1.17 \cdot 10^{-7}$	0.257	$2.0 \cdot 10^{-4}$	194.1	$20.8 \cdot 10^6$	$16.7 \cdot 10^8$
0.02	5	$4.14 \cdot 10^{-8}$	0.116	$7.0 \cdot 10^{-5}$	485.2	$13.0 \cdot 10^7$	$11.7 \cdot 10^9$
0.01	10	$2.0 \cdot 10^{-8}$	0.060	$3.0 \cdot 10^{-5}$	971.4	$52.0 \cdot 10^7$	$48.6 \cdot 10^9$

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