Self-focusing of high-power radiation of a femtosecond laser pulse with controlled parameters in the atmosphere

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Different methods to control the filamentation zone location of high-power femtosecond laser radiation at its self-focusing in the atmosphere are considered. These are spatial beam focusing, frequency beam modulation, and beam profiling. Solving the nonlinear Schrödinger equation, the range, within which the radiation parameters are to be modified to enable the control of nonlinear beam focus, is determined from the standpoint of the most effective transport of light energy to the receiver.

Introduction

In a wide range of atmospheric optics problems, the problems of laser energy transportation through the atmosphere to a receiver with minimum losses are now highly important. First of all, this implies optimization of the geometric size of a light beam at the receiving equipment while the conserving, if possible, the time and spectral properties of the radiation. In contrast to conventional linear regime of laser pulse propagation through the air, when one only needs to compensate for the beam diffraction and its spreading due to atmospheric turbulence along the source–receiver path, what is performed by a proper selection of the focal length of the transmitting optical system, the high-power laser radiation passes through the atmosphere in the self-action mode, i.e., it undergoes the amplitude and phase distortions due to nonlinear variation of optical characteristics of the medium itself.[1] In that case the effects of thermal defocusing, stimulated Raman scattering due to vibrational-rotational transitions of nitrogen and oxygen molecules, as well as the wind beam refraction essentially hamper conservation of the laser beam energy and size along the propagation path.

One of the solutions to this problem could be reduction of the laser radiation duration emitted while keeping the same energy. The sources of femtosecond-duration laser radiation, created over the past several years, can generate light pulses of several tens and hundreds of femtosecond duration with the peak power up to several petawatt. That high-power radiation propagates through the atmosphere in the filamentation regime along the optical path under conditions of a “frozen” medium, because the effects of thermal blooming and wind refraction cannot affect the radiation parameters.[2]

However, other factors appear in this situation, which adversely affect the characteristics of the laser pulses propagated. The filamentation of high-power radiation is accompanied by the formation, along the propagation path, of a plasma channel generated due to multiphoton gas ionization that can, first, significantly decrease the efficiency of the radiation energy transfer along the path and, secondly, essentially transform its time and spectral properties.[3] Besides, after being nonlinearly focused at the output from the filamentation zone a light beam acquires high divergence, which far exceeds the initial diffraction one, and sharply increases the beam size with the distance.[4]

Thus in order to optimize the energy transfer of high-power femtosecond radiation through the atmospheric path, it is necessary, first of all, to place a receiver close to a nonlinear beam focus, i.e., in the beginning of the filament. And vice versa, at a specified geometry of the path it is essential that a nonlinear focus be at the receiver’s zone, and to achieve this goal one should be capable of controlling the spatial position of the beam filamentation zone.

Below we consider three methods of achieving this task: control of the beam focusing along the path, the frequency modulation of the radiation, and formation of the intensity profile across the beam.

1. Beam focusing

One of the most traditional methods, which allows one to achieve concentrating the light energy somewhere in space, is to modify the curvature of the radiation phase front at the exit from the light source, or, in other words, the laser beam focusing. For a beam with the Gaussian spatial profile of the light field amplitude, characterized by the beam radius R_0, at the e^{-1} level of the intensity distribution maximum, and by the radius of the phase front curvature F' (focal length of the optical system), the square of the beam radius as a function of the distance z of the beam propagation in vacuum is described by the known expression:

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$$\frac{R^2(z)}{R_0^2} = \left(\frac{z}{2}\right)^2 + \left(1 - \frac{z}{F}\right)^2, \quad (1)$$

and its intensity is maximum at the point of a diffraction focus $z_F = z'_F / L_R = F/(1 + F^2/4)$. Here the following normalization of the values is used:

$$z = z'/L_R, \quad F = F'/L_R,$$

where $L_R = kR_0^2/2$ is the Rayleigh beam length, $k = 2\pi/\lambda$.

The Kerr effect favors the change of the medium refractive index n under the action of the light wave electric field. As a result the wave phase φ at the point $(\mathbf{r}_\perp, z)$ takes a shift $\delta\varphi_K$ proportional to the local field intensity $I(\mathbf{r}_\perp, z)$: $\delta\varphi_K(\mathbf{r}_\perp, z) = kzn_2I(\mathbf{r}_\perp, z)$, where n_2 is the coefficient characterizing the nonlinear response of the medium. It is evident that for a Gaussian beam the action of the Kerr effect is equivalent to its focusing by a nonaberrational spherical lens with the variable focal length. In this case within the framework of the theory of standard self-focusing,[5] Eq. (1) is transformed to the following form:

$$\frac{R^2(z)}{R_0^2} = (1 - \eta)\left(\frac{z}{2}\right)^2 + \left(1 - \frac{z}{F}\right)^2, \quad (2)$$

where $\eta = P_0/P_c$, P_0 is the initial radiation power, $P_c = 2\pi/(k^2n_0n_2)$ is the critical power of self-focusing. Thus at $\eta = 1$ the nonlinear Kerr lens compensates for the beam diffraction spreading, and at $\eta > 1$ results in its collapse ($R \to 0$) at the point of nonlinear focus:

$$z_{NF} = \left(\frac{1}{z_F} + \frac{1}{z_K}\right)^{-1} \simeq \frac{F}{\sqrt{\eta - 1}(F/2) + 1}, \quad (3)$$

where $z_K = 2/\sqrt{\eta - 1}$.

It should be noted that if the theory accounts the self-focusing of nonlinear (multiphoton) absorption of the atmosphere as well as of the plasma nonlinearity, then no beam collapse happens and, as a result, instead of the collapse the beam forms a waveguide propagation channel near the optical axis – the filament, in other words.

One can see from Eq. (3) that the nonlinear beam focus is always located before its diffraction focus, and, hence, at a fixed radiation power it is possible, by varying the parameter F, to move, within certain limits, the position of the light filament along the path.

Figure 1 shows for different values of the focal length F the calculated dependences on the propagation distance of the geometric (determined by the level e^{-1} from the maximum of transverse distribution of beam energy density) $R(z)$ and effective (power) radii $R_{eff}(z)$ of a femtosecond pulse with a Gaussian spatial-temporal intensity profile and the following parameters: the wavelength $\lambda_0 = 810$ nm, the duration $t_p = 80$ fs, the initial radius $R_0 = 1$ mm. The basis for numerical calculations was the nonlinear Schrödinger equation (NSE) describing the electromagnetic wave propagation in a medium in the approximation of slowly varying field amplitude supplemented by the rate equation for the concentration of free electrons of the plasma (see, for example, Refs. 4 and 6). The nonlinear Schrödinger equation allows for the light wave diffraction in the presence of the air dispersion as well as the basic physical mechanisms of medium nonlinearity for the ultra-short radiation pulses: instantaneous and inertial Kerr effect, radiation absorption and refraction by plasma, formed as a result of the multiphoton gas ionization.

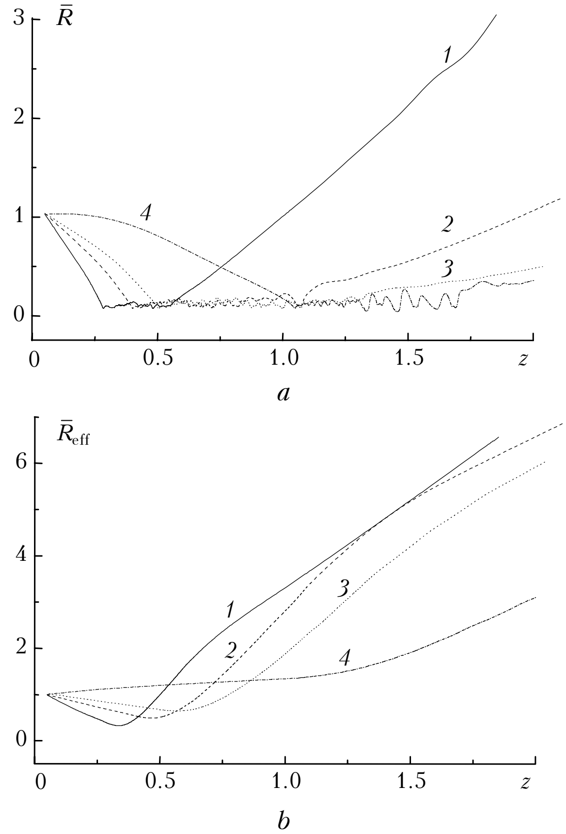


Fig. 1. The dependence of normalized values of geometrical $\bar{R} = R(z)/R_0$ (*a*) and the effective $\bar{R}_{eff} = R_{eff}(z)/R_0$ (*b*) radii of a femtosecond pulse with $\eta = 5$ and $F = 0.5$ (*1*); 1 (*2*); 2 (*3*), and –2 (*4*) on the distance along the propagation path.

The effective beam radius $R_{eff}(z)$ was commonly determined as a functional of the optical field intensity at each point of space $(\mathbf{r}_\perp, z)$ and time t[1]:

$$R_{eff}(z) = \left[\frac{1}{E(z)}\int_{-\infty}^{\infty} dt' \iint_{\mathbf{R}_\perp} d^2\mathbf{r}_\perp I(\mathbf{r}_\perp, z; t') \left|(\mathbf{r}_\perp - \mathbf{r}_{gr})\right|^2\right]^{1/2}, \quad (4)$$

where

$$E(z) = \int_{-\infty}^{\infty} dt' \iint_{\mathbf{R}_\perp} d^2\mathbf{r}_\perp I(\mathbf{r}_\perp, z; t')$$

is the total energy of the light pulse;

$$\mathbf{r}_{gr}(z) = 1/E(z) \int_{-\infty}^{\infty} dt' \iint_{\mathbf{R}_\perp} d^2\mathbf{r}_\perp \left[\mathbf{r}_\perp I(\mathbf{r}_\perp, z; t')\right]$$

is the radius-vector of the beam center of gravity. According to its definition, the effective radius determines the transverse size of the region where about 60% of the total beam energy is concentrated.

This figure shows that the decrease of the parameter F shifts the filament to the initial point of the path; in this case, in the region of nonlinear focus the degree of beam energy concentration increases (the effective radius decreases, Fig. 1*b*).

From the practical point of view the most interesting is the situation when a beam has previously been defocused ($F < 0$) so that $z_F < 0$ and $z_{NF} > z_K$, i.e., the onset of filamentation is shifted beyond the point of the collapse predicted by the theory of Kerr self-focusing for a collimated radiation at a given radiation power. It should be noted that the same effect can also be obtained by a specially selected small-scale modulation of the initial phase front of the beam.[7]

Theoretically the limiting value of the phase front radius of curvature, when the filament can yet be formed, is determined by the inequality:

$$|F| \geq |F_c| = 2/\sqrt{\eta - 1} \equiv z_K. \quad (5)$$

As the focal length approaches a given critical value, the distance $z_{NF} \to \infty$, and the effective beam radius grows infinitely.

Hence, using the focusing/defocusing of a beam it is possible to control the position of the waist of a high-power femtosecond radiation beam in the medium.

2. Radiation frequency modulation

The other method to control the femtosecond pulse filamentation is based on making use of the dispersion characteristics of the propagation medium itself. The idea consists in the use of originally long (subpicosecond) laser pulses with the initial peak power below the critical self-focusing power ($P_0 < P_c$) with a linear frequency modulation (chirping). The frequency modulation of the radiation leads in the medium with the frequency dispersion to the time compression of such a pulse as it propagates through the medium.[2] Thus the initially long pulse travels through the beginning of the path in the linear regime, without self-focusing, and then, being gradually compressed in time, passes into the regime of Kerr focusing accompanied by the formation of a filamentary structure. This technique has been already well approbated and has made it possible to obtain filaments at distances up to 200 m (Ref. 8) on horizontal paths in the atmosphere and at altitudes up to 20 km (Ref. 9) on the vertical paths.

Consider a chirped pulse with the Gaussian time profile:

$$U(\mathbf{r}_\perp, z = 0; t) = \tilde{U}(\mathbf{r}_\perp, 0)\exp\left\{-\frac{t^2}{2t_p^2}(1 + ib)\right\}, \quad (6)$$

where $\tilde{U}(\mathbf{r}_\perp, 0)$ is the transverse profile of the beam electric field envelope, Gaussian over the spatial coordinate $\mathbf{r}_\perp$ with the initial curvature of the phase front F; b is the parameter of chirping (frequency variation within the pulse limits $\delta\omega_0 = 4b/t_p$); t_p is the initial pulse duration. Now find the conditions imposed on the parameter value b, under which the initially “subcritical” pulse ($\eta_0 = P_0/P_c(z = 0) \leq 1$) converts to the self-focusing regime ($\eta(z) > 1$) on a slant atmospheric path in the region of the beam focal waist $z = 1$ ($F = 2$).

The solution of nonlinear Schrödinger equation for a beam with a temporal profile (6) for the path segment where the beam propagates linearly ($P_0 < P_c$) after integration over the beam cross section ignoring the plasma generation and Kerr effect for the medium with normal dispersion yields the following inequality[10]:

$$\eta^2(z) = \left[\frac{P_0(z)}{P_c(z)}\right]^2 = G^{-2}(z)\left[D_0^2\bar{G}^2 + \left(1 - D_0 b\bar{G}\right)^2\right] \geq 1, \quad (7)$$

where

$$D_0 = L_R \left.\frac{k''_\omega}{t_p^2}\right|_{z=0}; \quad \bar{G} = 1/L_R \int_0^{L_R} G(z')dz'; \quad k''_\omega = \partial^2 k/\partial\omega^2$$

is the dispersion of the pulse group velocity; $G(z)$ is the altitude model of k''_ω. In solving this inequality relative to the parameter b, we obtain the condition sufficient for “time self-focusing” along a slant path:

$$|b| \geq |b_c| = \frac{1}{D_0\bar{G}}\left(1 - \sqrt{\eta_0^2 G^2(z) - D_0^2\bar{G}^2}\right); \quad \mathrm{sign}(b) < 0. \quad (8)$$

The possibility itself of passing to the regime of supercritical powers ($\eta > 1$) due to the temporal pulse compression in the zone $z \approx L_R$ follows from non-negativity of the square root in Eq. (8) and is determined by the condition necessary for “temporal self-focusing”:

$$P_0(0)t_p^2/R_0^2 \geq P_c(0)k_0 k''_\omega(0)\bar{G}. \quad (9)$$

Equation (9) relates the initial parameters of a laser pulse to the physical characteristics of the medium it propagates through. For example, a 800-fs pulse of Ti-Sapphire laser with a beam of radius $R_0 = 4$ cm ($L_R =$ $= 6$ km) and the subcritical initial power $P_0 = 0.94P_c$ will turn into the self-focusing regime on the vertical path close to a geometric focus $z' \approx L_R$ at the initial negative chirping $|b| \geq 3$ ($\delta\omega_0 \simeq 15$ THz). In this case the altitude model of the behavior of the dispersion factor k''_ω was chosen in the exponential form[10]:

$$k''_\omega(h) = k''_\omega(0)G(h) = k''_\omega(0)\exp(-h/h^*),$$

where $k''_\omega(0) = 0.21$ fs^2/cm, and $h^* = 6.8$ km is the altitude of the inhomogeneous atmospheric layer.

Figure 2 shows numerically calculated results on self-focusing process in the air for a chirped femtosecond pulse with the initial power, which is almost twice lower than the critical one ($\eta_0 = 0.6$).

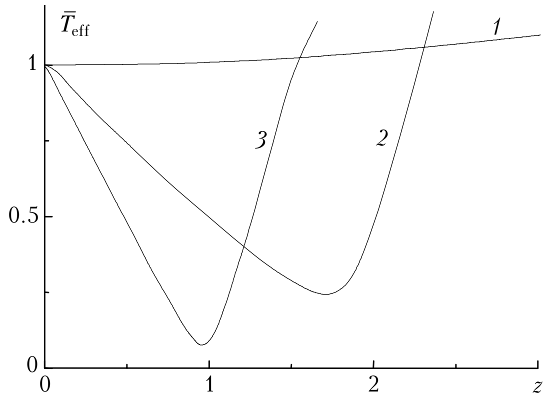
Figure 2 shows the variation along z of the geometric beam size and of the mean (effective) duration of the laser pulse T_l, which is determined in a way to similar to that used in the case of the effective radius (4):

$$T_{\text{eff}}^2(z) = \frac{1}{E(z)} \iint\limits_{\mathbf{R}_\perp} \mathrm{d}^2\mathbf{r}_\perp \int\limits_{-\infty}^{\infty} \mathrm{d}t' I(\mathbf{r}_\perp, z; t')(t' - t_0)^2,$$

where

$$t_0(z) = 1/E(z) \iint\limits_{\mathbf{R}_\perp} \mathrm{d}^2\mathbf{r}_\perp \int\limits_{-\infty}^{\infty} \mathrm{d}t' I(\mathbf{r}_\perp, z; t') t'$$

is the position of the time gravity center of the pulse.



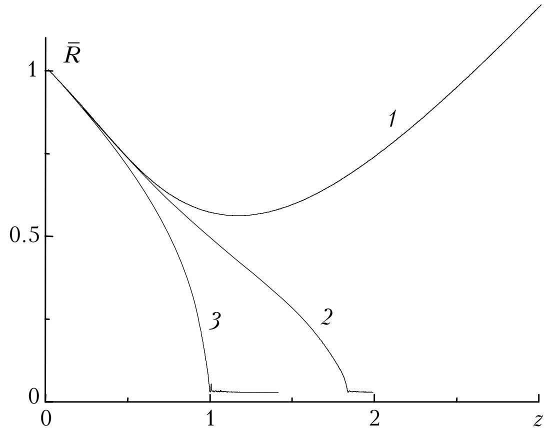


Fig. 2. The evolution along the horizontal path of normalized mean laser pulse duration $\bar{T}_{\text{eff}} = T_{\text{eff}}(z)/t_p$ (*a*) and the geometric radius $\bar{R} = R(z)/R_0$ (*b*) of a beam with "subcritical" initial power $\eta_0 = 0.6$, the initial duration $t_p = 200$ fs, and the radius $R_0 = 5$ mm ($F = 2$) at different degree of linear frequency modulation $b = 0$ (*1*); -10 (*2*); -20 (*3*).

It follows from Fig. 2 that because the initial power of a light pulse is lower than the critical one then in the absence of chirping the regime of its propagation is close to a linear one (curve *1*). However, the power of a chirped pulse increases as it propagates along the path and the pulse can already undergo self-focusing (curves *2* and *3*). In the case of the radiation parameters shown in Fig. 2 the threshold degree of the frequency modulation $|b_c|$ is about 8 according to Eq. (8).

One can also consider the case when the initial power of the light pulse exceeds the critical value ($\eta_0 > 1$) and it is required to shift the nonlinear beam focus to the right along the axis z. In this case, according to the condition of the problem the frequency dispersion of the medium must affect the modulated radiation pulse inversely: not decreasing its duration but increasing it instead, i.e., reducing the power as the radiation propagates along the path. This is achieved by changing the sign of chirping (the parameter b) from negative to positive. The numerically calculated evolution of the beam radius and the mean pulse duration in this case is presented in Fig. 3.

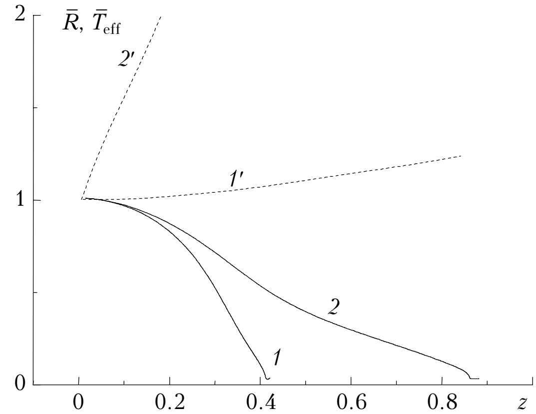


Fig. 3. The geometric beam radius $\bar{R}$ (*1*, *2*) and the mean pulse duration $\bar{T}_{\text{eff}}$ (*1′*, *2′*) with $\eta_0 = 10$, the initial duration $t_p = 80$ fs and $R_0 = 5$ mm ($F = \infty$) at different degree of linear frequency modulation $b = 0$ (*1*); +17 (*2*).

Hence similar to the beam focusing the pulse frequency modulation is capable of giving rise to its self-focusing and formation of filaments, whose position on the path can be controlled by changing the depth and sign of chirping.

3. Self-focusing of profiled light beams

A prerequisite for the control of nonlinear focusing of femtosecond radiation pulses using the variation of its spatial intensity profile is, first of all, the difference in the dynamics of linear diffraction of such beams. As known, free radiation diffraction is described by the following equation:

$$\frac{\partial \tilde{U}(\mathbf{r}_\perp, z)}{\partial z} = -\frac{1}{2ik_0} \nabla_\perp^2 \tilde{U}(\mathbf{r}_\perp, z), \tag{10}$$

where $\tilde{U}(\mathbf{r}_\perp, z)$ is the slowly varying complex amplitude of the electric field of a light wave.

As the initial conditions for Eq. (10) we shall consider the axisymmetric beams with a spatial profile presented by the following generalized dependence:

$$\tilde{U}(r,z=0)=\tilde{U}_0(q,p)\,\mathrm{e}^{-\frac{1}{2}\left[\frac{r^2}{R_0^2}\right]^q}\left[1-\mathrm{e}^{-\left[\frac{r^2}{R_0^2}\right]^q\left(p^{-2q}-1\right)}\right]^{1/2}, \quad (11)$$

where q is the parameter determining the beam shape; $p=R_{sh}/R_0$ is the shading parameter; R_{sh} is the radius of the beam shadow part at the intensity level e^{-1}, $\tilde{U}_0$ is the initial radiation amplitude selected from the condition of equality of the total radiation energy of different profiles entering the medium. Equation (11) enables us to preset the beams of the Gaussian ($g=1$, $p=0$), tubular (ring) ($q\geq 1$, $p>0$), and super-Gaussian profiles ($g>1$, $p=0$).

Figure 4 shows the results on comparison of free diffraction for the above-mentioned types of beams.

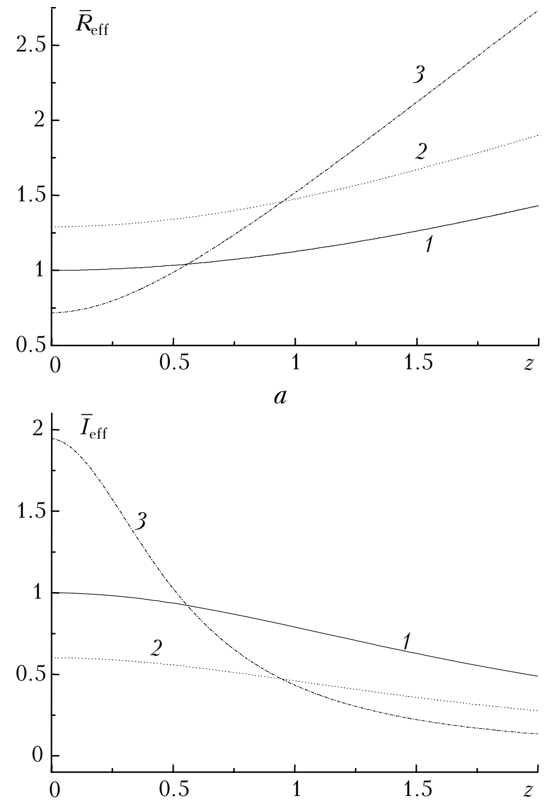


Fig. 4. The dependence of the effective radius (*a*) and mean intensity (*b*) of collimated beams of Gaussian (*1*), ring (*2*) with $p=0.5$, and super-Gaussian ($q=8$) cross section (*3*) on the space variable.

Figure 4 shows the variation of the effective beam radius $R_{eff}(z)$ and its mean intensity $I_{eff}(z)=P_0/[\pi R_{eff}^2(z)]$ at the diffraction length $L_D=2L_R$. The data presented in the figure are normalized, for the purpose of illustration, to the corresponding values of the Gaussian beam parameters in the beginning of the path:

$$\bar{R}_{eff}(z)=R_{eff}(z)/R_0;\ \ \bar{I}_{eff}(z)=(8\pi/c)I_{eff}(z)/\left|\tilde{U}(1,0)\right|^2.$$

It is evident that the beam with quasiuniform intensity distribution has the least value of the initial effective radius and the highest mean intensity. Therefore, the diffraction of such a beam takes place more actively. On the contrary, tubular beams are characterized by a more smooth variation of intensity along the path as compared with a beam of Gaussian profile. This predetermines the differences in self-focusing of such beams.

Figure 5 shows the evolution of dimensional parameters of the beams of Gaussian, ring, and super-Gaussian cross sections at their propagation in the air. The initial radiation power equaled five critical ones. Figure 5 shows that the nonlinear lens formed by a beam of super-Gaussian profile ($z_{NF}\simeq 0.5$) has the least focal length, then follows a Gaussian beam ($z_{NF}\simeq 0.75$), and finally the tubular beams have the longest self-focusing distance ($z_{NF}\simeq 1.5$).

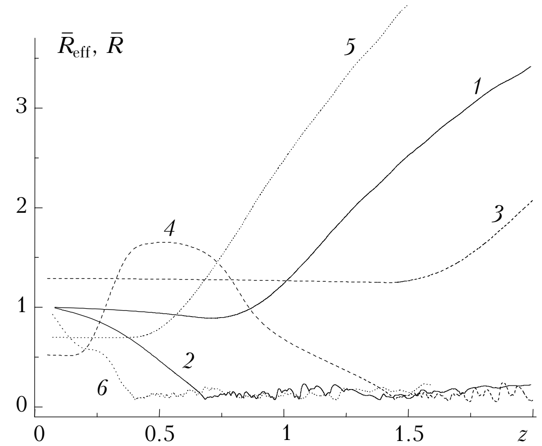


Fig. 5. Effective (*1*, *3*, *5*) and geometric radii (*2*, *4*, *6*) of collimated beams of Gaussian (*1*, *2*), ring (*3*, *4*) with $p=0.5$, and super-Gaussian ($q=8$) cross section (*5*, *6*) along the path at $\eta=5$.

If we introduce a concept of mean laser beam power $P_{eff}=I_{eff}\pi R_0^2$, then at a constant initial effective pulse duration T_{eff} the values of P_{eff} in the beginning of the propagation path for the beams of the ring and super-Gaussian profiles, as follows from Fig. 4, will differ from a Gaussian beam by two times toward a decrease in the power and its increase, respectively. Now using Eq. (3) and changing the real initial power P_0 of a beam by its mean value P_{eff}, we can obtain calculated values of the coordinate of the nonlinear focus of a beam of an arbitrary type:

$$z_{NF}=\frac{F}{(F/2)\sqrt{\eta\bar{R}_{eff}^{-2}(z=0)-1}+1}. \quad (12)$$

Hence, for the tubular beam we have from Eq. (12) that $\bar{R}_{eff}=1.29$ and $z_{NF}=1.43$, and for the super-Gaussian beam $\bar{R}_{eff}=0.69$ and $z_{NF}=0.61$ that correlates with the calculated data shown in Fig. 5.

Note that an extraordinary behavior of a geometric radius of the ring beam along the path (curve *4* in Fig. 5) is associated with the transformation, as the diffraction is being developed, of the initially bimodal intensity distribution ($\bar{R}$ is determined by the ring radius, see Fig. 6) first to a corona-like profile ($\bar{R}$ is determined by the largest transverse scale) and then to a quasi-Lorentzian unimodal profile.

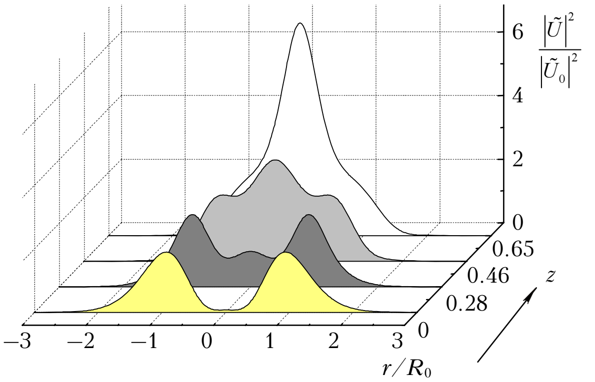


Fig. 6. The cross section of relative intensity of a collimated ring beam ($p = 0.5$) along the path at $\eta = 5$.

Conclusion

Thus for purposes of increasing the light energy concentration in the plane of the receiving equipment we have considered three methods of control of the spatial position of nonlinear focus of a femtosecond laser radiation pulse propagated through the atmosphere in the filamentation regime. These methods are the following: the spatial beam focusing, the frequency modulation of the radiation, and the beam profiling (pre-formation of the intensity distribution across the beam). For each of the considered versions the relations have been obtained, which enabled us to assess the coordinates of a nonlinear focus depending on the radiation parameters.

The following regularities have been determined:

1. To obtain the nonlinear beam focus at a maximum distance from the beginning of the optical path, it is necessary either to use a previously defocused radiation ($F < 0$) or to use the frequency-modulated radiation with the chirping parameter $b < 0$. In the first case the initial radiation power should essentially exceed its critical value for the Kerr self-focusing in the medium ($\eta \gg 1$), while in the second case this ratio should have the opposite sign: $\eta < 1$.

2. Another method of achieving the above-mentioned goal consists in changing the initial transverse profile of the laser beam intensity. The beams with the ring intensity distribution possess the largest self-focusing distance, while the super-Gaussian beams have the shortest nonlinear focus, all other factors being the same.

3. For the first two versions of optimization of light energy transfer, from the above-mentioned ones, the limiting values exist restricting the range of variation of radiation parameters, using which the control of the nonlinear beam focus can be performed. Thus for spatial focusing such a limit is the value of the focal length F_c provided by Eq. (5), for the radiation modulation such a limit is the chirping parameter b_c determined by the expression (6).

All the above-mentioned versions of the control of the position z_{NF}, in principle, can be reduced to the universal relationship, which is as follows

$$z_{NF} = 2\left[\sqrt{\frac{\eta(z)}{\bar{R}^2_{eff}(z=0)} - 1} + \frac{2}{F}\right]^{-1}.$$

Here F is the initial curvature of the phase front, the beam intensity profile is accounted for by the initial effective radius R_{eff}, and the radiation frequency modulation enters into the equation through the parameter $\eta(z)$, defined by Eq. (7).

Acknowledgments

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