## Pecularities in the vertical statistical structure of temperature, humidity and wind fields in the atmospheric boundary layer of Western Siberia. Part 2. Characteristics of vertical correlation

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Based on data of long-term observations at eight aerological stations, the peculiarities in inter-level correlation of the temperature, humidity and wind in the atmospheric boundary layer over the territory of Western Siberia are described. The problem of small parametric description of vertical structure of the fields of these meteorological parameters using the principal eigenvectors of their correlation matrices is under study.

### Introduction

The peculiarities of vertical distribution of mean values and rms deviations of the temperature, humidity and wind in the atmospheric boundary layer (ABL) over the territory of Western Siberia were treated in Ref. 1. However, it is known that the statistical description of vertical structure of any meteorological field is not complete if the peculiarities of its inter-level correlations are not considered.

In this paper, which is a continuation of Ref. 1, the peculiarities of inter-level correlations of the temperature, humidity (mass fraction of water vapor), and orthogonal components of wind velocity in the atmospheric boundary layer are under study. Besides, the problems are considered, how typical are these peculiarities for the whole territory of Western Siberia and to what extent they can be disturbed depending on the season, height of the initial level, and physicalgeographic conditions of an individual region. Such an important question (from the viewpoint of smallparametric description of the vertical structure of random meteorological fields) is also under consideration as representation of these fields in the form of a sum of principal natural orthogonal functions, which at a maximal information content (as compared to the initial correlation matrix) have a greater spatial and temporal stability.

## 1. Peculiarities of inter-level correlation of temperature, humidity, and wind

Let us dwell on basic regularities and peculiarities typical of vertical (inter-level) correlation links between temperature, humidity, and wind in the ABL and try to determine how general they are for the whole territory of Western Siberia or, on the contrary, to what extent they depend on local physical-geographic conditions. In this case, as in the analysis of background characteristics and variability,<sup>1</sup> the peculiarities of inter-level correlations will be considered for the temperature and humidity independently of zonal and meridional components of wind velocity.

# 1.1. Inter-level correlation of temperature and humidity

Figure 1 plots the inter-level correlations of temperature and humidity (mass fraction of water vapor) in the atmospheric boundary layer (up to a height of 1600 m), estimated by data of five-year (2001–2005) observations at two typical aerological stations: Salekhard and Omsk, presenting the polar region and the South of Western Siberia.

According to the matrix of coefficients of interlevel correlation, the diagonal line corresponds to the correlation coefficient values equal to unit; isopleths represent the coefficients of inter-level correlation. Because of symmetry of autocorrelation matrices the distribution of these coefficients for the temperature (above the diagonal) and the humidity (under the diagonal) is given in one plot.

Analysis of Fig. 1 shows that the correlation of variations of the temperature and humidity between the ground and all upper levels of the atmospheric boundary layer are positive and weaken as the distance between correlated levels increases. Such a behavior of the correlations is observed everywhere and in both seasons under consideration.

The same regularity is also typical for the correlation coefficients  $r_{tt}(h_i, h_j)$  and  $r_{qq}(h_i, h_j)$  calculated between any initial level and all upper levels of the atmospheric boundary layer.



Fig. 1. Plots of inter-level correlation of the temperature (above the diagonal) and humidity (under the diagonal) for typical stations of Western Siberia: January (a), July (b).

Along with this regularity, vertical correlations of the temperature and humidity in the atmospheric boundary layer are characterized bv some peculiarities. In particular, in winter the coefficients of correlation between the variations of temperature and humidity near the ground (or at a 100 m height) and at all higher levels decrease rapidly with height inside the low 600 m layer, where inter-level correlations of these meteorological parameters are strongly disturbed due to the occurrence of highpower surface inversions; and at higher altitudes they decrease more slowly. For example, near Salekhard the temperature correlation coefficient  $r_{tt}(h_0, h_i)$ decreases by 0.35 (from 1.00 at the ground to 0.65 at a 600 m height), and in the layer 600-1200 m (the same distance between correlated levels), it changes by 0.14 (from 0.65 to 0.50).

In summer, as opposite to winter, in the absence of strong near-ground inversions, the extinction of inter-level correlations for the temperature and humidity with increasing distance between intercorrelated levels proceeds with lower intensity and more uniformly (for example, if in winter near Salekhard the temperature correlation coefficient  $r_{tl}(h_0, 1600 \text{ m})$  is 0.46, then in summer it is about 0.70). However, in the south of Western Siberia, where in summer a 100 m layer of surface inversion has been revealed,<sup>1</sup> the attenuation of inter-level correlations between the temperature and humidity variations near ground and at upper levels has an irregular character: very active in the lower 400 m layer and more slow higher this layer. So, for example, in the vicinity of Omsk the temperature correlation coefficients  $r_{tt}(0, 400 \text{ m}) = 0.795$ , and  $r_{tt}(0, 800 \text{ m}) = 0.725$ , i.e., in the layer 0–400 m this coefficient decreases by 0.205, and between 400 and 800 m - only by 0.070.

#### 1.2. Inter-level wind correlation

To analyze main regularities and peculiarities of inter-level correlations of wind, presented by its orthogonal components, we use Fig. 2, where the distribution of coefficients of inter-level correlation of zonal and meridional wind velocities at different heights of the ABL, is plotted, obtained for the same aerological stations (Salekhard and Omsk).

Analysis of Fig. 2 shows that the same general regularity is characteristic of zonal and meridional components of wind velocity, as for temperature and air humidity: the inter-level correlation of orthogonal components of wind velocity is also positive



Fig. 2. Plots of inter-level correlation of zonal (over the diagonal) and meridional (under the diagonal) components of wind velocity for typical stations of Western Siberia: January (*a*), July (*b*).

throughout the atmospheric boundary layer, universally and regardless of the season, and it decreases with increasing the distance between the correlated levels. However, in winter the greatest decrease of inter-level correlation of zonal and meridional wind, estimated between the ground level and all higher levels, is observed not in 600 m layer, but in 300–400 m layer, i.e., the wind is characterized by faster decrease of  $r_{UU}(h_0, h_j)$  and  $r_{VV}(h_0, h_j)$  values with height.

Besides, the wind also is characterized by different variation of inter-level correlations with height in winter. In particular, (as compared to the summer) a faster decrease of inter-level correlations of zonal and meridional components of wind velocity with increase of distance between the correlated levels is observed. This is conditioned by the fact that in winter the inter-level wind correlations are essentially disturbed because of the occurrence throughout of strong surface inversions, which in summer are not observed.

All the above-mentioned regularities (positive correlation, its weakening with the increase of distance

between the correlated levels and also the growth of correlation closeness from winter to summer) are characteristic also for inter-level correlation estimated between the other initial (registered) levels and all the upper levels of the atmospheric boundary layer.

## 2. Results of small-parametric representation through natural orthogonal functions of vertical structure of random fields of temperature, humidity, and wind in the atmospheric boundary layer

In the present-day statistical investigations of spatial (including vertical) structure of random fields of meteorological parameters, they are optimally represented as a sum of natural orthogonal functions (eigenvectors of correlation matrices of the field specified in a chosen fixed system of points or heights).<sup>2–4</sup> Such a representation, as opposed to other

methods (for example, the expansions into the Chebyshev orthogonal polynomials, trigonometric functions or Legendre polynomials), presents a possibility to dispense with predefined expansion function, since it is determined statistically from actual peculiarities of the random field under study. Besides, the expansion of any random field into natural orthogonal functions has a faster rate of convergence. This allows a separation of most significant and stable characteristics from the initial information about this field, as well as an exclusion from consideration of small details and noises. Finally, the principal eigenvectors resulted from such expansion possess (as compared to the initial correlation matrix) a larger spatial and temporal stability.

All the above-mentioned advantages allow the use of natural orthogonal functions for solving such important practical problem as small-parametric description of a vertical structure of random meteorological fields in the atmospheric boundary layer, which for Western Siberia has not been solved up to the present. This Section is devoted to solution of this problem.

In the analysis of the obtained results we use eigenvalues and eigenvectors, calculated by the data of autocorrelation matrices  $||R_{tt}||$ ,  $||R_{qq}||$ ,  $||R_{UU}||$ , and  $||R_{VV}||$  for all considered stations of Western Siberia, using the system of equations

$$\sum_{j=1}^{k} R_{ij} F_{\alpha j} = \lambda_{\alpha} F_{\alpha i} \quad (i = k = 10), \tag{1}$$

where  $R_{ij}$  denotes the elements of the correlation matrix  $||R_{ij}||_{\xi}$ , calculated for a given meteorological parameter  $\xi$  (in our case, the temperature, humidity, and the orthogonal components of wind velocity);  $F_{\alpha i}$ are the components of eigenvectors of the same correlation matrix (where  $\alpha = 1, 2, ..., n$  is a number of expansion member, and i = 1, 2, ..., n is a number of the eigenvector component);  $\lambda_{\alpha}$  are eigenvalues of the matrix  $||R_{ij}||_{\xi}$  being the variances of the expansion coefficients  $C_{\alpha}$  of a certain random function  $\Phi(z)$  into eigenvectors of the same correlation matrix (here  $\Phi(z)$  is the k-dimensional random vector (vertical profile), given at the finite number of levels  $z_i$ ).

To choose the principal eigenvectors of the correlation matrix  $||R_{ij}||_{\xi}$ , providing an optimal and exact description of the vertical structure of the random meteorological field, as well as to estimate the admissible error, the following criterion was used

$$d_m = \sum_{\alpha=1}^m \lambda_\alpha / \mathrm{Tr} \| R_{ij} \|_{\xi}$$
(2)

(here  $\operatorname{Tr} \left\| R_{ij} \right\|_{\xi} = \sum_{\alpha=1}^{n} \lambda_{\alpha}$  is the trace of the correlation

matrix  $||R_{ij}||_{\xi}$  characterizing the share, contributed by the first (principal) vectors to the total variance. A description in detail of the procedure of obtaining natural orthogonal functions is given in Refs. 2 and 4.

Consider the behavior of natural orthogonal functions of vertical profiles of temperature, humidity, and wind, mainly, their statistical stability and information content. Figure 3 shows two first eigenvectors of correlation matrices of temperature, humidity, zonal and meridional components of wind velocity for stations Salekhard, Khanty-Mansiisk, Omsk, and Novosibirsk. Table presents the eigenvalues  $\lambda_{\alpha}$  of the above matrices for the same stations, as well as the values of the criterion  $d_m$  characterizing the share of the first m natural

orthogonal functions in the total variance  $\sum_{i=1}^{n} \lambda_{\alpha}$ .

It is seen that the first and second eigenvectors of the correlation matrices  $||R_{tt}||$ ,  $||R_{qq}||$ ,  $||R_{UU}||$ , and  $||R_{VV}||$  have general characteristics, independent of meteorological parameter, season, and geographic position of the station. Thus, for example, the first eigenvectors  $F_1$ , obtained for the above-mentioned matrices retain positive sign of their components throughout the atmospheric boundary layer. From the physical point of view, such altitude behavior of vectors  $F_1$  is fully regular, because reflects the positive inter-level correlation links characteristic of random variations of the temperature, humidity, zonal and meridional components of wind velocity throughout the atmospheric boundary layer (see Figs. 1 and 2). Note that in winter and in summer the eigenvectors  $F_1$  at all stations of Western Siberia form a family of curves, being in good agreement with each other and having insignificant spread (no more than 0.10), mainly for the component  $F_{11}$ .

The second eigenvectors  $F_2$ , preserving, as vectors  $F_1$ , spatial-temporal stability, have a more complicated character of distribution of the components with height, namely, they change their sign, regardless of meteorological parameter, close to the level 400–600 m. In this case, the vector  $F_2$  passes through zero from negative to positive values.

As for the eigenvalues  $\lambda_1$  and  $\lambda_2$ , the sum of which includes from 92 to 98% of the overall variance, they, depending on the season and geographical position of station, vary slightly (for  $\lambda_1$ ) or considerably (for  $\lambda_2$ ).

In conclusion note that the high spatial-temporal stability of the first two eigenvectors and fast convergence of the expansion into natural orthogonal functions allows a significant reduction (by 5 times) of the amount of statistical information, necessary for description of the vertical structure of random fields of temperature, humidity, and wind in the atmospheric boundary layer of Western Siberia. As well, they open fresh opportunities for their use in the problems of objective classification and statistical simulation of meteorological fields.



Fig. 3. Eigenvectors of correlation matrices of the temperature, humidity, zonal and meridional wind for the stations Salekhard (1), Khanty-Mansiisk (2), Omsk (3), and Novosibirsk (4): January (a), July (b).

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α	Salekhard		Khanty-Mansiisk		Omsk		Novosibirsk	
	$\lambda_{lpha}$	$d_m$	$\lambda_{lpha}$	$d_m$	$\lambda_{lpha}$	$d_m$	λα	$d_m$
January								
Temperature								
1	8.10	81	7.94	79	7.76	78	7.70	77
2	1.41	95	1.39	93	1.71	95	1.92	96
Humidity								
1	8.31	83	8.52	85	8.48	85	8.62	86
2	1.11	94	0.98	95	1.01	95	0.86	95
Zonal component of wind velocity								
1	8.08	81	8.38	84	8.24	82	8.22	82
2	1.26	93	1.15	95	1.15	94	1.26	95
Meridional component of wind velocity								
1	7.67	77	8.34	83	8.58	86	8.03	80
2	1.62	93	1.11	94	0.75	93	1.20	92
July								
Temperature								
1	9.23	92	9.21	92	9.00	90	8.90	89
2	0.60	98	0.61	98	0.74	97	0.91	98
Humidity								
1	9.19	92	9.12	91	8.97	90	8.83	83
2	0.40	96	0.48	96	0.48	94	0.57	94
Zonal component of wind velocity								
1	8.47	85	8.20	82	8.36	84	8.40	84
2	0.92	94	1.09	93	0.96	93	1.03	94
Meridional component of wind velocity								
1	8.21	82	8.42	84	8.61	86	8.11	82
2	1.16	94	0.88	93	0.77	94	1.24	94

Table. Eigenvalues of correlation matrices of the temperature  $\lambda_{\alpha}$  (°C)<sup>2</sup>, humidity  $\lambda_{\alpha}$  (‰)<sup>2</sup>, zonal and meridional components of wind velocity  $\lambda_{\alpha}$  (m/s)<sup>2</sup> and a share (%) of the considered variance  $d_m$ , calculated for some stations of Western Siberia

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