PROPAGATION THROUGH STOCHASTIC SCATTERING MEDIA WITH STRONGLY FLUCTUATING SCATTERING PARAMETERS

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A technique is developed for calculating moments of arbitrary order for radiation transmitted through or reflected from a layer of stochastic scatterers with strongly fluctuating scattering parameters. Contrast statistical properties are analyzed for an image transmitted through such a layer.

A method was developed in Ref. 1 for solving the stochastic equation of transfer in the parabolic approximation, based on the assumption of local homogeneity (the horizontal scale of macro inhomogeneities in the medium is much large than the width of the Green's function for the equation of transfer). The assumption of normally distributed scattering parameters was used to calculate the first and second moments of the Green's function. It is obvious that this assumption is true only when variations in the scattering parameters are sufficiently small (weak fluctuations).

In this paper, we study the properties of propagation through a randomly inhomogeneous medium when the variations are fairly large (strong fluctuations).

Within the framework of the method used in Ref. 1, stochastic realizations of the stochastic layer diffusion coefficient at the point p are given by

$$T(\rho) = \exp\left[-\int_0^{z_s} k^*(r_{eu})du/\mu_0\right] \tag{1}$$

and a stochastic realization of the stochastic layer local optical transfer function (OTF)[2] is

$$T(\omega;\rho) = \exp\left[-\int_0^{z_s} \sigma^*(\rho;u)F(\omega u)du\right], \tag{2}$$

$$F(p)=1-Q(p),\quad Q(p)=\frac{1}{2}\int_0^{\pi/2} \gamma\, i_0(\gamma)\, I_0(p\gamma)\, d\gamma. \tag{3}$$

A stochastic realization of the brightness of normally incident and observed radiation reflected from the layer is

$$I(\rho;N) = \frac{\sigma_0(\pi)}{2k_0^*}\left[1-\exp\left[-2k_0^*\int_0^{z_s}\eta(\rho;u)\,du\right]\right]. \tag{4}$$

Here $k^*(r)$ is the spatial distribution of the effective absorption index of the medium $\varepsilon(r) - \sigma^*(r)$, where $\varepsilon(r)$ is the attenuation index, $\sigma^*(r)$ is the effective scattering index,[4] z_s is the layer thickness, $\{r_{eu}\} = (\rho - a\ (z_s - u);\ u)$, $a = \Omega_{0\perp}/\mu_0$. Ω_0 is the unit vector defining the direction of incidence on the layer, $\Omega_{0\perp}$ is the projection of Ω_0 into the plane of the layer, μ_0 is the cosine of the angle between the vector Ω_0 and the z-axis, $i_0(\gamma)$ is the scattering phase function of the medium, $\sigma_0(\pi)$ is the index of backscattering by macroinhomogeneities, k_0^* is the effective absorption index of a macroinhomogeneity, $\eta(\rho;\ u)$ is an indicating function which is equal to unity within a macroinhomogeneity and zero outside, J_0 is the zero-order Bessel function, and ω is the spatial frequency.

Using expressions (1)–(4), we can determine the expressions for moments of arbitrary order of the optical field characteristics in question. It is necessary to know the characteristic functionals of the random processes $k^*(r) - \Phi_k[v]$, $\sigma^*(r) - \Phi_\sigma[v]$ and $k^*(r) = k_0^*\eta(\rho;u) - \Phi_z[v]$ to calculate the moments of $T(\rho;\ N)$, $T(\omega;\ \rho)$ and $I(\rho;\ N)$, respectively. Then

$$\langle T^n\rangle = \Phi_k[ni],\quad \langle T^n(\omega)\rangle = \Phi_\sigma[niF],$$

$$\langle I^n(N)\rangle = g^n\sum_{m=0}^{n} C_n^m\,(-1)^m\,\Phi_z[im], \tag{5}$$

where

$$C_n^m = \frac{n!}{m!\,(n-m)!},\quad g = \frac{\sigma_0(\pi)}{2k_0^*},\quad i = \sqrt{-1}$$

Fairly simple expressions for characteristic functionals can be obtained for Poisson-distributed scattering parameters[5]. We know that for a Poisson random process,

$$\Phi_k[F] = \exp\left\{\frac{\bar{M}}{z_s}\int_0^{z_s} dt\left[\chi_k\left(\int_0^{z_s}\eta_0(t-t')F(t')\,dt'\right]-1\right]\right\}, \tag{6}$$

where $\bar{M}$ is the mean number of macroinhomogeneities on a segment of length z_s, $\chi_k(v)$ is the characteristic function of the random quantity k_0^*

$$\eta_0(u) = \begin{cases} 1, & \text{if } 0<u<u_0 \\ 0, & \text{if } u<0,\ u>u_0 \end{cases}$$

and u_0 is the macroinhomogeneity scale length. Expressions for the functionals $\Phi_\sigma[v]$ and $\Phi_z[v]$ can be obtained similarly.

For example we adduce the expressions for the moments $\langle T^n \rangle$ and $\langle I^n(N) \rangle$, which are valid for a medium whose effective absorption thickness is governed by an exponential distribution function:

$$\tau^*_{kh} = k^*_0 u_0: \quad \langle T^n \rangle = \exp -\left[\frac{\bar{M} n \langle \tau^*_{kH} \rangle}{1+n \langle \tau^*_{kH} \rangle} \right]$$

where $\langle \tau^*_{kH} \rangle$ is the mean effective absorption thickness for macroinhomogeneities. In that case the expressions for $\langle I^n(N) \rangle$ are given by (5), where $\Phi_z(im) = \exp\,[-2\bar{M}\langle \tau^*_{kH} \rangle m/(1 + 2m\langle \tau^*_{kH} \rangle)]$.

Let the scattering phase function of the medium be the small-angle approximation to the Henyey-Greenstein function:

$i_0(\gamma) = 2\alpha(\alpha^2 + \gamma^2)^{-3/2}$, where $\alpha = (1 - \bar{\mu})\bar{\mu}^{-1/2}$,

and $\bar{\mu}$ is the mean cosine of the scattering angle. Than the expressions for $\langle T^n(\omega) \rangle$ can be obtained quite easily:

$$\langle T^n(\omega) \rangle = \exp\left\{ -\bar{M}\left[1 - \exp\left[n \langle \tau^*_{\sigma H} \rangle \varphi(\omega^*) \right] \right] \right\}$$

$$\varphi(\omega^*) = (\omega^*)^{-1}\left[Ei(\mathcal{L}_n) - Ei(\mathcal{L}_n e^{-\omega^*}) \right]$$

$$\mathcal{L}_n = n \langle \tau^*_{\sigma H} \rangle (1 - e^{-\omega^* \theta})(\omega^* \theta)^{-1}$$

where $\omega^* = \alpha\omega z$, $\theta = u/z$, $Ei(x) = \int_{-\infty}^{x} t^{-1} e^{-t} dt$ is the exponential integral, and $\langle \tau^*_{\sigma H} \rangle$ is the mean effective scattering thickness for macroinhomogeneities.

A knowledge of the moments of the optical, field characteristics under consideration enables one to calculate their distribution function, for example, by expanding this function in orthogonal polynomials[6].

The main shortcoming of the Poisson random: process model is that a Poisson distribution for the number of inhomogeneities is; only valid when $\bar{M}$ is sufficiently large. For small $\bar{M}$, the Poisson model simply fails to hold. Therefore, let us consider another model for a stochastic scattering layer.

This model enables one to analyze the radiation field statistical characteristics: in stochastic media whose macroinhomogeneity scale length is of the same order as the layer thickness.

Let us consider the coefficient of diffuse transmission as an example. Calculations of transmission in the forward direction and of reflection will give similar results.

The random quantity $\tau^*_k = \int_0^{z_s} k^*(r_{eu}) \frac{du}{\mu_0}$ entering into (1) can be written as $\tau^*_k = \sum_{j=1}^{M} \tau^*_{kj}$, where τ^*_{kj} is the random value of the effective optical depth of inhomogeneity j, and M is the random number of macroinhomogeneities along the vector r_{eu}. Assuming the τ^*_{kj} to be statistically independent, and supposing that inhomogeneity j is realized with the probability p_j, one can write down the characteristic function of the random quantity τ^*_k as[6]

$$\Phi(v) = \sum_{j=1}^{\infty} p_j \chi^j_\tau(v),$$

where $\chi\tau(v)$ is the characteristic function of the random quality τ^*_{kj}. In a number of cases P_j is unknown, while the macroinhomogeneity size-distribution function $f(\Delta)$ is known. The function $\Phi(v)$ can then be obtained approximately if P_j 0 when $j > 2$. Indeed, if this condition is fulfilled, then

$$P_2 = \int_0^{z_s} f(\Delta)d\Delta, \quad P_1 = 1 - P_2.$$

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