# DEFORMATION OF COHERENT OPTICAL PULSES PROPAGATING ALONG SLANT PATHS IN A RESONANTLY ABSORBING ATMOSPHERE 

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#### Abstract

This paper concerns the analysis of optical pulse deformation occurring in the atmosphere when such pulses propagate along slant paths in the presence of resonant absorption. The analysis of deformation presented in the paper deals with Gaussian pulses of different durations. The atmosphere is considered in this study to be an inhomogeneous vertically stratified medium. The influence of resonant absorption by $\mathrm{H}_{2} \mathrm{O}$ vapor on optical pulse parameters is demonstrated for ruby laser radiation (at $\lambda=0.69438 \mu \mathrm{~m}$ ). Calculations of the optical wave field have been made using the geometrical optics approach. It is shown that the deformations of light pulses can be quite significant, and are determined by the direction of propagation, angular beam width, detuning of incident radiation from resonance, and the initial pulse duration.


## INTRODUCTION

The deformation of optical pulses propagating along slant paths in a resonantly absorbing atmosphere have certain peculiarities caused by variations of the medium's optical properties along the paths. Thus, for example, the energy losses of pulses propagating In a resonantly absorbing medium are determined by the ratio of absorption line width $\gamma$ to the spectral width $\delta_{\mathrm{r}}$ of the emitted radiation (Ref. 1), and will therefore depend on the range along the slant path because of narrowing of the absorption with increasing height. A shift 'in center frequency of the absorption line induced by pressure can also alter the medium absorption coefficient ${ }^{2}$. Specific features of light propagation along slant paths are not only manifested by changes in the beam's energy parameters but also by significant distortion of the pulse shape resulting from atmospheric refraction In the spectral region of selective atmospheric absorption ${ }^{3,4}$. Note that the foregoing effects are, as a rule, taken into consideration in connection with their Influence on local optical properties of the atmosphere ${ }^{2,5}$, as well as in propagation problems involving narrow-band radiation ${ }^{3,6}$ (when $\delta_{r} / \gamma \ll 1$ ).

This paper presents an analysis of coherent pulse deformation in beams propagating along slant atmospheric paths in the presence of resonant absorption. We treat the case of Gaussian pulses of different durations.

The atmospheric volume is modeled In this study a plane stratified Inhomogeneous medium, the parameters of which are varied according to the standard statistical models of the atmosphere ${ }^{7}$. A
ten-kilometer thick layer of the atmosphere was Investigated with water vapor as a resonantly absorbing component for radiation at $\lambda=0.69438 \mu \mathrm{~m}$. The calculations took into account the charge in shape, width and center frequency of the absorption line, and in of the water vapor number density with the altitude. Similarly, the resonant component of the mediums refractive index varied according to the Kramers-Kronig relationships ${ }^{3}$. The nonresonant component of the refractive index was calculated using the formula ${ }^{8}$

$$
n_{0}(r)=1.0+58.2 \times 10^{-6}\left(1+7.52 \times 10^{-3} \lambda^{-2}\right) P(r) / T(r) \text {, }
$$

where $\lambda$ is the wavelength of radiation in $\mu \mathrm{m}, P(r)$ is the pressure (torr), and $T(r)$ is the temperature in K. Nonresonant losses are trivially taken into account, and will be neglected here.

## THE PROPAGATION MODEL

The radiation field in a medium can be described in the geometrical optics approximation by

$$
\begin{align*}
& E(t, r(s))=\frac{e^{i \omega t}}{2 \pi} \int_{-\infty}^{\infty} \varepsilon(0, v) \exp \left\{-i k \int_{0}^{s}\left[n_{0}\left(s^{\prime}\right)+\right.\right. \\
& \left.\left.+2 \pi S_{0} \times N\left(s^{\prime}\right) G\left(v, s^{\prime}\right)\right] d s^{\prime}+i v t\right\} d v, \tag{1}
\end{align*}
$$

where $\varepsilon(0, v)$ is the Fourier transform of slowly varying complex amplitude of the field at the medium's input, $\omega$ is the carrier frequency of the field, $S_{0}$ is the intensity of line absorption per unit number
density $N(r)$ of the absorbing gas. The functions $\operatorname{JmG}(v, r)$ and $\operatorname{Re} G(v, r)$ describe the absorption line shape and the resonant component of the refractive index of the medium (anomalous dispersion region), respectively. The integration in the exponential term of Eq. (1) is carried out over the real ray determined by the eikonal equation for the wave,

$$
\begin{equation*}
\Delta \psi(\nu, r)=n_{0}(r)+2 \pi S_{0} N(r) \operatorname{Re} G(\nu, r) \tag{2}
\end{equation*}
$$

The solution of this equation is represented by Snell's law ${ }^{9}$.

Equation (1) is valid, taking Eq. (2) into account, for band-limited functions $\varepsilon(r, v)$ in a small neighborhood of и (the condition of slowly changing amplitudes $)^{10}$, and neglecting diffraction effects. Further-more, the following condition ${ }^{11}$ must be satisfied:

$$
\begin{equation*}
\left|S_{0} N(r) \operatorname{Jm} G(\nu, r)\right| \ll n_{0}(r) . \tag{3}
\end{equation*}
$$

Note that a weak dependence of the absorption line intensity on temperature is neglected in Eq. (1) because, e.g., for midlatitudes in summer the temperature change occurring in the 10 km atmospheric layer causes only a $6 \%$ change in intensity, while its change due to number density variation can exceed $190 \%$.

## QUALITATIVE ANALYSIS OF THE PROPAGATION PROCESS

Consider some specific features following from Eq. (1) for the case of an inhomogeneous resonantly absorbing atmosphere.


FIG. 1. The optical depth along a slant path in the atmosphere as a function of frequency $v$, calculated for the summer atmospheric model at mid-latitudes'; curves 1 are for $\theta=0$, curves 2 are for $\theta=40^{\circ}$; a) shows the data for downward propagation and b) for the upward propagation.

The time required for a quasimonochromatic wave packet ( $S_{\mathrm{r}} / \gamma \ll 1$ ) to travel along a path of the length $L$ is given by (see, for example, Ref. 12)
$\tau \equiv \int_{0}^{\mathrm{L}} \frac{d s}{V_{\mathrm{g}}(s)}=\int_{0}^{\mathrm{L}} \frac{n_{0}(s) d s}{c}+\frac{v}{c} \int_{0}^{\mathrm{L}} \frac{\partial n_{\mathrm{p}}(\nu, s)}{\partial v} d s$
where $V_{\mathrm{g}}$ is the group velocity of the pulse in the medium, $n_{\mathrm{r}}$ is the resonant part of the medium's refractive index, $s$ is the coordinate along the ray. For slant atmospheric paths the spatial position of the rays for different spectral components of the pulse will depend on the frequency, due to refraction, i.e., $s=s(v)$.Figure 1 presents the computational results for the imaginary part of the phase function from Eq. (1), which determines the accumulated phase of different spectral components of a pulse. As is seen from the figure, the time required for a wave packet (strictly speaking, its maximum) to travel along a slant path depends on both its angle of incidence and the direction of propagation. This can be explained by the fact that in downward propagation, atmospheric refraction reduces the frequency gradient of the function $\int n_{\mathrm{r}}(v, s(v)) d s(v)$, caused by the dependence of $n_{\mathrm{r}}$ on the frequency ( $n_{\mathrm{r}}=n_{\mathrm{r}}(v)$ ), while in the opposite case of upward propagation this gradient is increased by the atmospheric refraction.

For of pulses of arbitrary bandwidth, one can obtain the asymptotic form of the integral in Eq. (1) at large $s$ by using, for example, the method of stationary phase ${ }^{13}$, since the principal contribution then comes from points $v_{1}$ of the phase function where
$\varphi(v, s) \equiv \frac{1}{c} \frac{\partial}{\partial v} \int_{0}^{s} v n_{\mathrm{p}}\left(v, s^{\prime}\right) d s^{\prime}=\frac{1}{c} \int n_{0}\left(s^{\prime}\right) d s^{\prime}-t$
One can see from Figure 2 that the number of stationary points is determined by the pulse bandwidth $S_{\mathrm{p}}$ and by the frequency offset from resonance. If there are two or more stationary points then one can expect oscillations in the envelope of pulse intensity $I(t, r)$ due to the interference of contributions from different stationary points.


FIG. 2. Asymptotic evaluation of the integral in Eq. (5). Here $B(v, s)=A(v) \exp \left\{-i k\left[\psi^{0}(v)+\right.\right.$
$\left.\left.+2 \pi S_{0} \int_{0}^{\mathrm{s}} N\left(S^{\prime}\right) \operatorname{Im} G\left(v, s^{\prime}\right) d s^{\prime}\right]\right\}$

It is also seen from Figure 2 that the pulse duration in the medium is determined by $\max _{v} \varphi(v, s)-\min _{v} \varphi(v, s)$. Clearly, then, in downward propagation the duration of a pulse will be reduced for slant paths, compared to the case of a vertical path. For upward propagation, the situation is the opposite, i.e., the duration of a pulse will be greater than for a vertical path.

## NUMERICAL RESULTS

The calculation of $G(v, s)$, taking into account the transformation of the absorption line profile with height, has been carried out using Simpson's rule with the Runge correction term ${ }^{14}$. The integrand of Eq. (1) is then replaced by a periodic function with a period considerably exceeding that of $\varepsilon(0, v)$. After sampling the integrand, the corresponding Fourier series is calculated using the fast Fourier transform algorithm.

The results of calculations illustrating the deformation of a coherent Gaussian pulse for different frequency offsets from resonance at $S_{\mathrm{p}} / \gamma(0)=0.3$ are presented in Figs. 3a and 4a. For a frequency shift $\Delta=\gamma(0)=0.1 \mathrm{~cm}^{-1}$, the group velocity of the light


FIG. 3. Deformation of coherent (a) and incoherent (b) Gaussian pulses propagating along slant atmospheric paths. The calculations were made for the summer mid-latitude atmospheric model. Here $\Delta=-0.1 \mathrm{~cm}^{-1}$, $\tau=30 \mathrm{~cm}$. Curves 1 and 2 are obtained for downward propagation at incidence angles $0^{\circ}$ and $40^{\circ}$ respectively; curve 3 represents the data for upward propagation at $\theta=40^{\circ}$; $I(t, 0)_{\max }=0.767$.

The influence of atmospheric turbulence on the characteristics of pulsed radiation can be assessed by taking into account the following considerations. The
pulse in the atmospheric ground layer is equal to the phase velocity and the shape of the pulse should not be distorted. However, at $h>0$ for this frequency shift from resonance, the medium has normal frequency dispersion because of narrowing of the line profile with increasing altitude. On the whole, this results in a displacement of the pulse maximum toward its trailing edge. For $\Delta=0$, the medium is characterized by anomalous frequency dispersion over the spectral range occupied by the components of the pulse within the whole atmospheric layer up to 10 km in height. In that case the relationship $V_{\mathrm{g}}(\mathrm{r})>n_{0}(\mathrm{r}) / \mathrm{c}$ is valid and the maximum of the pulse is displaced toward its leading edge. It should be noted here that $\mathrm{S}_{\mathrm{p}} / \gamma(h)$ increases with increasing altitude, and as a consequence, all the above conclusions based on the group velocity are valid only qualitatively ${ }^{12}$. It is readily seen from the figures that the pulse shape is very sensitive to the direction of propagation. Thus, in the case of downward propagation of a pulse through the atmosphere, pulse shape distortions are smaller for larger angles of incidence, while in the opposite case of upward propagation, the distortions are greater for larger angles of incidence.


FIG. 4. Deformations of coherent (a) and incoherent (b) Gaussian pulses propagating along slant atmospheric paths. The calculations were made for the summer atmospheric model at mid-latitudes. Here $\Delta=0, \quad \tau=30 \mathrm{~cm}$. Curves 1 and 2 are obtained for downward propagation at incidence angles $0^{\circ}$ and $40^{\circ}$, respectively, and curve 3 represents the data for upward propagation at $\theta=40^{\circ}$; $I(t, 0)_{\max }=0.767$.
coherence of an optical wave is reduced by passage through a randomly inhomogeneous medium. The optical wave can then be represented as a superposition
of coherent and noncoherent components ${ }^{15}$. Results presented in Figs. 3b and 4b show that the shape of the incoherent component of a pulse is not changed by propagation through a randomly inhomogeneous medium. In general the influence of atmospheric turbulence on the characteristics of a light pulse is minor (see, e.g., Ref. 16).

The calculations show that when $S_{\mathrm{p}} / \gamma(0)>1.0$, the pulse distortions depend only slightly on the frequency shift from resonance. This is illustrated by the data in Fig. 5 which show the calculated pulse shapes for $S_{\mathrm{p}} / \gamma(0)=3.0$ and $\Delta=0$. It is seen from this figure that for this case more pronounced oscillations are observed in the pulse shape. Since the maximum values of intensity $I(t, r)$ are approximately the same for the cases presented in Figures 3 to 5, the energy losses of a pulse propagating through the medium are inversely proportional to the number of oscillation peaks in the pulse shape.


FIG. 5. Deformation of coherent Gaussian pulses propagating along slant atmospheric paths. The calculations were made for a summer mid-latitude atmospheric model. Here $\Delta=0, \tau=3 \mathrm{~cm}$. Curves 1 and 2 are obtained for downward propagation at incidence angles $0^{\circ}$ and $40^{\circ}$ respectively; curve 3 represents the data for upward propagation at $\theta=40^{\circ} ; l(t, 0)_{\max }=0.767$.

Note that ail the characteristic features of the pulse shape transformations revealed in the computational data are in good agreement with the qualitative analysis in section 3 of this paper.

Specific features of the deformation of coherent optical pulses should be taken into account in the propagation of light through the atmosphere, as well as when selecting optimal conditions for sensing of the gaseous atmospheric constituents along slant paths.

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## REFERENCES

1. L. Allen and J. Eberly, Optical Resonance and Troo-Level Atoms (Mir, Moscow, 1978).
2. V.V. Zuev, Yu.N. Ponomarev, A. M. Solodov, et al., Optics Letters 10, 318, (1985).
3. Yu.V. Kistenev and Yu.N. Ponomarev, Izv. Vyssh. Uchebn. Zaved. Fizika No. 8, 21-25 (1987). 4. M.V. Kabanov, Yu.V. Kistenev, and Yu.N. Ponomarev, Izvestia Akad. Nauk SSSR, ser. Fiz. Atmosfery i Okeana 24, 566 (1988).
4. O.K. Voitsekhovskaya, Yu.S. Makushkin, V.N. Maritchev, et al., Izv. Vyssh. Uchebn. Zaved. Fizika No. 1, 62 (1977)
5. V.P. Lopasov, YU.S. Makushkin, A.A. Mitsel', and Yu.N. Ponomarev, Izv. Vyssh. Uchebn. Zaved., Fizika, No. 2, 7 (1978).
6. F.X. Kneizya, F.R. Shettle, W.O. Gallery, et al., Atmospheric Transmittance/Radiance: Computer Code Lowtran 5, Environment Res. Paper, AFGL-TR-80-0067 No. 697 (1980).
7. Laser Beam Propagation in the Atmosphere, Ed. by D. Strohben (Mir, Moscow, 1981).
8. M.B. Vinogradova, O.V. Rudenko and A.P. Sukhorukov, Theory of Waves (Nauka, Moscow, 1979).
9. Yu.A. Kravtsov and Yu.I. Orlov, Geometrical Optics of Inhomogeneous Media (Nauka, Moscow, 1980).
10. Yu.A. Kravtsov, Izv. Vyssh. Uchebn. Zaved., Radiofiz. 10, No. 9-10, 1283 (1967).
11. M.D. Crisp, Physical Review A1, No. 6, 1604 (1971); A4, No. 5, 2104 (1971).
12. M.B. Fedoryuk, The Saddle-Point Method (Nauka, Moscow, 1977).
13. A.I. Plis, N.A. Slivina. Laboratory Practical Work on Higher Mathematics (Vysshaya shkola, Moscow, 1983)
14. V.D. Moskalev. Theoretical Principles of Opticsphysics Research (Mashinostroenie, Leningrad, 1987) 16. V.A. Donchenko, E.V. Lugin. Electrodynamics and Propagation of Waves (Izdat.TGU,Tomsk, 1980)
