DEPENDENCE OF THE MULTIPLE SCATTERING CONTRIBUTION TO LIDAR RETURNS ON THE INTEGRAL PARAMETERS OF THE CLOUD PARTICLE SIZE SPECTRUM

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Multiple scattering from clouds with different particle size spectra is calculated in order to assess its input to lidar returns. It is shown that at certain receiving angles, independent of the distribution type the level of multiple scattering is determined by one of the integral parameters of the spectrum: $\sqrt[n]{\langle r^n \rangle}$ *(n = 2–5) or* $r_{32} = \langle r^3 \rangle / \langle r^2 \rangle$. *It is also shown that at large enough receiving angles the multiple scattering contribution depends weakly on both the spectrum type and the size of the particles. Some possibilities are explored of applying these effects to solving inverse problems of laser sounding of a cloudy medium.*

During laser sounding of optically dense media, such as clouds and fogs, aimed at retrieving either the extinction coefficient or the transparency profile along the beam path, the problem arises of accounting for the effects of multiple scattering (MS). The input from MS to lidar returns depends both on the extinction coefficient profile and the cloud particle size spectrum. Hence in general one needs to solve the combined problem of retrieving the size spectrum and the optical property profiles as functions of distance along the beam path. The problem thus formulated is extremely difficult and so far practical techniques for its solution have not been found. An alternative in this case is to seek for simplified formulations of the problem. One of such possible simplifications consists in an approximate parameterization of the particle size spectrum. The problem is then reduced to retrieving the extinction coefficient profile together with the profile of some parameter (or a set of parameters) of the particle size spectrum. Note that the retrieval of the parameters of the size spectrum may be of separate interest.

Envisaging the possibility of such a parameterization, a numerical simulation was performed in the present study of the dependence of MS input to lidar returns on the integral parameters of cloud particle size spectra. Two such parameters were chosen: $\sqrt[n]{\langle r^n \rangle}$ $n = 2–5$, and $r_{32} = \langle r^3 \rangle / \langle r^2 \rangle$, where the averaging is done over the size spectrum. The calculations followed a technique based on the small-angle approximation for various model spectra, similar to actual size distributions observed in stratiform clouds.[1, 2]

As is well known, stratiform clouds most often have relatively narrow droplet size distributions, which are described quite well by the γ-distribution:

$$f(r) = Ar^{\mu} \exp\left(-\frac{\mu r}{r_0}\right), \quad (1)$$

where the spectral modal radius r_0 = 4–6 μm; and the characteristic value of the parameter μ may be taken to be μ = 6.

At the same time, at the cloud edges and also inside the clouds during the later stages of their development different spectra may be observed, both wider than spectrum (1) at μ = 6, and bimodal. To simulate these, either log-normal distributions or γ-distributions of the form (1) or combinations of them were used. It was then assumed that the size variation of the Individual modes in the overall distribution was limited to the range 1.5–10 μm.[3] Specific values of the distribution parameters are presented in Table I. Spectra 1, 7, 8, and 9 are unimodal and the rest are bimodal. Spectrum 8 is described by the log-normal distribution;

$$f(r) = Ar^{-1} \exp\left[-\frac{\left(\ln \frac{r}{r_0}\right)^2}{\mu}\right]. \quad (2)$$

Spectra 1, 7, and 9 are γ-distributions of type (1). Spectra 1, 7, 8, and 9 display wide particle size distributions in which their modal radii r_0 and effective radii r_4 (or r_{32}) differ by a factor of several units.

Bimodal spectra 2–5 were made up from two γ-distributions, and spectrum 6 — from two lognormal distributions. Individual modes in spectra 2–5 were pushed to the very edges of the modal radius range, assumed above for stratiform clouds. In that case the effective particle size should vary from spectrum to mode-to-mode variation of the number densities (i.e.,

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the value of N_2/N_1). As for particle size, the finest sizes were encountered in spectrum 6 (r_{32} = 1.8). It contained numerous fine droplets and water-coated condensation nuclei.

TABLE I.

Model spectra parameters used in the calculations

N	r_{01}	μ_1	r_{02}	μ_2	N_2/N_1	r_{32}	r_4
1	1	0.652	—	—	—	5.60	4.51
2	2	6	8	6	$1.56\cdot10^{-2}$	4.80	4.18
3	1.5	8	8	8	$1.35\cdot10^{-2}$	4.54	3.62
4	1.5	8	8	8	$1.25\cdot10^{-3}$	2.29	2.33
5	1.5	8	8	8	$5.26\cdot10^{-2}$	7.39	4.97
6	0.3	1.47	1.5	0.058	$7.1\cdot10^{-2}$	1.8	1.33
7	4	2	—	—	—	9.60	8.42
8	1	0.3	—	—	—	9.75	7.7
9	1	2	—	—	—	7.23	4.65

Thus, the chosen model size spectra were quite representative for the task of numerical simulation of MS input to lidar returns as a function of variations in the spectral shape and the effective particle size.

MS returns from homogeneous layers were calculated. Such a choice was determined, on the one hand, by the available experimental and computational data.[4, 5] According to these, depending on the physical conditions under which the clouds were formed, various vertical profiles of the extinction coefficient may be observed, so that it appears impossible to choose some such profile as average (presumably such a profile would be statistically more representative than a homogeneous one). On the other hand, computational results from Ref. 6 have demonstrated that variations in the extinction coefficient profile at a fixed optical sounding depth do not produce any qualitative changes in the dependence of the MS input on the receiving angle, so it may be easily accounted for by introducing some effective parameter, functionally related to the profile shape. Therefore, to a first approximation, the effect of varying the cloud particle size on the MS input may be investigated using the homogeneous layer model. The experimental geometry selected for the calculations was monostatic. The sounding wavelength was chosen to be λ = 0.532 μm. The parameters varied were the optical depth τ, the geometric parameter $\tau_0 = z_0\sigma$ (here z_0 is the layer-to-layer distance and σ is the extinction coefficient), and the receiving angle is $2\phi_r$. The value of τ varied from 1 to 6. In reality this would correspond to the case of sounding the optically thick lower-layer clouds or to throughput sounding of such clouds during their formative or dissipative stages, and also to throughput sounding of the majority of upper-layer cloud forms. (According to data from Ref. 7 the median value of the optical thickness for stratiform and Sc (straticumulus) clouds of the upper layer is τ_{med} = 6.)

Typical results from these calculations are presented in Figs. 1 and 2, which show the dependences of the MS input to the backscattering returns $\lg(P/P_1)$ on the integral parameters r_3 (Fig. 1a), r_5 (Fig. 1b), and r_{32} (Fig. 1c). Here P is the total backscattered signal power and P_1 is the power of the single-scattering signal component.

Receiving angles are shown in the graphs. Integral parameters are given in microns. The parameter τ_0 is taken to be equal to 10. The solid lines correspond to the case of the investigated spectrum of type I at μ = 6. The individual points refer to the different spectra described in Table I.

It is seen from Figs. 1 and 2 that the degree of scatter of these points around the solid-line curves depends on the receiving angle. At certain receiving angles these points cluster quite densely around their respective curves; in other words, irrespective of the shape of the size spectrum the value of $\lg(P/P_1)$ is determined by the integral parameters r_3, r_5, and r_{32}. A similar picture is also observed for the parameters r_2 and r_4.

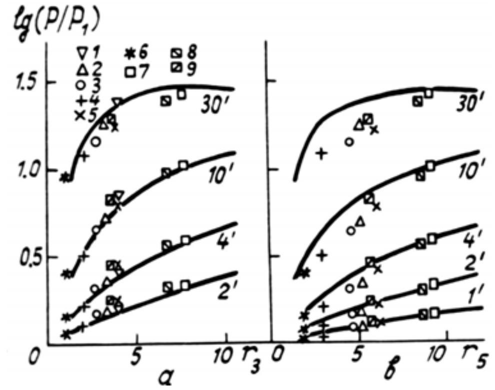

FIG. 1. Dependence of the MS input to lidar returns on the integral parameters of the particle size spectra r_3 (a) and r_5 (b).

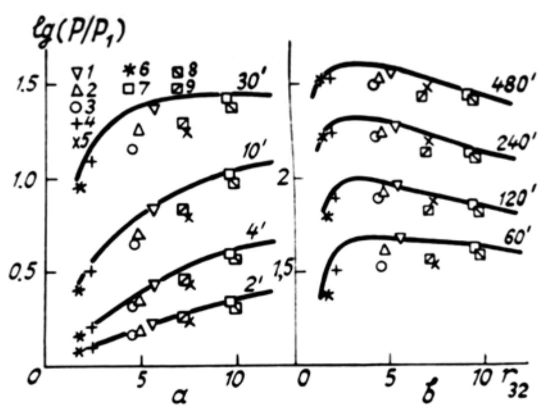

FIG. 2. Dependence of MS input to lidar returns on the integral parameters r_{32} of the size spectra.

Dependences similar to those presented in Figs. 1 and 2 are also obtained for other values of the parameters τ and τ_0. Note that value of the receiving angles, ϕ_c, at which this relationship between $\lg(P/P_1)$ and the spectral moments is the closest, depends but weakly on τ and is determined primarily by the value of the parameter τ_0. Table II lists the approximate values of the angle ϕ_c for $\tau_0 = 1$ 1 and 10, and all the integral parameters considered. It may be seen from Table II that while τ_0 remains constant, the value of ϕ_c decreases with increasing order of the moments. This fact may also be interpreted conversely: at smaller receiving angles the order of the spectral moment that is most clearly related to the MS input should increase.

Let us analyze the relationships outlined above. Of considerable importance is the fact that the processes of radiation propagation in both the forward and the backward directions for the chosen τ and ϕ_r occur within small angles.[1] When radiation is scattered into small angles, its directional scattering coefficient $S(\gamma)$ is related to the spectral moments. Thus, for the diffractional scattering angles $\gamma \le \gamma_d \simeq 2\rho_{\text{eff}}^{-1}$ (ρ_{eff} is the effective value of the Mie parameter for the given spectrum), the value of $S(\gamma)$ is determined by a combination of the moments $\langle r^4 \rangle$ and $\langle r^6 \rangle$ (Refs. 8 and 9). Choosing, for the case at hand, ρ_3 through ρ_5 as our ρ_{eff} we have $\gamma_d = (2.8–3.2) \cdot 10^{-2}$. In the range of large scattering angles, roughly from 10° to 45°, scattering by cloud droplets is adequately described by geometric optics, wherefore the value of $S(\gamma)$ is approximately proportional to $\langle r^2 \rangle$ (Ref. 10). Apparently, these relationships may be extrapolated to intermediate angles, where the transition from diffractional to geometric optics takes place.

TABLE II.

Multiple scattering input vs integral particle size: angles of best correlation

τ_0	r_2	r_3	r_4	r_5	r_{32}
1	90	60	30	6	4
10	30	20	4	1	2

The above properties of $S(\gamma)$ are reflected in the MS processes that determine the MS input to lidar returns. Forward scattering may be characterized by an average angle of scattering into the forward hemisphere (i.e., along the trajectories ending with the photons entering the receiver): $\bar{\gamma}(\phi_r)$. The value of $\bar{\gamma}(\phi_r)$ was estimated using the Monte-Carlo technique. In particular, the local double estimate technique was employed.[11] In that case the value of $\bar{\gamma}(\phi_r)$ is estimated from the relation:

$$\bar{\gamma}(\varphi_{\text{пр}}) = \Big(\sum_{n,m} \bar{\gamma}_{n,m} \Phi_{n,m}\Big) \times \Big(\sum_{n,m} \Phi_{n,m}\Big)^{-1},$$

where $\Phi_{n,m}$ is the input to total local double estimate from the signal coming from the mth node of the nth trajectory; $\bar{\gamma}_{n,m}$ is the angle of scattering into the forward hemisphere, averaged over the trajectory segment corresponding to the mth node. It appears that for the case $\tau_0 = 10$ the values of $\bar{\gamma}(\phi_r)$ depend weakly on τ for $\tau > 1$, and amount to 0.018, 0.025, 0.046, and 0.065, respectively, for $\phi_r = 1'$, $4'$, $20'$, and $30'$. Thus for $\phi_r = 1'$ (in this range the MS input is closely related to $\langle r^5 \rangle$) the angle $\bar{\gamma}(\phi_r)$ should be in the range $\gamma < \gamma_d$, where the value of $S(\gamma)$ is determined by $\langle r^4 \rangle$ and $\langle r^6 \rangle$. Correspondingly, for $\gamma_r = 20'$ and $30'$, where the MS input is related to $\langle r^3 \rangle$ and $\langle r^2 \rangle$, the angle should fall into the transitional range between diffractional and geometric optics.

The above estimates of $\bar{\gamma}(\phi_r)$ demonstrate that the close relationships between the MS input to lidar returns and the spectral moments are based on similar relationships between the respective light-scattering properties of an elementary volume in the forward scattering direction. It should be noted that the trajectories of the photons forming the signal include, besides a number of forward scattering ones, one backscattering one.[1, 2] Similar estimates made for the trajectory-averaged backscattering angle demonstrate that its value is significantly larger than the forward scattering value of $\bar{\gamma}(\phi_r)$, so that backscattering is not a governing factor in the formation of the above dependences.

Let us now consider those features in the obtained dependences of the MS input on the integral size parameters of the various spectra which are important in formulating and solving our inverse problems. As an example we choose the dependence on the parameter r_{32}. This choice is determined by the fact that retrieving the r_{32} profile is equivalent to retrieving the liquid water content profile W. (Indeed, since $W \sim r_{32}\sigma$, such a representation is of high practical importance).

Figure 2 presents the dependences of MS input to lidar returns on the parameter r_{32} for a wide range of receiving angles. It may be seen from the figure that for narrow receiving angles a quite clearly expressed dependence of $\lg(P/P_1)$ on r_{32} is observed. Increasing ϕ_r to 30′ weakens this dependence; further increase of ϕ_r makes it somewhat stronger again, though generally this dependence of $\lg(P/P_1)$ on r_{32} remains weak in this range. Taking such features of the behavior of $\lg(P/P_1)$ into account, two different types of its dependence on the integral particle size in the respective chosen receiving angle ranges can be recommended for practical application to laser sounding of cloudy medium. One of them (type I) corresponds to a rigid dependence of $\lg(P/P_1)$ on the integral particle size while the latter varies with spectral shape with this dependence of $\ln(P/P_1)$ on the integral particle size being of a clearly expressed and regular character. Using a relationship of type I, the problem of the approximate retrieval of the extinction coefficient $\sigma(z)$, together with one of the integral sizes, can be formulated.

The second type of relationship (type II) is realized at those receiving angles for which the dependence of $\lg(P/P_1)$ on particle size and spectral shape is the loosest. In that case a simpler problem may be formulated of the approximate retrieval of the $\sigma(z)$ profile, while accounting for MS effects, irrespective of the spectral shape and the integral particle size.

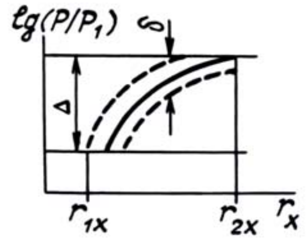

FIG. 3. On estimating the parameters δ and Δ

Let us now look more closely at the problem of selecting the proper receiving angles in both cases. Figure 3 shows the dependence of the $\lg(P/P_1)$ on one of the integral sizes r_x (either r_2–r_5 or r_{32}), thus generalizing the data from Figs. 1 and 2. As in those figures the solid line represents the dependence for some standard case, a spectrum of such shape that it may be used as a basic input in a model simulation when solving the inverse problem. We assume that this dependence is monotonic in the interval (r_{1x}, r_{2x}). Let us denote the total scatter in this interval by Δ. The dashed lines indicates the tube of width δ into which most points corresponding to non-standard shape spectra will fall. This width δ, provides an estimate of the possible deviation of $\lg(P/\mathrm{P}_1)$ from the model computed dependence due to variations of the cloud particle size spectrum. For the parameter r_{32} the value of δ was estimated from the computational data displayed in Fig. 2. Note that the largest deviation from the curve was not included in the estimate. The values of δ, $D = \delta/\Delta$, and $\Delta_s = \Delta+\delta$ thus obtained are listed in Table III.

The parameters that are important for problems of type I are D and Δ. The smaller the value D, the lower the systematic errors of retrieving the integral size parameter $r_x(z)$ attendant to variations in the spectral shape. The value of Δ characterizes the sensitivity of the result to Instrumental errors: the higher the value of Δ for a fixed level of the instrumental error, the lower the error in estimating $r_x(z)$. However we may, in order to provide for a more accurate joint retrieval of r_x and $\sigma(z)$, select a receiving angle from a range in which the MS input depends only loosely on r_x but where its relationship with r_x is still felt, i.e. such that the value of Δ_s is low. It can be seen from Table III that D becomes minimal for $\phi_r \le 10'$ at $\tau_0 = 1$, and for $\phi_r \le 4\le$ at $\tau_0 = 10$. Consequently, narrow receiving angles are informative for a problem of type I, and our set of receiving angles should be chosen from among them. If we are dealing with a problem of type II, the parameter Δ_s becomes decisive. The lower this parameter is, the weaker will be the influence of changes in the size spectrum on the results of determining the value of $\sigma(z)$. It follows from Table III that for $\tau_0 = 1$ the parameter Δ_s becomes minimal when $\phi_r \le 4'$ or $\phi_r \ge 120'$; while for $\tau_0 = 10$, the same happens for $\phi_r \ge 60'$. The case $\phi_r \le 4'$ will not be considered, since it is applicable only when $\tau_0 \le 1$. Moreover, in such a case the contribution of MS will in general be small, which should result in low Δ_s. Hence the range that is suitable for solving problem of type II for $\tau_0 \le 10$ is that of receiving angles of at least several degrees. Note that for such receiving angles the MS component of the signal is more than an order of magnitude larger than the single-scattering component. At $\tau_0 > 10$ the MS input becomes approximately invariant with respect to the parameter $\tau_d = \tau_0\phi_r$, so that at higher τ_0 narrower receiving angles may be used to solve type-II problems. However, it follows from this invariance feature that the MS input remains large there. An important advantage of using such receiving angles is the following; the level of the signal received from deep layers at high τ is increased by orders of magnitude due to MS input, and, respectively, the dynamical range of the signal is reduced.

TABLE III.

Values of δ, Δ, and D, determining the character of the dependence of $lg(P/P_1)$ on r_{32}

φ_w, min	τ_0 = 1			τ_0 = 10		
	δ	Δ_s	D	δ	Δ_s	D
2	—	—	—	0.02	0.36	0.055
4	0.015	0.19	0.079	0.05	0.52	0.096
10	0.05	0.38	0.13	0.13	0.55	0.24
30	0.12	0.45	0.27	0.17	0.29	0.58
60	0.17	0.36	0.47	0.13	0.17	0.76
120	0.16	0.21	0.76	0.08	0.20	0.40
240	0.10	0.15	0.67	0.10	0.23	0.43
480	0.08	0.19	0.42	0.10	0.22	0.45
600	0.07	0.20	0.35	—	—	—

Let us consider a concrete example of such a dependence. The Monte-Carlo simulation of a C1 model cloud, sounded at $\lambda = 0.7$ μm, demonstrates that up to $\tau = 10$ (i.e., the parameter $\tau_0 = 10$) choosing $\phi_r = 0.05$ should yield lidar returns within a range of approximately 3.5 orders of magnitude for a homogeneous layer.[12] Moreover, if it is taken into account that actual clouds have transitional edge zones in which the value of cr increases, and that the maximum return is observed not for $\tau = 0$, but for $\tau = 1$, then the actual range of variation of the returns should fall within only 3 orders of magnitude. Signal registration within such a range is quite possible with modern wide-range receivers.

Thus, operating within the range of those receiving angles in which the MS input to the backscattered lidar returns is at its maximum permits cloud sounding up to rather high optical depths. The problem of spatial resolution is also quite important for such soundings, since this resolution gradually deteriorates due to blooming of the propagating light pulse. Monte-Carlo estimates demonstrate that even up to the asymptotic regime of light propagation such spatial resolution remains at a level acceptable in practice.

In particular, it was found for the above case that for $\tau \le 6$ the relative spatial resolution $\Delta z/(z-z_0) \le 0.25$ (here Δz is the effective spatial length of the sounding light pulse after propagating the distance $z-z_0$ within the cloud, so that $\tau = \sigma(z - z_0)$. A more detailed analysis of this aspect of the problem will be presented elsewhere.

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