THERMAL BLOOMING OF LASER BEAMS ALONG INHOMOGENEOUS PATHS IN A TURBULENT ATMOSPHERE

V.A. Banakh, N.N. Kerkis, and I.N. Smalikho

Institute of Atmospheric Optics,
Siberian Branch of the USSR Academy of Sciences, Tomsk
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The effect of turbulence on the distribution of the average intensity of a partially coherent laser beam propagating in the atmosphere under conditions of thermal blooming along vertical and inclined paths is studied. It is shown that in calculations of the thermal distortions of laser beams with visible and IR wavelengths in the atmospheric turbulence must be taken into account for many situations of practical importance. Formulas that permit finding from the values of a collection of functions at the boundary of the effective layer of the atmosphere x_e the dimensions and the displacement of a beam on any path $x > x_e$ without direct calculation of the average intensity distribution are derived.

In theoretical studies of the thermal blooming of beams propagating on vertical paths the effect of the atmospheric turbulence on the distribution of the average intensity in the plane of observation was neglected.[2–4] However it may be necessary to take atmospheric turbulence into account in the analysis of the propagation of a beam on long inclined paths for short-wavelength lasers and in the case of intense turbulent pulsations of the index of refraction of air.[1]

In this paper the average intensity of a partially coherent laser beam propagating in a turbulent atmosphere along vertical and inclined paths under conditions of thermal blooming is studied based on a numerical solution of the equation for the mutual coherence function of the field of the wave. The effect of turbulence on the characteristics of the beam under different conditions of propagation is analyzed.

We shall employ the following approach to calculate the average intensity $\langle I(x, \vec{\rho})\rangle$ of a beam under conditions of steady blooming. The entire path x is subdivided into N layers. In the planes containing the front boundary of each layer, the attenuation of the field of the wave $U_j(x, \rho)$, occurring owing to absorption, and the turbulent distortions of the field are taken into account in the phase-screen approximation. Under the assumptions that the field along the propagation path is 6-correlated and the turbulent pulsations of the refractive index have a Kolmogorov spectrum[5] the following equations[9] can be derived for the coherence function

$$\Gamma_j(x', R, \rho) = \left\langle U_j\left[x', R + \frac{1}{2}\rho\right] U_j^*\left[x', R - \frac{1}{2}\rho\right]\right\rangle$$

from the parabolic equation for the field of the wave[1] after the induced temperature $T(x, \rho)$ is replaced in it by the average value $\langle T(x, \rho)\rangle$:

$$\left[2ik\frac{\partial}{\partial x'} + 2\nabla_R\nabla_\rho\right]\Gamma_j(x', \vec{R}, \vec{\rho}) + k^2\frac{\partial\varepsilon(x')}{\partial T}\times$$
$$\times\left[\left\langle T\left[x_{j-1}, \vec{R} + \frac{1}{2}\vec{\rho}\right]\right\rangle - \left\langle T\left[x_{j-1}, \vec{R} - \frac{1}{2}\vec{\rho}\right]\right\rangle\right]\times$$
$$\times\Gamma_j(x', \vec{R}, \vec{\rho}) = 0 \qquad (1)$$

with the boundary condition at the interface between the two layers

$$\Gamma_j(x_{j-1}, \vec{R}, \vec{\rho}) = \Gamma_{j-1}(x_{j-1}, \vec{R}, \vec{\rho})\exp\left[-\int\limits_{x_{j-1}}^{x_j} dx'\alpha_\alpha(\vec{x})\right]\times$$
$$\times\exp\left[-(1,46k^2\int\limits_{x_{j-1}}^{x_j} dx' C_n^2(x')^{6/5}\vec{\rho}^2\right], \qquad (2)$$

where $k = 2\pi/\lambda$; λ is the wavelength; $\partial\varepsilon/\partial T$ is the derivative of the dielectric constant of air with respect to the temperature; α_a is the absorption coefficient; C_n^2 is the structure constant of the turbulent pulsations of the refractive index; $x' \in [x_{j-1}, x_j]$; $j = 1, 2, \ldots, N$; $x_0 = 0$; $x_N = x$. The quadratic approximation[6] was employed in the second exponential in Eq. (2).

According to the results of Ref. 14 the fluctuations of the induced temperature $\tilde{T} = T - \langle T\rangle$ can be neglected in deriving Eq. (1) in the calculation of the average intensity of the beam, if the condition for fluctuations of the intensity owing to turbulent pulsations of the refractive index to be weak is satisfied in the induced thermal lens, formed primarily in the bottom 1–3 km of the troposphere,[1] This situation practically always arises on vertical propagation paths.

In the case of a partially coherent gaussian beam we shall give the boundary conditions for the first layer in Eq. (1) in the form[7]

$$\Gamma_0(0,\vec{R},\vec{\rho}) = I_0 \exp\left[-\frac{\vec{R}^2}{a_0^2} - \frac{\vec{\rho}^2}{4a_0^2}\left(1 + \frac{a_0^2}{a_k^2}\right)\right], \tag{3}$$

where I_0 is the maximum value of the intensity; α_c is the coherence radius; and, a_0 is the radius of the beam.

Retaining only the linear term in the expansion of the temperature difference in Eq. (1) in a Taylor series in $\vec{\rho}$ and Fourier transforming Eq. (1)

$$J_j(x',\vec{R},\vec{\chi}) = \int_{-\infty}^{+\infty}\int d^2\rho\cdot\Gamma_j(x',\vec{R},\vec{\rho})e^{-i\vec{\chi}\vec{\rho}}, \tag{4}$$

we arrive at the equation of radiation transfer,[8] after solving which Γ_j can be represented in dimensionless variables[9] as

$$\Gamma_j(x_j,\vec{R},\vec{\rho}) = \frac{P^2}{\pi}\int_{-\infty}^{+\infty}\int d^2\chi J_j \times$$

$$\times\left[x_{j-1},\tilde{\vec{R}}(x_{j-1}),\frac{\partial}{\partial x'}\tilde{\vec{R}}(x_{j-1})\right]e^{iP\vec{\chi}\vec{\rho}}, \tag{5}$$

where $\tilde{\vec{R}}$ satisfied the equation

$$\frac{d^2}{dx'^2}\tilde{\vec{R}}(x') =$$

$$= \frac{\nu(x')}{\sqrt{\pi}}\nabla_{\tilde{\vec{R}}}\int_0^{\infty}d\tau\left\langle I\left[x_{j-1},\tilde{\vec{R}}(x') - \frac{\vec{V}_\perp(x')\tau}{|\vec{V}(x')|}\right]\right\rangle \tag{6}$$

with the boundary conditions $\vec{R}_j(x) = \vec{R}$, $\frac{\partial}{\partial x'}\cdot\tilde{\vec{R}}(x_j) = \chi$, $x' \in [x_{j-1}, x_j]$.

In Eq. (5) $p = L_d/R_{nl}$, where $L_d = k\,a_0^2/(1 + a_0^2/a_k^2)^{1/2}$ is the effective diffraction length;

$$R_{nl} = \left[\frac{\sqrt{\pi}}{2}\left|\frac{\partial\varepsilon(0)}{\partial T}\right|\frac{\alpha_a(0)I_0}{\rho(0)C_p a_0|\vec{V}_\perp(0)|}\right]^{-1/2} \tag{7}$$

is the effective thermal blooming length in the plane $x' = 0$; $\rho(0)$ and C_p are the density and heat capacity of air, respectively; $\vec{V}_\perp(0) = \{V_{z\perp}(0);\ V_{y\perp}(0)\}$ is the component of the wind velocity vector, perpendicular to the axis of propagation x', in the plane of the source of radiation.

Since $\frac{\partial\varepsilon(x')}{\partial T} : \frac{\rho(x')}{T_a(x')}$, where T_a is the absolute temperature of the air, the function $\upsilon(x')$ in Eq. (6) will be determined as follows:[3]

$$\nu(x') = \frac{\alpha_a(x')T_a(0)|\vec{V}_\perp(0)|}{\alpha_a(0)T_a(x')|\vec{V}_\perp(x')|}. \tag{8}$$

The average intensity was calculated using the numerical scheme presented in Ref. 9. The models of altitude profiles of the meteorological and optical parameters of the atmosphere for middle latitudes were employed.[10–12] The turbulent fluctuations of the index of refraction were taken into account based on the results presented in Ref. 13, where models of the altitude behavior of $C_n^2(x')$, which correspond to the best conditions for propagation of light when C_n^2 assumes its minimum values and the worst conditions when C_n^2 assumes its maximum values, are presented. A third profile $C_n^2(x')$ corresponding to the geometric average of the first two profiles is constructed for the average conditions.[13]

The absorption coefficient $\alpha_a(x')$, the temperature $T_a(x')$, and the transverse components of the wind velocity $V_{\perp z}(x')$ and $V_{\perp y}(x')$ in Eq. (8) are presented in a coordinate system (tied to the beam) whose origin lies at the source of radiation. We denote the absorption coefficient by α_a^0, the absolute temperature of the air by T_a^0, and the components of the wind velocity along the z axis (west-east direction) and along the y axis (south-north direction), represented in a coordinate system whose origin lies on the earth's surface, by V_{0z} and V_{0y}.

It is not difficult to obtain the following relationships:

$$\alpha_a(x') = \alpha_a^0(H_0 + x'\cos\theta); \tag{9}$$

$$T_a(x') = T_a^0(H_0 + x'\cos\theta); \tag{10}$$

$$V_{\perp z}(x') = V_{0z}(H_0 + x'\cos\theta)[1 - (1 - \cos\theta)\cos^2\varphi] -$$

$$- V_{0y}(H_0 + x'\cos\theta)\sin\varphi\cos\varphi(1 - \cos\theta) \tag{11}$$

$$V_{\perp y}(x') = V_{0y}(H_0 + x'\cos\theta)\cdot[1 - (1 - \cos\theta)\cdot\sin^2\varphi] +$$

$$+ V_{0z}(H_0 + x'\cos\theta)\sin\varphi\cos\varphi\cdot(1 - \cos\theta); \tag{12}$$

where H_0 is the altitude of the radiation source above the earth's surface; θ is the zenith angle; and, x' is the normalized coordinate.

From Eqs. (11) send (12) one can see that the transverse components of the wind velocity $V_{\perp z}$ and $V_{\perp y}$ depend not only on θ but also on the angle φ (see Fig. 1).

It follows from Eqs. (7), (11), and (12) that the effective thermal blooming length in the plane of the source depends on the altitude H_0 and the angles θ and φ. We denote by $\chi(\theta, \varphi)$ the ratio of $R_{nl/\theta=0}$ to R_{nl}. Then

$$\chi(\theta,\varphi) = \left[1 - \sin^2\theta \times \frac{(V_{0z}(H_0)\cos\varphi + V_{0y}(H_0)\sin\varphi)^2}{V_{0z}^2(H_0) + V_{0y}^2(H_0)}\right]^{1/2}. \quad (13)$$

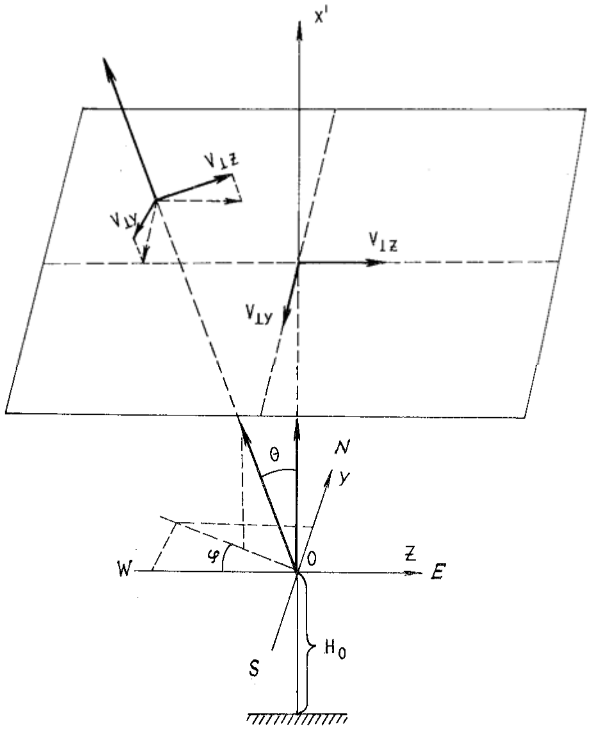

FIG. 1. Diagram of the propagation of a laser beam in the atmosphere.

The quantity $\chi(\theta, \varphi)$ assumes its maximal value $\chi = 1/\sqrt{\cos\theta}$ (for which the greatest nonlinear distortions of the beam occur) when the angle $\varphi = \mathrm{arctg}\,[V_{0y}(H_0)/V_{0z}(H_0)] = \varphi_1$, and its minimum value ($\chi = 1$) when $\varphi = \varphi_1 + \pi/2 = \varphi_2$. Therefore, as the angle θ varies, for example, from 0 to 60° the length R_{nl} decreases by a factor of $\sqrt{2}$ for the case $\varphi = \varphi_1$ (the thermal distortions of the beam increase by approximately the same factor) and remains constant for the case $\varphi = \varphi_2$.

Figures 2 and 3 show the results of the calculation of the distribution of the average intensity, normalized to the maximum value $\langle I \rangle_{max}$, of a beam propagating along a vertical path x = 20 km for large nonlinearity parameters ($P^2 \gg 1$) and different turbulent conditions, obtained for the summer model of the absorption coefficient.[11] The calculation was performed for two wavelengths (10.6 and 1.064 μm) in cases when the nonlinearity length R_{nl} had the same value (R_{nl} = 7 km) and different values (R_{nl} = 7 km for 10.6 μm and R_{nl} = 22 km for 1.064 μm) but the radiation power in the plane of the source was the same. This difference is determined by the fact that the energy absorption coefficients of the beam at these wavelengths differ approximately by an order of magnitude.[11,12]

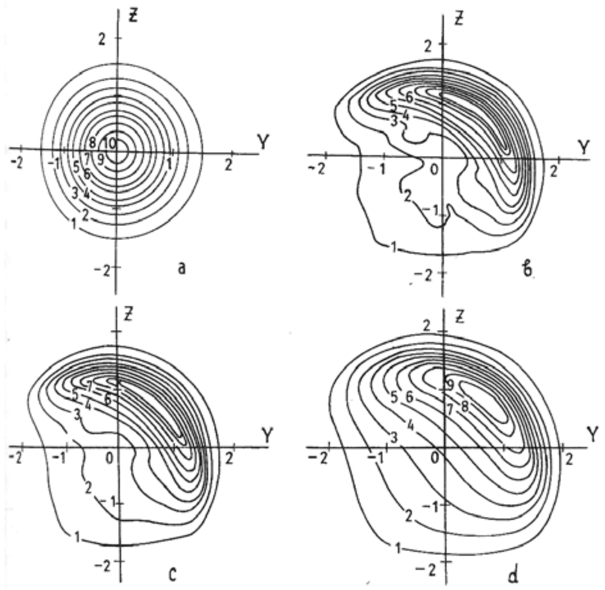

FIG. 2. The distribution of the average intensity of a laser beam with the wavelength 10.6 μm in the plane of the radiation (a) and at a distance x = 20 km for the best (b), average (c), and worst (d) turbulent conditions of propagation of light for R_{nl} = 7 km.

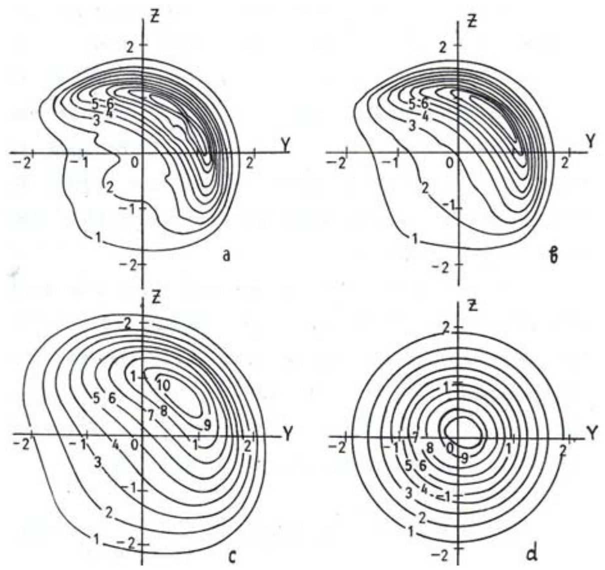

FIG. 3. The distribution of the average intensity of a laser beam with the wavelength 10.6 μm for R_{nl} = 7 km and the best (a), average (b), and worst (c) conditions and for R_{nl} = 22 km and the worst conditions (d).

The calculations showed that for the best conditions of propagation of light (Figs. 2b and 3a) the turbulence has virtually no effect on the distribution of the average intensity of the beam. As one can see from Figs. 2b and 3a, the thermal distortions for the same values of R_{nl} for beams with different wavelengths are virtually identical. If $R_{nl} = 22$ km, then, conversely, the thermal distortions on the 20 km path can be neglected. Turbulence has an appreciable effect on the distribution of the average intensity only under the worst conditions (Figs. 2d, 3c, and 3d). As expected, the turbulent spreading of the beam is stronger for a laser at the wavelength 1.064 μm (compare Figs. 2d and 3c).

We analyzed the average intensity of beams for extended paths $x \gg 20$ km when a large portion of the path passes outside the distorting layer of the atmosphere. We shall determine the effective thickness of the distorting layer x_e, equal to the distance along the direction of propagation at which the values of the absorption coefficient and (or) the structure constant of the refractive, index decrease to 10^{-3} of the maximum values in the plane $x' = 0$. In particular, for a vertical path and a wavelength of 10.6 μm the absorption coefficient a drops by three orders of magnitude at a distance $x = x_e = 60$ km in the case of the summer model. At the same time, for $\lambda = 1.064$ μm the ratio $\alpha_a(x')/\alpha(0)$ is equal to10^{-3} already at altitudes $x' \approx 10$ km,[12] while the values of $C_n^2(x')/C_n^2(0)$ have not yet dropped to this level. In this case x_e was set equal to 20 km, where $C_n^2(x')$ also already satisfies the imposed requirement.

For $x > x_e$ it can be assumed that the beam propagates in a homogeneous medium. Then the following relations can be easily derived for the displacement vector of the energy center of gravity of the beam $\vec{R}_c(x)$ and the squared effective dimensionless radius of the beam $g^2(x)$:

$$\vec{R}_c(x) = \vec{R}_c(x_e) + \vec{\mu}(x_e)(x - x_e); \tag{14}$$

$$g^2(x) = g^2(x_e) + \gamma(x_e)(x - x_e) + \beta(x_e)(x - x_e)^2, \tag{15}$$

where

$$\vec{\mu}(x_e) = \frac{1}{P_0(x_e)}\iint\limits_{-\infty}^{+\infty} d^2R\vec{D}(x_e, R) < I(x_e, \vec{R}) > ; \tag{16}$$

$$\gamma(x_e) = \frac{2}{P_0(x_e)}\iint\limits_{-\infty}^{+\infty} d^2R(\vec{R} - \vec{R}_c(x_e))\vec{D}(x_e, \vec{R}) < I(x, \vec{R}) > ; \tag{17}$$

$$\beta(x_e) = \frac{1}{P_0(x_e)} \times$$

$$\times\iint\limits_{-\infty}^{+\infty} d^2R\ [\vec{D}(x, \vec{R}) + \vec{G}^2(x_e, \vec{R}) - \vec{\mu}^2(x_e)] < I(x_e, \vec{R}) > \tag{18}$$

$$P_0(x_e) = \iint\limits_{-\infty}^{+\infty} \alpha^2R^2 < I\ (x_e, \vec{R})>, \quad \vec{D}(x_e, \vec{R}) = \{D_z, D_y\};$$

$$\vec{G}(x_e, \vec{R}) = \left\{\frac{B_z^{1/2}}{\sqrt{B_zB_y - B_{zy}^2}}, \ \frac{B_y^{1/2}}{\sqrt{B_zB_y - B_{zy}^2}}\right\}.$$

The values of B_z, B_y, B_{zy}, D_z, and D_y are calculated numerically when the coherence function Γ_j is calculated in the plane x_e.[9] Thus after the coefficients $\vec{\mu}$, β, and γ are determined from the formulas (16)–(18) and substituted into Eqs. (14) and (15) the width and displacement of the beam on any path $x > x_e$ can be found.

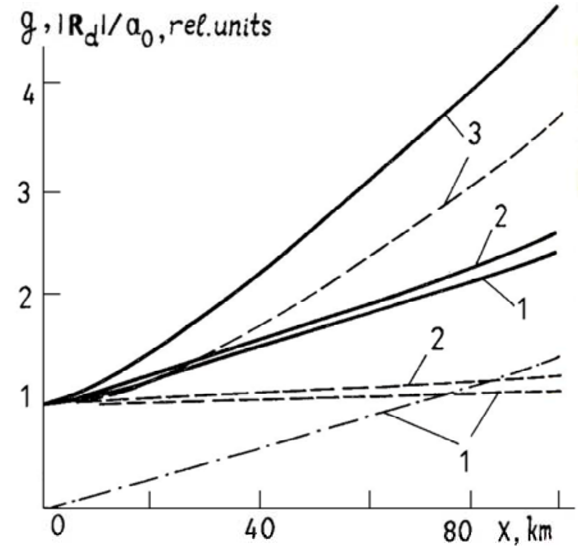

FIG. 4. The dimensionless effective betrn radius g Csolid and dashed lines1 and the normalized modulus of the displacement vector of the beam $\left|\vec{R}_\alpha\right| / a_0$ (dot-dashed curve) as a function of the path length x for $H_0 = 0$ km and $R_{nl} = 7$ km (solid and dot-dashed lines) and $R_{nl} = 22$ km (dashed lines) for the best (1), average (2), and worst (3) turbulent conditions for propagation of light.

It follows from the result of the calculations based on the formula (15) that the spreading of the 10.6 μm beam on extended paths with $R_{nl} \le 7$ km in the case of the best and average conditions of propagation (relative to C_n^2) is determined primarily by the thermal distortions, and for the worst conditions nonlinearity and turbulence make comparable contributions to the spreading of the beam. As the wavelength decreases ($\lambda = 1.064$ μm) the effect of turbulence increases and for $R_{nl} = 22$ km the effective radius of the beam g(x) increases practically completely owing to turbulence. Nonlinearity and turbulence make an almost additive contribution to the squared effective radius of the beam $g^2 \approx g_{nl}^2 + g_t^2$, where g_{nl}^2 is the radius of the beam in the absence of turbulent fluctuations of the refractive index of air and g_t^2 is the turbulent correction to the squared beam radius for a linear medium,[6] proportional to $\lambda^{-2/5}$.

The approximation $g^2 \approx g_{nl}^2 + g_t^2$ will also be valid for the best turbulent conditions of propagation of light for any values of R_{nl}. The calculations show, however, that for $x \gg 20$ km and $R_{nl} = 7$ km this approximation underestimates the value of $g^2(x)$ and the relative error $S = [g^2/(g_{nl}^2 + g_t^2) - 1] \cdot 100\%$ is equal to ~ 3% for the average conditions and ~ 27% for the worst conditions. The quantity S can significantly exceed these values for shorter nonlinearity lengths R_{nl} and on inclined propagation paths.

Figure 4 shows the integral parameters of the radiation beam with $\lambda = 1.064$ as a function of x in the case of vertical propagation. One can see that for the worst turbulent conditions of propagation of light and $R_{nl} = 7$ km at an altitude of 100 km the effective radius of the beam increases by almost a factor of five. As one can see by comparing the continuous curves 1 and 3, neglecting the turbulent pulsations of the refractive index in this case would result in an underestimation of this quantity by approximately a factor of two.

Thus we have shown in this paper that under the worst conditions of propagation of light the effect of turbulence on the distribution of the average intensity of the beam must be taken into account even for vertical propagation paths and the infrared region of the spectrum. The nonlinearity and turbulence make a nonadditive contribution to the spreading of the beam. As the wavelength of the laser radiation decreases the effective radius of the beam can increase significantly owing to turbulence on vertical paths not only for the worst but also for the average conditions of propagation of light. For this reason, in order to describe correctly the thermal distortions of laser beams with visible and IR wavelength, in the atmosphere the effect of turbulence must be taken into account in many practically important situations. If the numerical values of a definite collection of functions at the boundary of the effective layer of the atmosphere x_e are known, the formulas (14)–(18) permit determining the dimensions and displacement of a beam on any path $x > x_e$ without directly calculating the average intensity distribution.

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