POTENTIAL CHARACTERISTICS OF THE WFR ALGORITHM FOR OBSERVATIONS OF EXTENDED TARGETS

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The qualitative characteristics of the WFR algorithm are analyzed for signal reception from extended targets with diffusely reflecting surface. The limiting performance characteristics of the algorithm, which are reached in the regime of multiple sensing of the target, are estimated.

The efficiency of optical information systems (i.e., systems of communication and remote sensing) significantly depends on the possibilities for the delivery of optical radiation energy through the turbulent atmosphere and on the available means of controlling the shape of the wavefront of the emitted field at the transmitting aperture of such systems. The construction principles and quality of algorithms designed to control energy transport through the turbulent atmosphere during observations of point-size targets have been studied in ample detail.[1–4] In particular, it was demonstrated in Ref. 5 that under certain conditions an optimal algorithm for controlling the wavefront shape is that of the wavefront reversal (WFR). However under actual conditions one most often has to deal with extended objects as the observation targets. Meanwhile, it seems that considerably fewer studies have been dedicated to such extended objects. Results from statistical modeling of the phase conjugation algorithm for certain particular cases of the sensing of extended targets are presented in Refs. 2 and 3.

This study analyzes the FWR algorithm, the receiving part of which is constructed on the basis of a multichannel heterodyne receiver. Its transmitting part is constructed according to the following scheme: driving generator (DG), multichannel amplitude-phase modulator (M), and optical quantum amplifier (OQA), with the DG simultaneously playing the role of a heterodyne in the receiver. WFR itself is performed at the intermediate (radio) frequency, using a set of phase converters. Analytical expressions are obtained for estimating the algorithm quality when the signal is received from an extended target with a diffusely reflecting surface. The limiting performance characteristics of control, which are attained in the regime of the multiple target sensing, are also estimated.

It is convenient to select as the figure of merit of the algorithm during the mth sensing step the coefficient E_m of the efficiency of use of the sounding energy ($m = 1, \dots, n$, $n = T/T_0$; T is the duration of the observation interval, T_0 is the sensing period). This coefficient is equal to the ratio between the coefficients of energy transmission to the target in the actual situation under study K_m and under ideal conditions K_1. The latter are understood to combine a homogeneous propagation medium, a point reflector sitting in the isoplanar area of the target, and a noise-free receiver.

When observing an extended target it is natural to consider two different means of transmission of energy to it:

a) energy is transmitted to a given point (z, ρ_0) of the target:

$$E_m(z,\rho_0) = K_m(z,\rho_0)/K_1(z,\rho_0); \tag{1}$$

b) energy is transmitted to the target as a whole:

$$E_m = K_m/K_1 \tag{2}$$

In expressions (1) and (2) we have introduced the following notations:

$$K_m(z,\rho_0) = \frac{\langle H_m(z,\ \rho_0)\rangle}{\frac{1}{W}\int\limits_{(m-1)T_0}^{mT_0} dt\ \frac{1}{S_a}\int\limits_{\Omega} \langle U_{0m}^2(r,\ t)\rangle d^2r},$$

$$K_m = \frac{\int\limits_{\Omega_0}\langle H_m(z,\ \rho)\rangle d^2\rho}{\frac{1}{W}\int\limits_{(m-1)T_0}^{mT_0} dt \int\limits_{\Omega} \langle U_{0m}^2(r,\ t) d^2r};$$

$$K_1(z,\ \rho_0) = \frac{S_a^2}{\lambda^2 z^2};\qquad N_0 = \frac{S_a S_0}{\lambda^2 z^2};$$

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$K_1 = \dfrac{N_0}{1+N_0}$; $H_m(z, p) = \dfrac{1}{W}\int\limits_{(m-1)T_0}^{mT_0} U_m^2(z, \rho, t)dt$ is the energy density of the emitted field $U_m(z, \rho, t)$ at the point (z, ρ) of the image plane; $U_{0m}(r, t)$ is the field emitted during the mth sensing step; S_a and S_0 are the areas of the transceiving aperture Ω and the projection of the target surface upon the image plane Ω_Q; λ is radiation wavelength; W is the wave impedance of the homogeneous propagation medium. The angular brackets denote averaging over the statistical ensembles of realizations of the refractive index of the turbulent atmosphere, the Fresnel reflectance coefficient of the rough surface, and the receiver noise.

The field emitted by the N-channel transmitter during the mth period may be written as

$$U_{0m}(r, t) = \mathrm{Re}\, U(t - (m - 1)T_0)\hat{A}_m(r, (m - 1)T_0)e^{-i\omega} \quad (3)$$

where $U(t)$ is the regular temporal modulating function, normalized to the condition $T_{ef} = \int\limits_0^{T_0} |U(t)|^2\, dt$, which determines the effective duration of the sensing pulse; $\hat{A}_m(r, t)$ is the amplitude-phase control function; and ω is the reference frequency. The control function $\hat{A}_m(r, t)$ formed with the use of the driving generator and the modulator has the form

$$\hat{A}_m(r, t) = \gamma E^*_{sg}(r)\hat{V}^*_{jm}(t), \quad r \in \Omega_j, \quad j = 1, \ldots, N \quad (4)$$

where $E_{sq}(r)$ is the complex amplitude of the DG radiation; γ is the gain coefficient of the OQA; $V_{jm}(t)$ is the complex amplitude of the signal exiting the intermediate frequency filter (IFF) of the jth channel; * is the conjugation signal; and Ω_j is the jth subaperture of the system. The signal at the intermediate frequency ω_0 is a conditionally Gaussian process with mean value $\mathrm{Re}\,\overline{\hat{V}}_{jm}(t)e^{i\omega_0 t}$ and the spatial correlation function of the form $B_{ijm}(t) = \delta^2_{\gamma m}(t)\delta_{ij}$, such that

$$\bar{\hat{V}}_{jm}(t) = \frac{a}{2W}\sum_{l=1}^{m}\int\limits_{(l-1)T_0}^{lT_0} d\tau h(t - \tau) \times$$

$$\times \int\limits_{\Omega_j} E^*_{sg}(r)X_{nl}(r, t)V_l(r)d^2r\,, \quad (5)$$

$$\delta^2_{jm}(t) = \frac{1}{2}\nu_{sg}(1 + \nu_t/\nu_{sg} + \alpha N_f)\sum_{l=1}^{m}\int\limits_{(l-1)T_0}^{lT_0} h_2(t - \tau)d\tau, \quad (6)$$

here $\alpha = \eta / h\omega$; η is the quantum efficiency of the photodetector (PD), $\hbar\omega$ is a quantum of energy at the signal frequency; $h(t)$ is the envelope of the IFF impulse response; ν_t and ν_{sq} are the electron flux intensities at the output of the PD produced by the PD dark current and the DG radiation; N_f is the spectral density of the external background;

$$X_{nm}(r, t) = U(t - (m - 1)T_0 - 2z/c) \times$$
$$\times \exp\left\{\frac{ik}{2z} r^2 - ik\theta_0 r\right\}$$

is the unperturbed regular component of the signal reflected by the target; $\Theta_0 = \rho_0/z$ is the angular coordinate of the target; and $V_m(r)$ is the complex amplitude of the random component of the signal. Assuming that the target lies completely in the isoplanar area with respect to the aperture Ω and that the turbulent atmosphere can be described as sin ampl itude-phase screen situated close to the aperture, we have

$$V_m(r) = E_m(r)\cdot\exp\left\{\psi(z, r, \rho_0; (m - 1)T_0 + 2z/c)\right\} \quad (7)$$

where $E_m(r)$ is the complex amplitude of the signal, determined by the statistical nature of the reflected signal; it is formed by the diffusely reflecting target surface and is further randomized by the turbulent atmosphere along the forward beam path of the sensing pulse; $\psi(z, r, \rho_0; (m - 1)T_0 + 2z/c) = \psi_m(z, r, \rho_0)$ is the random run-on of the complex phase, accumulated along the reverse path of the mth signal in the turbulent atmosphere.

Taking relations (3)–(7) into account and assuming that the spatial distribution of the DG radiation and the impulse response of the IFF are in agreement with regular modulation of the signal and that the forward plus backward propagation time delay along the signal path considerably exceeds the correlation interval for the atmospheric channel, we may express relations (1) and (2) in the form

$$E_{m+1}(z, \rho_0) = \frac{\langle R_{xx}(z, \rho_0)\rangle + \langle R_x(z, \rho_0)\rangle/q_m}{\langle R_x\rangle + N/q_m} \quad (8)$$

$$E_{m+1} = (1 + N_0)\frac{\dfrac{1}{S_0}\int\limits_{\Omega_0}\langle R_{xx}(z, \rho)\rangle d^2\rho + \dfrac{1}{S_0}\int\limits_{\Omega_0} R_x(z, \rho)\rangle d^2\rho/q_m}{\langle R_x\rangle + N/q_m} \quad (9)$$

Here

$$q_m = \frac{\dfrac{a}{2W}T_{ef}\int\limits_{\Omega}\langle|E_m(r)|^2 d^2r}{1 + \upsilon_T/\upsilon_g + aN_f} = q_0\frac{E_m(z, \rho_0)}{1 + N_0E_m(z, \rho_0)}$$

is the signal-to-noise ratio during the mth sensing interval; q_Q is the signal-to-noise ratio for propagation of the signal through a homogeneous medium and reflection from a point target with effective scattering surface (ESS) equal to the ESS of our extended target;

$$R_x = \frac{1}{N}\sum_{j=1}^{N}\frac{|x_{jm}|^2}{2W};\quad R_X(z,\rho) = \frac{1}{N}\sum_{j=1}^{N}|X_{jm}(z,\rho)|^2;$$

$$R_{xX}(z,\rho) = \left|\frac{1}{N}\sum_{j=1}^{N}\frac{x^{*}_{jm}X_{jm}(z,\rho)}{2W}\right|^2;$$

$$x_{jm} = \frac{1}{\Gamma_m^{1/2}}\,\frac{1}{\Delta}\int\limits_{\Omega_j}E_m(r)\exp\left\{\psi_m(z,r,\rho_0)\right\}d^2r \tag{10}$$

$$X_{jm}(z,\rho) = \frac{1}{\Delta}\int\limits_{\Omega_j}\exp\left\{-\frac{ik}{z}(\rho-\rho_0)r + \psi_m(z,\rho_0,r)\right\}d^2r$$

where $\Delta = \dfrac{S_a}{N}$ is the subaperture area;

$$\Gamma_m = \Gamma_m(r,r) = \langle|E_m(r)|^2\rangle/2W,$$

and $\Gamma_m(r_1, r_2) = \langle E_m(r_1)E^*_m(r_2)\rangle / 2W$ is the coherence function of the field $E_m(r)$.

Generally speaking, explicit expressions for figures of merit (8) and (9) cannot be found. For the situation that is the most interesting for the operation of adaptive systems ($N_{0m}/\sqrt{N} \le N_0/\sqrt{N} \ll 1$, $N_a/N \le 1$ and $N_a \gg 1$), quadratic approximation of the structure function of the complex phase, and Gaussian approximation of the coherence function $\Gamma_m(r_1, r_2)$. we have

$$E_{m+1}(z,\rho_0) \cong \langle R_a\rangle\frac{1/(1+N_{0m}) + 1/\langle q_m\rangle}{1+N/\langle q_m\rangle} \tag{12}$$

$$E_{m+1} \cong (1+N_0)\langle R_a\rangle\frac{1/(1+N_{0m}+N_0) + 1/\langle q_m\rangle}{1+N/\langle q_m\rangle} \tag{13}$$

$$N_{0m} \cong \frac{N_0}{1+N_0E_m(z,\rho_0)}, \tag{14}$$

Here $\langle R_a\rangle = \dfrac{1}{1+N_a/N}$; $\langle q\rangle = q_m\langle R_a\rangle$; $N_a = \dfrac{S_a}{\pi\rho_a^2}$ and N_{0m} is the number of coherence spots of the return radiation at the receiving aperture, produced respectively by atmospheric interference (ρ_a is the coherence radius of the spherical wave propagating from the target to the aperture) and by the diffusely reflecting surface of the iluminated part of the extended target during the mth sensing step; note that

$$E_1(z,\rho_0) = \langle R_a\rangle/N; \tag{15}$$

$$E_1 = \frac{1+N_0}{N}\cdot\frac{1}{S_0}\int\limits_{\Omega}\langle R_X(z,\rho)\rangle d^2\rho = (1+N_0)/(N+N_0+N_a). \tag{16}$$

It follows from the last two expressions that the optimal number of channels for the first "non-adaptive" step is one. Therefore

$$E_1(z,\rho_0) = 1/(1+N_a);\quad E_1 = 1/(1+N_a/(1+N_0));$$
$$N_{01} = N_0/(1+N_0/(1+N_a)) \tag{17}$$

Note that expressions (12) and (13) for the figures of merit are easily transformed into the respective expressions for a point target ($N_{0m} = N \ll 1$), (these are presented in Ref. 4) and also for the case of radiation transmitted to extended target with a dominating speckle point ($N_{0m} \ll 1$).

On the basis of the obtained relations it is possible to answer the question of the limiting performance characteristics of the WFR adaptive systems.

In the case of a large enough number of channels in the system ($N_a/N \ll 1$) and a high enough signal-to-noise ratio ($\langle q_m\rangle / N \gg 1$) the energy losses during the transfer of radiation to the extended target result from intense multiplicative interference, caused by the target surface roughness; this may lead to considerable distortion of the measured wavefront. Looking into this aspect of the problem, let us determine the limiting values of the figures of merit $E_m(z, \rho_0)$ and E_m in the stationary regime (for $m \to \infty$). For $\langle q_m\rangle / N$ and $N_a/N \ll 1$ the stationary value of the parameter N_{0m} must according to expression (14) satisfy the equation

$$N_{0\infty} = N_0/(1+N_0/(1+N_{0\infty}))$$

from which it follows that

$$N_{0\infty} = \frac{1}{2}\left[\sqrt{1+4N_0} - 1\right].$$

Note also that

$$E_\infty(z,\rho_0) \cong 1/\left[1+\frac{1}{2}\left[\sqrt{1+4N_0}-1\right]\right] \tag{18}$$

$$E_\infty \cong 1/\left[1+1/2(1+N_0)\right]\left[\sqrt{1+4N_0}-1\right]. \tag{19}$$

In this case it is easy to obtain an explicit expression for the current value of the parameter N_{0m} as a function of the number of the sensing step, m:

$$N_{0m} = N_{0\infty} + \frac{N_0}{K_\infty^{-2(m-1)}\left[\dfrac{K_\infty}{1-K_\infty^2} - \dfrac{N_0}{N_{01}-N_{0\infty}}\right] - \dfrac{K_\infty}{1-K_\infty^2}} \tag{20}$$

where $K_\infty = K_1 E_\infty$ is the transmission coefficient of energy transfer in the stationary regime for the problem of the delivery of radiation to an extended target.

Tables I and II present the figures of merit (18) and (19) for certain typical situations (nominator) and their corresponding values for the non-adaptive system (denominator), the latter given by expressions (17).

TABLE I

$E_\infty(z, \rho_0)$

N_a	$N_0 = 2$	$N_0 = 10$	$N_0 = 100$
10	$\frac{0.5}{0.1}$	$\frac{0.27}{0.1}$	$\frac{0.1}{0.1}$
100	$\frac{0.5}{0.01}$	$\frac{0.27}{0.01}$	$\frac{0.1}{0.01}$

TABLE II

E_∞

N_a	$N_0 = 2$	$N_0 = 10$	$N_0 = 100$
10	$\frac{0.75}{0.23}$	$\frac{0.8}{0.52}$	$\frac{0.91}{0.91}$
100	$\frac{0.75}{0.03}$	$\frac{0.8}{0.1}$	$\frac{0.91}{0.5}$

Thus, in the course of the multiple sensing of an extended target under such conditions there occurs a gradual "forgetting" of the initial situation (the latter is characterized by the state of the turbulent atmosphere, i.e., by the number of coherence spots AM. As a result, the maximum figure of merit of the adaptation possible is determined by the characteristics of the observed target only (by the parameter N_0). When the inequality $N \le N_a$ is satisfied the adaptation yields a significant gain in the figure of merit. This effect is explained by the fact that under such conditions the system aperture may be approximately represented as a set of N_{0m} surfaces, so that reasonably good compensation of the wavefront distortions produced by atmospheric instabilities is achieved within each of them by means of WFR. Fields from various parts of the aperture are incoherently summed at the target surface. When distortions from different parts of the surface are completely compensated, the expression for the figure of merit (12) takes the form $E_m(z, \rho_0) \cong (1 + N_{0m})$. On the other hand, in the absence of control we have $E_m(z, \rho_0) \cong 1/(1 + N_a)$.

Since large values of $N_0 \gg 1$ -correspond to $N_{0\infty} \approx \sqrt{N_0}$, a noticeable gain is already achieved for $N_0 \approx N_a$. Naturally, if a dominating speckle is present at the surface of the target ($N_{0m} \ll 1$), the fields at this surface, emitted by any region or regions of the aperture, will be completely in phase; the gain obtained in this case will be maximal. In the opposite situation, when $N_0 \gg N_a$, adaptation is absent.

When estimating the efficiency of the WFR adaptive system, it is always important to know what number of soundings will yield the above limiting performance characteristics and hence provide the needed gain from such an adaptive system. To find it, let us determine the threshold number of soundings m, such that for E_{m+1} the figure of merit will differ from its limiting value E_∞ by no more than some small value $\varepsilon \ll 1$:

$$1 - E_{m+1}/E_\infty \le \varepsilon,$$

In this case $E_{m+1} = 1/(1 + N_{0m}/(1 + N_0))$. Using expression (20), we obtain from this inequality

$$m \ge 1 - \frac{1}{\ln K_\infty^2} \ln \frac{1 + (1 - K_\infty^2)\cdot(1 - \varepsilon)/\varepsilon}{1 + (1 - K_\infty^2)\, N_0/K_\infty(N_{01} - N_{0\infty})} \tag{21}$$

The threshold values of m, obtained for $\varepsilon = 0.05$ under conditions similar those assumed in Tables I and II are presented in Table III.

Calculations show that the time needed for setting up of the stationary regime, which is characterized by the limiting values of these characteristics, depends on the extension of the target (on the value of N_0) and increases as it increases. The only exception is the case in which the adaptive regime cannot be reached at all ($N_{01} \le N_{0\infty}$). In our present study this corresponds to the situation in which $N_0 \gg N_a$.

TABLE III

N_a	$N_0 = 2$	$N_0 = 10$	$N_0 = 100$
10	2	3	1
100	2	4	7

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