# ON THE INTERSECTION OF DIFFERENT SCATTERING PHASE FUNCTIONS IN THE UV REGION OF THE SPECTRUM 

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#### Abstract

The displacement of the angle of intersection of the relative total luminance indicatrices with one another as well as with the spherical and Rayleigh luminance functions is studied with the help of model calculations of the luminance indicatrices of the daytime sky in the UV-region of the spectrum. The contribution of each term of the luminance indicatrix (aerosol, Rayleigh, and multiple scattering) to the drift of these intersection angles is determined. It is shown that in the UV-part of the spectrum multiple-scattering effects can cause the angles of intersection of the indicatrices to follow a loop-shaped trajectory.


## INTRODUCTION

Phenomenological and statistical studies of the luminance of the clear daytime sky in different regions of the spectrum have also included a search for special points in the sky, in particular, the regions of the sky where the luminance does not depend on the form of the scattering phase function. This made it easier to model mathematically the scattering processes in the special regions and it has made it possible to find the correlations between an integral quantity - the light-scattering optical thickness of a dispersed medium $\tau$ -

$$
\begin{equation*}
\tau=2 \pi \int_{0}^{\pi} \mu(\varphi) \sin \varphi d \varphi \tag{1}
\end{equation*}
$$

and the directional light-scattering coefficient $\mu(\varphi)$ for a definite scattering angle $\varphi=\varphi_{0}$, employed, for example, in the nephelometric method for determining $\tau$. These questions, as a rule, are related, since the angle $\varphi_{0}$ is often chosen at the point of intersection of the real observed indicatrix

$$
\mu_{s p}\left(\varphi_{0}=\varphi_{s p, o b}\right)
$$

where the relation

$$
\begin{equation*}
\tau_{o b}=4 \pi \mu_{o b}\left(\varphi_{o}\right) \tag{2}
\end{equation*}
$$

holds. The quantity $\tau_{\mathrm{ob}}$ is defined by the formula(1), in which the scattering phase function $p$ is replaced by the luminance indicatrix $\mu_{\mathrm{ob}}$.

Experience has shown, however, that the position of the angles $\varphi_{\text {sp,ob }}$ and $\varphi_{0}$, which at first appeared to be a constant for any scattering phase functions, ${ }^{1-3}$
depends on the region of the spectrum where the measurements are performed, the layers of the atmosphere which are studied, and the optical states of the atmosphere. ${ }^{4-7}$

According to observations in the visible part of the spectrum the relative luminance indicatrices $f_{\mathrm{ob}}(\varphi)$, equal to $\mu_{\mathrm{ob}} \varphi / \tau_{\mathrm{ob}}$, are practically equal to one another for scattering angles $\varphi \geq 40^{\circ}$ and intersect with the spherical indicatrix at $\varphi_{\text {sp,ob }} \sim 57^{\circ}$ (Refs. 1-3); the closest correlation between the integral $\tau_{\mathrm{ob}}$ and the vector $\mu_{\mathrm{ob}}\left(\varphi_{0}\right)$ was observed at the same angle $\varphi_{0}=\varphi_{\mathrm{sp}, \mathrm{ob}} \sim 57^{\circ}$. In later works ${ }^{4-6}$ it was shown that the angle of intersection $\varphi_{\text {sp,ob }}$ of real luminance indicatrices and the spherical indicatrix as well as with $\mu_{\mathrm{ob}}\left(\varphi_{0}\right)$ migrates insignificantly as a function of the zenith distance of the sun $Z_{\odot}$ and the physical-geographical conditions of observation. For the surface layer, however, in the same spectral range $\varphi_{0} \sim 45^{\circ}$ while $\varphi_{\text {sp,ob }} \sim 50 \div 55$.

The values of $\varphi_{0}$ and $\varphi_{\text {sp,ob }}$ in the near-IR region of the spectrum are the same. ${ }^{\text {? }}$ Our measurements of the sky luminance in the UV-region give completely different values: $\varphi_{0}=\varphi_{\text {sp,ob }} \sim 60^{\circ}$ (Ref. 8).

It is natural to conjecture that this discrepancy in the values of $\varphi_{0}$ and $\varphi_{\mathrm{sp}, \text { ob }}$ determined by the ratio of the single-scattering component $\mu_{1}=\mu_{\mathrm{a}}(\varphi)+\mu_{\mathrm{R}}(\varphi)$ and the multiple-scattering component $\mu_{2}(\varphi)$ in the total luminance indicatrix

$$
\mu_{o b}(\varphi)=\mu_{1}(\varphi)=\mu_{1}(\varphi)+\mu_{2}(\varphi)
$$

as well as by the contribution of aerosol scattering $\mu_{\mathrm{a}}(\varphi)$ and Rayleigh scattering $\mu_{\mathrm{R}}(\varphi)$. In the surface layer of air the Rayleigh and multiple-scattering processes play a small role, and the scattering in this layer is mainly of an aerosol character. In the visible part of the spectrum the multiple-scattering effects
also are not determining. This suggests to us that the ultraviolet region of the spectrum is most favorable for studying changes in the position of the angle of intersection of different indicatrices both with one another $\left(\varphi_{\mathrm{i}}\right)$, with the spherical indicatrix ( $\varphi_{\mathrm{sp}, \mathrm{ob}}$ ) and the Rayleigh indicatrix ( $\varphi_{R, \text { ob }}$ ). Each component of the luminance indicatrix is manifested most clearly here.

In this paper the mechanism of migration of the angles $\varphi_{\mathrm{i}}, \varphi_{\mathrm{sp}, \mathrm{ob}}$, and $\varphi_{\mathrm{R}, \text { ob }}$ is studied in detail as a function of the contribution of different components of the total luminance. The computed and observed UV luminance indicatrices $f(\varphi)$ are analyzed. In these functions the Rayleigh component makes the largest contribution, the multiple-scattering component is significant, and under summer conditions reflected
light makes a small contribution to the sky luminance. ${ }^{8}$ For this reason the form of the observed indicatrices $f_{\mathrm{ob}}(\varphi)$ is insensitive to variations of the aerosol, and it responds to the aerosol only near the sun. According to our experimental data, starting at scattering angles of the order of $25-30^{\circ}$ and up to $\varphi \sim 160^{\circ}$, the relative luminance indicatrices $f_{\mathrm{ob}}(\varphi)$ are identical, and their form remains the same at different geographical points (Fig. 1). ${ }^{8}$ The sky luminance in the halo part is determined primarily by scattering by large aerosol particles and is approximated well by calculations based on the Young aerosol scattering functions with the particle-size distribution parameter $v^{*} \sim 2.5$, the upper limit of the particle radius $r_{\text {max }} \sim 5-10 \mu \mathrm{~m}$, and the real refractive index $n=1.5$ (Refs. 9-12).


FIG. 1. The relative luminance indicatrices obtained from observations:
a) $Z_{\odot} \sim 64^{\circ}$; dots - Assy Plateau, August 17, 1976, (a.m.), $\tau_{\mathrm{a}}=0.11 ; \lambda=313.5 \mathrm{~nm},\left(Z_{\odot}=64.15^{\circ}\right)$; circles - Kryzhanovka, September 6, 1972, (a.m.), $\tau_{\mathrm{a}}=0.75 ; \lambda=311 \mathrm{~nm},\left(Z_{\odot}=64.1^{\circ}\right)$; crosses Kirbaltabai, September 18, 1973, (p.m.), $\tau_{\mathrm{a}}=0.34 ; \lambda=311 \mathrm{~nm},\left(Z_{\odot}=63.6^{\circ}\right)$;
b) $Z_{\odot} \sim 72-73^{\circ}$; dots - Assy Plateau, August 20, 1976, (p.m.), $\tau_{\mathrm{a}}=0.09 ; \lambda=313.5 \mathrm{~nm}$, $\left(Z_{\odot}=72.3^{\circ}\right)$; circles - Kryzhanovka, September 8, 1972, p.m., $\tau_{\mathrm{a}}=0.38 ; \lambda=311 \mathrm{~nm}$, ( $Z_{\odot}=72.7^{\circ}$ ); crosses Kirbaltabai, September 17, 1973, (p.m.), $\tau_{\mathrm{a}}=0.27 ; \lambda=311 \mathrm{~nm}$, ( $Z_{\odot}=71.9^{\circ}$ ).

The observed UV luminance indicatrices $f_{\mathrm{ob}}(\varphi)$, as we have established in Refs. 8-11, intersect with the spherical indicatrix $f_{\mathrm{sp}}(\varphi)$ near the scattering angle $\varphi_{\text {sp,ob }} \sim 60^{\circ}$. Additional studies ${ }^{14}$ have shown that the location of $\varphi_{\mathrm{sp}, \mathrm{ob}}$ is related with the turbidity of the atmosphere, characterized by the aerosol optical thickness $\tau_{\mathrm{a}}$. For example, for the settlement of Kryzhanovka in the Odessa region, where the aerosol optical thickness, averaged over a season of observations, for the wavelength $\lambda \sim 337 \mathrm{~nm}$ $\bar{\tau}_{\mathrm{a}} \sim 0.6, \varphi_{\text {sp,ob }} \sim 57-59^{\circ}$; for the settlement of Kirbaltabaĭ and at the Astrophysical Institute of the Academy of Sciences of the Kazakhstan SSR, where
$\bar{\tau}_{\mathrm{a}} \sim 0.25,\left(\bar{\varphi}_{\mathrm{a}} \sim 0.24\right)$; and, for the high-mountain Assy-Turgen Plateau, where $\bar{\tau}_{\mathrm{a}} \sim 0.08, \varphi_{\text {sp,ob }} \sim 55^{\circ}$.

We analyzed the intersection of different indicatrices in the UV region of the spectrum with the help of model Monte-Carlo calculations of the scattering phase function $\mu_{\mathrm{ob}}(\varphi)$, performed at the Computer Center of the Siberian Branch of the Academy of Sciences of the USSR ${ }^{15}$ based on the Young aerosol indicatrices $\mu_{\mathrm{a}}(\varphi)$ with the particle-size distribution parameters $v^{*}=2$ and 4 ; this made it possible to place the real values of the indicatrices between the computed values. By varying the aerosol optical thickness $\tau_{\mathrm{a}}$ over the range $0.15-0.70$, the Rayleigh optical thickness $\tau_{\mathrm{R}}$
over the range $0.415 \div 1.110$, and the ozone absorption thickness $\tau_{\mathrm{oz}}$ over the range 001-1.110 it was possible to cover the wavelength range $373-307 \mathrm{~nm}$ under different physical-geographical conditions. The summer albedo $\alpha$ of the underlying surface is equal to 0.045 (Ref. 8). The zenith distances of the sun were set equal to $Z_{\odot}=63$, 72 , and $76^{\circ}$, which ensured that multiple-scattering effects make a different contribution to the luminance distribution along the sun's parallel of altitude.

The problem was to determine the effect of each component $\mu_{\mathrm{a}}(\varphi), \mu_{\mathrm{R}}(\varphi)$, and $\mu_{2}(\varphi)$ of the indicatrix $\mu_{\mathrm{ob}}(\varphi)$ on the position of the point of intersection of the luminance indicatrices with one another and with the spherical and Rayleigh functions, and ultimately to prove the empirical fact that near the angles $\varphi \sim 55-60^{\circ}$ the sky luminance in the UV region of the spectrum is independent of the form of the scattering phase function.

The investigations were organized as follows. The Young indicatrices were studied first and the Rayleigh indicatrices, followed by the multiple-scattering components, were added to them. Next the displacement of the point of 'intersection of the indicatrices owing to one or another term was traced. We employed this method in Refs. 14 and 16.

## INTERSECTION OF THE MODEL INDICATRICES IN THE UV REGION OF THE SPECTRUM WITH ONE ANOTHER

The Young aerosol indicatrices $f_{\mathrm{a}}(\varphi)=\mu_{\mathrm{a}}(\varphi) / \tau_{\mathrm{a}}$, employed in the calculations, with $v^{*}=2$ and 4 intersect in the forward hemisphere at $\varphi_{\mathrm{i}} \sim 18^{\circ}$. Rayleigh scattering shifts the angles $\varphi_{\mathrm{i}}$, into the region $22-26^{\circ}$. The indicatrices for the first scattering $f_{1}(\varphi)=\mu_{\mathrm{a}}(\varphi)+\mu_{\mathrm{R}}(\varphi) /\left(\tau_{\mathrm{a}}+\tau_{\mathrm{R}}\right)$ distinguished by the Young parameter $\left(v^{*}=2\right.$ and 4), intersect precisely here for all cases modeled when the ratio of the aerosol and Rayleigh optical thicknesses $\tau_{a} / \tau_{R}$ falls into the range 0.135-1.687.

The multiple-scattering contribution $\mu_{2}(\varphi)$ shifts $\varphi_{i}$ toward larger angles, so that the computed values

$$
f_{o b}(\varphi)=\mu_{a}(\varphi)+\mu_{R}(\varphi)+\mu_{2}(\varphi) / \tau_{o b},
$$

based on the Young functions with $v^{*}=2$ and 4 for these cases, intersect with one another already at $\varphi_{I}=25-50^{\circ}$, when the sun's zenith distance $Z_{\odot} \sim 63^{\circ}$. In addition, the angle $\varphi_{\mathrm{i}}$ is all the larger the larger the value of $\tau_{\mathrm{a}}$. However for total scattering thicknesses $\tau_{\mathrm{a}}+\tau_{\mathrm{R}}>1.5$ and $Z_{\odot}>75^{\circ}$ the angle of intersection $\varphi_{\mathrm{i}}$ traverses a loop and shifts in the opposite direction - toward $30-8^{\circ}$. Under these conditions the multiple-scattering effects are determining, the multiple-scattering indicatrices become virtually spherical, and the total functions $\mu_{\mathrm{ob}}(\varphi)$ for $v^{*}=2$ and 4 loose their asymmetry and converge closer to one another not only at middle but also at small scattering angles, so that $\varphi_{i}$ starts to decrease.

The drift of the angle $\varphi_{i}$ along a loop-shaped trajectory, owing to the increase in the contribution of the higher orders of scattering to $\mu_{\mathrm{ob}}(\varphi)$, makes it possible to explain the statistical average value $\varphi_{\mathrm{i}} \sim 25-30^{\circ}$ for all observed luminance indicatrices in the UV region. In the visible section of the spectrum, however, the scattering thicknesses $\tau_{1}$ are, as a rule, not large enough for the multiple-scattering effects to cause a return to small angles. Apparently, this is why the average statistical value of the angle $\varphi_{i}$ is equal to ~ $40^{\circ}$ (Ref. 3).

Comparison of the observed luminance indicatrix ${ }^{14}$ and the Monte-Carlo luminance indicatrices showed that in the UV region of the spectrum the observed absolute luminance indicatrices $\mu_{\mathrm{ob}}(\varphi)$ fall between the computed indicatrices; the computed indicatrices $\mu_{\mathrm{ob}}(\varphi)$, differing only by the value of the size distribution parameter of the aerosols $\left(2 \leq v^{*} \leq 4\right)$ in the region of scattering angles $\varphi_{\mathrm{i}} \leq \varphi<160^{\circ}$, are identical to within the limits of error in the calculation of $\mu_{\mathrm{ob}}(\varphi)$, which are set equal to the errors in the experimental determination of $\mu_{\mathrm{ob}}(\varphi)$. This is why in the spectral region under study it makes sense to talk about not the points of intersection of the observed relative indicatrices with one another but rather about the neighborhood of scattering angles in the forward hemisphere near $\varphi_{\mathrm{i}}$ where the observed $f_{\mathrm{ob}}$ (or model) indicatrices start to separate owing to the difference in the aerosol components.

## INTERSECTION OF THE MODEL FIRST-SCATTERING INDICATRICES WITH SPHERICAL AND RAYLEIGH INDICATRICES

To find the region of scattering angles where the sky luminance in the UV region of the spectrum does not depend on the form of the indicatrix the points of intersection of the model indicatrices $f_{\mathrm{ob}}(\varphi)$, based on the Young indicatrices $f_{\mathrm{a}}(\varphi)$ for $v^{*}=2$ and 4, with the spherical and Rayleigh indicatrices were determined. These points should obviously be located at angles $\varphi<\varphi_{\mathrm{i}}$.

It was established first that the Young first-scattering indicatrices intersect with the spherical and Rayleigh indicatrices at the angles $\varphi_{\mathrm{sp}, \mathrm{a}}$ and $\varphi_{\mathrm{R}, \mathrm{a}}$, presented in Table 1.

TABLE $I$.
The scattering angles $\varphi$ at which the model Young indicatrices ( $v^{*}=2$ and 4) intersect with the spherical and Rayleigh indicatrices.


Adding the Rayleigh component $\mu_{\mathrm{R}}(\varphi)$ to $\mu_{\mathrm{a}}(\varphi)$ shifts the point of intersection $\varphi_{\text {sp }, 1}$ of the total first-scattering indicatrix $f_{1}(\varphi)$ with the spherical function $f_{\text {sp }}(\varphi)$.

Figure 2 shows the displacement of $\varphi_{\mathrm{sp}, 1}$ as a function of the asymmetry $\Gamma_{1}$ of the scattering phase functions:

$$
\Gamma_{1}=\int_{0}^{\pi / 2} \mu_{1}(\varphi) \sin \varphi d \varphi / \int_{\pi / 2} \mu_{1}(\varphi) \sin \varphi d \varphi
$$

$\Gamma_{1}$ varies in accordance with the relative contributions of $\mu_{\mathrm{a}}$ and $\mu_{\mathrm{R}}$ to $\mu_{1}$ in the range from $\Gamma_{\mathrm{a}}=1$. Figure 2 shows that as Rayleigh scattering becomes more important the values of $\varphi_{\mathrm{sp}, 1}$ approach from $\varphi_{\text {sp,a }}\left(v^{*}=2\right.$ and 4), giving two branches of points (Table I), $\varphi_{\text {sp }, \mathrm{R}}=54.7^{\circ}$ as in the case of a purely Rayleigh indicatrix. For the model indicatrices which we analyzed the angle $\varphi_{1, \text { sp }}$ is equal to $51-53^{\circ}\left(v^{*}=2\right)$ and $54.7-59^{\circ}\left(v^{*}=4\right)$.


FIG. 2. The displacement of the point of intersection $\varphi_{\mathrm{sp}, 1}$ of the first-scattering indicatrices with the spherical indicatrix with a change in asymmetry: $\mathrm{v}^{*}=2$ (dark circles) and $\mathrm{v}^{*}=4$ (light circles).

As regards the intersection of model first-scattering indicatrices $f_{1}(\varphi)$ with the Rayleigh indicatrix $f_{\mathrm{R}}(\varphi)$, the angle $\varphi_{1, \mathrm{R}}$ falls into the range $46-52^{\circ}$ for $v^{*}=2$ and $60-63^{\circ}$ for $v^{*}=4$, i.e., it is determined primarily by the intersection of $f_{\mathrm{a}}(\varphi)$ with $f_{\mathrm{R}}(\varphi)$ (Table I). Figure 3 shows $\varphi_{1, \mathrm{R}}$ as a function of $\Gamma_{1}$.

## EFFECT OF MULTIPLE SCATTERING ON THE INTERSECTION OF DIFFERENT INDICATRICES

The addition of $\mu_{2}(\varphi)$, determined by the multiple scattering of light, to the asymmetric model
first-scattering indicatrices $\mu_{1}(\varphi)$ shifts the point of intersection $\varphi_{\text {sp,ob }}$ of the indicatrices $f_{\mathrm{ob}}(\varphi)$ with the spherical indicatrix into the region of angles larger than $\varphi_{\text {sp }, 1}$ (Fig. 4a, b, c, d). This shift is all the larger the larger the value of $\tau_{\mathrm{a}}$, but as $Z_{\odot}$ changes it can acquire a loop-shaped character, if $\tau_{\mathrm{R}}>\tau_{\mathrm{a}}$ or $\tau_{\mathrm{R}}>1$ (Figs. 4c and d). All cases covered by the model calculations ${ }^{14}$ give the intervals $52-75^{\circ}\left(v^{*}=2\right)$ and 58-75 ( $\left.v^{*}=4\right)$ for $\varphi_{\text {sp }, o b}$.



FIG. 3. The position of the point of intersection $\varphi_{\mathrm{R}, 1}$ of the first-scattering indicatrices with the Rayleigh indicatrix as a function of the asymmetry of the model indicatrices $v^{*}=2$ (dark circles) and $v^{*}=4$ (light circles).


FIG. 4. Drift of the point of intersection $\varphi_{\mathrm{sp}, \text { ob }}$ of the model indicatr ices $f_{\mathrm{ob}}(\varphi)$ with the spherical indicatrix accompanying a change of asymmetry $\Gamma_{\mathrm{ob}}$. $1-Z_{\odot}=63^{\circ}, 2-Z_{\odot}=72^{\circ}, 3-Z_{\odot}=76^{\circ}$; dark circles $-v^{*}=2$; light circles $-v^{*}=4$, a) $\tau_{\mathrm{a}}=0.70, \quad \tau_{\mathrm{R}}=0.415, \quad \lambda=373 \mathrm{~nm}$; b) $\tau_{\mathrm{a}}=0.30, \quad \tau_{\mathrm{R}}=0.975, \quad \lambda=311 \mathrm{~nm}$; c) $\tau_{\mathrm{a}}=0.70, \quad \tau_{\mathrm{R}}=1.11, \quad \lambda=306,8 \mathrm{~nm}$; d) $\tau_{\mathrm{a}}=0.15, \tau_{\mathrm{R}}=0.415, \lambda=373 \mathrm{~nm}$.

Table II also illustrates the fact that multiple scattering increases $\varphi_{\text {sp,ob }}$ for asymmetric indicatrices. The table gives the values of $\varphi_{\mathrm{sp}, \mathrm{ob}}$ for two elongated indicatrices $\mu_{\mathrm{ob}}(\varphi)$ VII and VIII according to the data from Ref. 17.

TABLE II.
The position of the angle of intersection of the indicatrices $f_{\mathrm{ob}}(\varphi)$ (VII and VIII, Ref. 17) and the Rayleigh indicatrix (Ref. 18) with the spherical indicatrix accompanying a change in the contribution of multiple scattering.

| indica- <br> trix | $\tau_{1}$ | $f_{1}$ |  | $\mathrm{Z}_{\odot}=60^{\circ}$ |  | $\mathrm{Z}_{\odot}=75^{\circ}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\varphi_{\mathrm{sp}, 1}^{0}$ | $\varphi_{\mathrm{R}, 1}^{0}$ | $\varphi_{\mathrm{sp}, 2}^{0}$ | $\varphi_{\text {sp,ob }}^{0}$ | $\varphi_{\mathrm{sp}, 2}^{0}$ | $\varphi_{\text {sp, ob }}^{0}$ |
| VII | $\begin{aligned} & 0.6 \\ & 0.8 \end{aligned}$ | 57.6 | 58.5 | $\begin{aligned} & 72 \\ & 73 \end{aligned}$ | $\begin{aligned} & 60 \\ & 61 \end{aligned}$ | $\begin{aligned} & 72 \\ & 73.7 \end{aligned}$ | $\begin{aligned} & 62 \\ & 64.7 \end{aligned}$ |
| VIII | $\begin{aligned} & 0.6 \\ & 0.8 \end{aligned}$ | 54.8 | 54.5 | $\begin{aligned} & 67 \\ & 68.5 \end{aligned}$ | $\begin{aligned} & 59 \\ & 60 \end{aligned}$ | $\begin{aligned} & 67.5 \\ & 69.5 \end{aligned}$ | $\begin{aligned} & 60 \\ & 62.2 \end{aligned}$ |
| $f_{\text {R }}$ | 0.4 $0.6$ $0.8$ | 54.7 | indi-catrices are identical at all points | 55 <br> 56.5 <br> 56 | 54.7 <br> 55 <br> 55 | 50 <br> 53.5 $52.5$ | 53 <br> 54.7 $53.5$ |

Continuing the analysis of Table II we note the following. In the case of scattering by molecules at
middle and large angles $\varphi$ the multiple-scattering component is substantially spherica ${ }^{18}$ and for this reason it does not change $\varphi_{\mathrm{sp}, \mathrm{R}(\mathrm{ob})}$ right up to $\mathrm{Z}_{\odot} \sim 75^{\circ}$. When the sun' s zenith distances reach $Z_{\odot} \geq 75^{\circ}$ the total indicatrix $\quad \mu_{\mathrm{R}(\mathrm{ob})}=\mu_{\mathrm{R}}+\mu_{2 \mathrm{R}} \quad$ approaches a spherical function, and $\varphi_{\mathrm{sp}, \mathrm{R}(\mathrm{ob})}$ shifts toward smaller angles: $\varphi_{\mathrm{sp}, \mathrm{R}(\mathrm{ob})}<\varphi_{\mathrm{sp}, \mathrm{R}^{\prime}}$. The last result is also true for the total indicatrices $\left(\mu_{R}+\mu_{2 R}\right)$ in the UV region of the spectrum, when $\mu_{2}\left(Z_{\odot}>80^{\circ}\right)$ becomes the dominant component and the asymmetry $\Gamma_{\mathrm{ob}}$ approaches unity, but does not reach it (the deep regime) ${ }^{3}$.

Since multiple scattering causes the indicatrix ( $\mu_{R}+\mu_{2 R}$ ) to approach a spherical form the conditions for intersection of the observed indicatrices $\mu_{\mathrm{ob}} / \tau_{\mathrm{ob}}$ with $\left(\mu_{\mathrm{R}}+\mu_{2 \mathrm{R}}\right) / \tau_{\mathrm{ob}}$ are close to the conditions for intersection of $\mu_{\mathrm{ob}} \tau_{\mathrm{ob}}$ with the spherical indicatrix: $\varphi_{\mathrm{sp}, \mathrm{ob}} \simeq \varphi_{\mathrm{R}(\mathrm{ob}), \mathrm{ob},}$

Thus the use of computed model indicatrices $\mu_{\mathrm{ob}}$ based on the Young indicatrices with $v^{*}=2$ and 4 for the UV region of the spectrum, when the scattering thicknesses fall in the range $0.4 \leq \tau_{\mathrm{R}} \leq 1.11$ and $0.15 \leq \tau_{\mathrm{a}} \leq 0.7$, covering the indicatrices $\mu_{\mathrm{ob}}$ which we actually observed, as well as the asymmetric phase functions VII and VIII (for anisotropic scattering and thicknesses $\tau_{1}=0.6$ and 0.8 ), modelled at the Institute of Atmospheric Physics of the Academy of Sciences of the USSR, ${ }^{17}$ showed that in the case when the scattering thicknesses $\tau_{1}$ are equal and the sun's zenith distances $\mathrm{Z}_{\odot}$ fall into the range $\sim 63-75^{\circ}$ the point of intersection of the model indicatrices $\mu_{\mathrm{ob}} / \tau_{\mathrm{ob}}$ with the spherical and Rayleigh indicatrices can drift over a quite wide range of scattering angles $\Delta \varphi_{\mathrm{ob}}\left({ }_{\mathrm{sp}, \mathrm{R}(\mathrm{ob})}\right)=52 \div 75^{\circ}$.

TABLE III.
The correlation $R$ between $\mu_{\mathrm{ob}}\left(60^{\circ}\right)$ and $\tau_{\mathrm{ob}}$ and the coefficient of proportionality $\gamma\left(60^{\circ}\right)$ between these quantities in the $U V$ absorption band of ozone, $\lambda<340 \mathrm{~nm}$.

| $\begin{gathered} \text { location } \\ \text { of } \\ \text { observation } \end{gathered}$ | $\mu_{\text {ob }}\left(60^{\circ}\right)$ | $\sigma \mu_{\text {ob }}\left(60^{\circ}\right)$ | $\tau_{\text {ob }}$ | $\sigma \tau_{0 b}$ | $R \mu_{o b} \tau_{\text {ob }}\left(60^{\circ}\right)$ | $\gamma\left(60^{\circ}\right)$ | $\sigma \tau\left(60^{\circ}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Assy Plateau } \\ 1976 \\ N_{\text {s } 1}=16 \end{gathered}$ | 0.15 | 0.02 | 2.01 | 0.21 | 0.97 | 13.0 | 0.4 |
| Astrophysics Academy of Sciences of Kazakhstan $\begin{gathered} \text { SSR } \\ N^{1971}=162 \\ \hline \end{gathered}$ | 0.38 | 0.18 | 4.86 | 2.25 | 1.00 | 12.9 | 0.4 |
| $\begin{gathered} \text { Kirbaltabaĺ } \\ 1973 \\ N^{1973}=43 \\ \hline \end{gathered}$ | 0.85 | 0.80 | 10.9 | 10.1 | 1.00 | 12.7 | 0.2 |
| $\begin{gathered} \text { Kryzhanovka } \\ N_{1}^{1972}=51 \end{gathered}$ | 2.43 | 2.74 | 30.6 | 34.3 | 1.00 | 12.6 | 0.2 |
| All locations of observation |  |  |  |  |  | 12.8 | 0.3 |

$N^{*}$ is the number of cases analyzed.

According to our data, however, the real indicatrices $\mu_{\mathrm{ob}}$ in the UV region of the spectrum exhibit a significantly smaller range of forms than the computed indicatrices analyzed here and, as we mentioned, they are close to the model luminance indicatrices based on the Young aerosol indicatrices with the distribution parameter $v^{*} \sim 2.5$. As a result, when the zenith distance of the sun's parallel of altitude is limited to $Z_{\odot}<75^{\circ}$ (where $\mu_{\mathrm{ob}}$ were measured) the interval $\Delta \varphi_{\mathrm{ob}(\mathrm{sp}, \mathrm{R}(\mathrm{ob}))}$ becomes narrower: $\sim 55-65^{\circ}$. This is why the average statistical value of the coefficient of proportionality $\gamma\left(60^{\circ}\right)$ between $\mu_{\mathrm{ob}}\left(60^{\circ}\right)$ and $\tau_{\mathrm{ob}}$ is close to 4ir, as in the case of the spherical indicatrix,

$$
\tau_{o b}=\gamma\left(60^{\circ}\right) \mu_{o b}\left(60^{\circ}\right) .
$$

This is illustrated in Table III, where the values of $\gamma\left(60^{\circ}\right)$ for the internal of the sun's zenith distances indicated above are presented for four locations of observations.

Analysis of Table III shows that the value of $\overline{\gamma\left(60^{\circ}\right)}$ ranges from 12.6 ( $\sim 4 \pi$ ) for Kryzhanovka near Odessa, where the indicatrix $\mu_{\mathrm{ob}}$ is substantially of an aerosol character, up to 13 at the high-mountain plateau of Assy, when the observed indicatrices are practically of the Rayleigh form and intersect with the spherical indicatrix at $\varphi_{\mathrm{i}} \sim 55^{\circ}$ and not at $60^{\circ}$. Another important conclusion that can be drawn from Table III and everything else said above is that in spite of the significant spread in $\sigma \mu_{\mathrm{ob}}\left(60^{\circ}\right)$ and $\sigma \tau_{\mathrm{ob}}$, the absolute values of the directional light-scattering coefficient $\mu_{\mathrm{ob}}\left(60^{\circ}\right)$, and the integral $\tau_{\mathrm{ob}}$, obtained using a formula analogous to (1), in the UV region of the spectrum for different geographical points relative values $\mu_{\mathrm{ob}}\left(60^{\circ}\right) / \tau_{\mathrm{ob}}$ are practically identical not only at different locations of observations, but they differ little from $\mu_{\text {sp }}\left(60^{\circ}\right) / \tau_{\mathrm{ob}}$ for the spherical luminance indicatrix and $\mu_{\mathrm{R}}\left(60^{\circ}\right) / \tau_{\mathrm{ob}}$ for the Rayleigh luminance indicatrix. This indicates that near $\varphi_{i} \sim 60$ the sky luminance in the UV region of the spectrum does not depend on the form of the indicatrix: here the real indicatrices intersect the spherical and Rayleigh indicatrices.

## CONCLUSIONS

1. Owing to the overall approach employed from the spectral viewpoint the results obtained in this work based on model calculations in the UV region of the spectrum are generally valid: the Rayleigh and multiple-scattering phase functions are added to the Young (naturally averaged) phase functions.
2. It was found that changing the ratio of the aerosol, Rayleigh, and multiple-scattering components, whose absolute contribution to the luminance indicatrix is determined by the optical thicknesses $\left(\tau_{\mathrm{a}}=0.15-0.7\right.$ and $\left.\tau_{\mathrm{R}}=0.45-1.11\right)$ and the sun's zenith distance ( $Z_{\odot}=63-76^{\circ}$ ), results in a
shift of the position of the angle of intersection $\varphi_{o b, s p(R)}$ of the computed indicatrices with the spherical indicatrix in the range $51-75^{\circ}\left(v^{*}=4\right)$ and with the Rayleigh indicatrix in the range $46-75^{\circ}\left(v^{*}=2\right)$ and $60-75^{\circ}\left(v^{*}=4\right)$, including also the first-scattering indicatrix. It was established that in the UV region of the spectrum the multiple-scattering effects can cause this angle to drift over a loop-shaped trajectory. As a rule, for the luminance indicatrix in which Rayleigh scattering does not predominate multiple-scattering corrections increase, as compared with the first-scattering indicatrices, the angle $\varphi_{\text {sp,ob }}$ of intersection of the total indicatrices with the spherical indicatrix. However when $\tau_{\mathrm{R}}>\tau_{\mathrm{a}}$ or $\tau_{\mathrm{R}}>1$ the angle $\varphi_{\text {sp,ob }}$ at first increases as $Z_{\odot}$ increases, but at zenith distances $Z_{\odot}>76^{\circ}$ it starts to move in the opposite direction - toward smaller angles. The behavior of the angle of intersection $\varphi_{i}$ of the luminance indicatrices $\mu_{\mathrm{ob}} / \tau_{\mathrm{ob}}$ for $v^{*}=2$ and 4 with one another is analogous; $\varphi_{i}$ ranges from $8^{\circ}$ to $50^{\circ}$.

3 . The variability of the observed indicatrices $f_{\mathrm{ob}}$ and the optical aerosol thicknesses $\tau_{\mathrm{a}}$ at a specific location is much narrower than in the case of the modeled indicatrices. In the UV region of the spectrum the form of the observed indicatrices is described well by the model Young indicatrix with the particle-size distribution parameter $v^{*} \sim 2.5$. For this reason the relative observed indicatrices $\mu_{\mathrm{ob}} / \tau_{\mathrm{ob}}$ are virtually identical at scattering angles $\varphi>\varphi_{\mathrm{i}}$ irrespective of the meteorological and geographical conditions of the measurements of the sky luminance, and they become separated only for $\varphi<\varphi_{\mathrm{i}}$. The angle of intersection $\varphi_{\mathrm{i}}$ of the observed indicatrices with one another falls into a narrow range: $25-30^{\circ}$.
4. Since variations of the aerosol are manifested in the UV region of the spectrum only in the halo part of the observed indicatrices, which makes an insignificant contribution to the observed indicatrices, which makes an insignificant contribution to the integral $\tau_{\mathrm{ob}}$, the value of the integral $\tau_{\mathrm{ob}}$ does not depend on the form of the scattering phase function; the relative observed luminance indicatrices $\mathrm{f}_{\mathrm{ob}}$ intersect with the spherical and Rayleigh indicatrices in the range of angles $\varphi_{\mathrm{ob},(\mathrm{sp}, \mathrm{R})}=55-60^{\circ}$; the luminance of the sky near these angles does not depend on the form of the scattering phase function, so that the nephelometric relation, relating the integral $\tau_{\mathrm{ob}}$ with the directional light-scattering vector $\mu_{\mathrm{ob}}$, has the same form (2) as for the spherical indicatrix.

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