OPTIMIZATION AND CALCULATION OF THE PARAMETERS OF A LASER ALTIMETER WITH INCOHERENT RECEPTION

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Received December 19, 1988

A graphic-analytical method is proposed for calculating the parameters of a pulsed laser altimeter with incoherent reception. An expression is derived for the signal-noise ratio at the output of the photodetector. This expression is used to determine the probability of correct detection of a signal based on the given probability of a false alarm and the tactical characteristics and specifitions of the altimeter. The observation interval is optimized in order to maximize the probability of correct detection of the signal and the value of the threshold is chosen. It is shown that the pulse detector processing scheme is insensitive to a 50% change in the observation interval. The error in measuring the range based on the interval between the leading edges of the sounding and reflected pulses is estimated. The potential accuracies of the range measurements are compared for different pulse detector processing schemes. The critical angle of inclination of the underlying surface, for which the power of the received signal at the input to the photodetector is equal to the threshold value, is found.

The use of a laser ranging system instead of radar on a spacecraft makes it possible to determine much more accurately the parameters of an orbit owing to the much narrower directional pattern and smaller widths of the transmitted pulses. The energy parameters of the existing pulsed lasers permit incoherent reception of the signal reflected from the underlying surface (US), using photomultipliers.[1,2] A signal is detected in the output circuit of the photomultiplier when the instantaneous voltage exceeds a threshold value. When the average number of photoelectrons detected simultaneously in the observation interval τ exceeds 10 the probability density of the number of photoelectrons is approximated by a Gaussian function.[1] We shall assume below that this condition is satisfied for both the signal and the signal + noise, but when applied to the statistics of only the noise photoelectrons this assumption gives a detection threshold that is too high.

The problem of optimizing the processing of optical signals with incoherent detection based on the criterion of maximum signal-noise ratio was studied in Ref. 2. As shown in Ref. 1, however, in the region of weak noise this criterion cannot always be used to estimate the efficiency of a ranging system, since the characteristics of detection of an optical signal depend on the absolute values of the average intensities of the optical signal and the noise.

In this paper the parameters of a satellite-borne laser altimeter, employing direct detection of the optical signal and a pulse detector filter with an amplitude-frequency characteristic of the form $[\sin(\omega\tau/2)]/(\omega\tau/2)$, where τ is the optimized observation interval (the width of the impulsive characteristic of the filter), are optimized using the procedure of detection based on the Neiman-Pearson criterion. We shall assume that the envelope of the signal reflected from the underlying surface is a Gaussian curve[3,4]

$$P_s(t) = P_{s0}\cdot\exp\left[-2t^2/\tau_{s0}^2\right], \quad (1)$$

where P_{s0} is the peak power of the received signal and τ_{s0} is the width of the signal at maximum slope, i.e., at the level 0.607. We shall study below not only the analog mode of operation of photomultiplier but also the mode in which charge is accumulated over the observation interval.[5] We shall represent the average number of signal photoelectrons as

$$N_s = \sqrt{2\pi}\,P_{s0}\tau_{s0}\varepsilon\cdot[\Phi(z) - 0.5]/h\nu, \quad (2)$$

where ε is the quantum efficiency of the photomultiplier, $\Phi(z) = (2\pi)^{-1/2}\int_{-\infty}^{z}\exp(-x^2/2)dx$ is the error function, and $z = \tau/\tau_{s0}$ is the normalized observation interval.

Using the expressions (1) and (2) the amount by which the threshold number of photoelectrons N_{th} exceeds the average number of the noise photoelectrons N_n in the observation interval can be written as

$$n_{th.s} = N_{th} - N_n = N_s f_1, \quad (3)$$

0235-6880/90/05 503-07 $02.00

where

$$f_1 = \left[\tau\cdot\exp\left(-T_{th}^2/2\tau_{s0}^2\right)\right]/\sqrt{2\pi}\,\tau_{s0}[\Phi(z)-0.5],$$

and T_{th} is the duration of the signal according to the chosen threshold level. Here and below we neglect the dark currents of the photomultiplier. The signal and noise power as well as the threshold power corresponding to the threshold voltage with pulse detector processing are shown in Fig. 1.

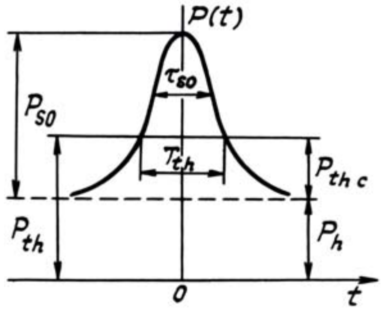

FIG. 1. The shape of the optical signals studied. The notation is explained in the text.

When ranging the underlying surface at the nadir from the maximum altitude of the orbit H_{max} the minimum power of the signal reflected from the underlying surface and received at the photocathode of the photomultiplier is equal to

$$P_{s0} = A\cdot E\cdot S_a, \qquad (4)$$

where E is the energy of the transmitter, S_a is the area of the receiving antenna, and A is a coefficient, which for a Lambertian surface is given by the expression

$$A = \sqrt{2}\cdot\rho_{min}\cdot K_{tr}\cdot\tau_{at}^2 \;/\; \pi^{3/2}\cdot H_{max}^2\cdot\tau_{s0}^2.$$

Here ρ_{min} is the lowest albedo of the underlying surface, K_{tr} is the transmittance of the receiving optical channel, and τ_{at} is the transmittance of the atmosphere.

The maximum value of the total noise power at the photocathode of the photomultiplier, owing to backscatterlng from the atmosphere and the external background radiation, is equal to

$$P_n = S_a(B\cdot E + \eta\cdot\Omega_r), \qquad (5)$$

where in the signal-scattering approximation $B = cK_{tr}\cdot\beta_\pi\tau_{at}^2\,/\,2H_{min}$ (here c is the velocity of light, β_π is the backscattering coefficient of the atmosphere near the underlying surface, and H_{min} is the minimum distance to the underlying surface); Ω_r is the solid angle of the field of view of the receiving antenna, $\eta = N_\lambda\cdot\Delta\lambda\cdot K_{tr}\cdot K(\varphi, \rho, \sigma_a)$ is a coefficient characterizing the external background level (here N_λ is the spectral power density of the solar irradiation at the boundary of the atmosphere, $\Delta\lambda$ is the bandwidth of the optical filter, and $K(\varphi, \rho, \sigma_a)$ is a coefficient that depends on the angle of the sun relative to the nadir φ, the albedo ρ of the underlying surface, and the attenuation coefficient σ_a in the atmosphere).

If the statistics of the noise and the signal + noise are Gaussian, then the probability of a false alarm P_{fa} and the probability of correct detection of the signal P_{cd} in the observation interval[1,4,7] have the form

$$P_{fa} = \frac{1}{\sqrt{2\pi}\,\sigma_n}\int_{N_{thr}}^{\infty}\exp\left\{\left[N-N_n\right]^2/2\sigma_n^2\right\}dN = 1-\Phi(U_1); \qquad (6)$$

$$P_{sd} = \frac{1}{\sqrt{2\pi}\,\sigma_\Sigma}\int_{N_{thr}}^{\infty}\exp\left\{\left[N-\left(N_s+N_n\right)\right]^2/2\sigma_\Sigma^2\right\}dN = \Phi(U_2), \qquad (7)$$

where the argument of the error function $\Phi(U)$ are equal to

$$U_1 = \left(N_{th}-N_n\right)/\sigma_n = n_{th.s}/\sqrt{FN_n}; \qquad (8)$$

$$U_2 = \left(N_s+N_n-N_{th}\right)/\sigma_\Sigma = \left(N_s-n_{th.s}\right)/\sqrt{F(N_s+N_n)}, \qquad (9)$$

$\sigma_n^2 = F\cdot N_n$, $\sigma_\Sigma^2 = F(N_s+N_n)$ are the variances of the number of photoelectrons associated with the external noise and with the signal + noise, respectively, and F is the noise factor of the photomultiplier. Using the expressions (8) and (9), together with Eq. (3), we find N_s:

$$N_s = F\left\{\frac{(1-f_1)^2}{[\Phi^{-1}(P_{cd})]^2} - \frac{f_1^2}{[\Phi^{-1}(1-P_{fa})]^2}\right\}^{-1}. \qquad (10)$$

The relative power threshold $f_0 = P_{th,s}/P_{s0}$ can be obtained from Eq. (1), substituting Eqs. (8), (4), and (3),

$$f_0 = \frac{\Phi^{-1}(1-P_{fa})}{A\cdot E}\sqrt{\frac{(\eta\Omega_r + B\cdot E)Fh\nu}{\varepsilon\tau S_a}}. \qquad (11)$$

The quantity $\Phi^{-1}(P_{cd})$ is determined from the expression (10) using the expressions for f_0 and (11). After some transformations we obtain

$$\Phi^{-1}(P_{cd}) = \sqrt{\frac{\sqrt{2\pi}\,AS_aE\tau_{s0}^2\varepsilon}{Fh\nu}}\times$$

$$\times\left\{\frac{\Phi(z)-M\sqrt{z}-0.5}{\sqrt{\Phi(z)+\dfrac{(\eta\Omega_r+BE)z}{\sqrt{2\pi}AE}-0.5}}\right\} \qquad (12)$$

where

$$M=\frac{\Phi^{-1}(1-P_{fa})}{AE}\sqrt{\frac{(\eta\Omega_r+BE)Fh\nu}{2\pi\varepsilon S_a\tau_{s0}}}\,. \qquad (13)$$

In the expression (12) $A\cdot E/(\eta\cdot\Omega_r+B\cdot E)=q^2$ is the power signal-to-noise ratio at the input of the photodetector. Its maximum value, when there is no external background radiation, is limited by the backscattering noise and is equal to $q^2_{\max}=A/B$. We shall seek the maximum value of $\Phi^{-1}(P_{cd})$, which depends on z, as the maximum of the function y, determined by the expression in the braces in Eq. (12)

$$y=\frac{\Phi(z)-M\sqrt{z}-0.5}{\sqrt{\Phi(z)+z/\sqrt{2\pi}q^2-0.5}}\,. \qquad (14)$$

For the values $q^2\gg 1$ and $z\le 3$ it can be assumed that

$$z \ll \sqrt{2\pi}\,q^2[\Phi(z)-0.5]. \qquad (15)$$

The solution of Eq. (14), obtained by a numerical method under the condition (15), is presented in Fig. 2. One can see from it that the maximum of y is not sharp and as M decreases it shifts to the right and its value increases. It should be noted that $M=0$ corresponds to the probability of correct detection of the signal $P_{cd}=0.5$.

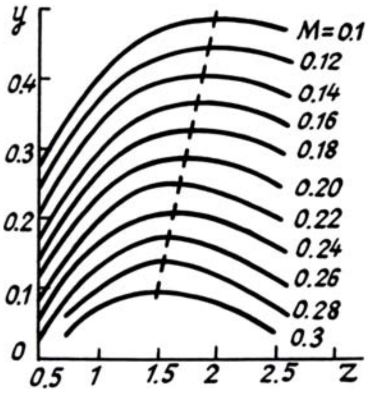

FIG. 2. The function y given by Eq. (14) *versus the normalized interval of observation z for different values of the parameters M for signal-to-noise ratio* $q^2\ge 100$.

Figure 3 shows plots of z_{opt} versus M and q^2. For $q^2\ge 100$ the curves merge into one curve. Figure 4 shows $y_{\max}$ versus M for different values of q^2. For negative values of $y_{\max}$ the probability of correct detection of the signal $P_{cd}<0.5$. Decreasing M increases P_{cd}. Thus, when calculating the parameters of the altimeter for fixed value of M and q^2, z_{opt} can be found from Fig. 3 and $y_{\max}$ can be found using Fig. 4.

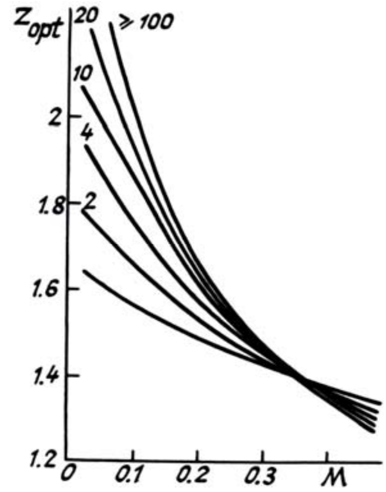

FIG. 3. The optimal observation interval z_{opt} *versus the parameter m for different value of the signal-to-noise ratio.*

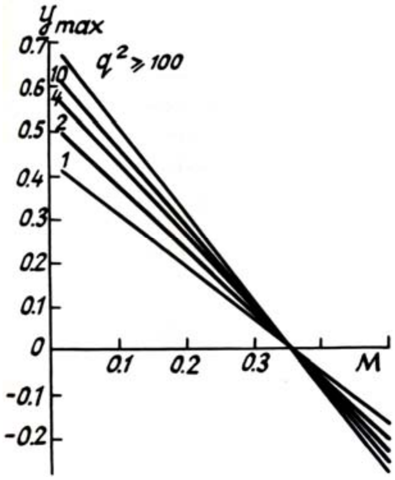

FIG. 4. $y_{\max}$ *versus M for different values of the signal-to-noise ratio.*

Determining z_{opt} permits optimizing the width of the transmission band of the electric filter at the output of the photodetector.[1,6] Once z_{opt} is known it is possible to find the characteristics of detection based on the Neiman-Pearson criterion, i.e., given the false-alarm probability P_{fa} or given $\phi^{-1}(1-P_{fa})$, to find the maximum probability of correct detection of the signal P_{cd} or $\phi^{-1}(P_{cd})$.

We shall represent the expression (12) and (13), substituting (4) and (14), in the following form:

$$\Phi^{-1}(P_{cd}) = y\cdot\sqrt{2E_{s0}\varepsilon/Fh\nu}\ ; \qquad (16)$$

$$M = \Phi^{-1}(1 - P_{fa}) \sqrt{Fh\nu/\sqrt{8\pi}E_{s0}q^2\varepsilon}\ , \tag{17}$$

where $E_{s0} = \sqrt{\pi/2} \cdot P_{s0} \cdot \tau_{s0}$ is the total energy of the received signal at the input of the photodetector. As one can see from Eqs. (16) and (17), the probability of correct detection of the signal P_{cd} depends on the total energy of the signal E_{s0}, the quantum efficiency, and the noise factor of the photomultiplier as well as on the signal-to-noise ratio and the false-alarm probability through y. Thus the relations (16) and (17), together with the plots presented in Figs. 3 and 4, permit calculating the probability of correct detection of the signal and choosing the time constant z_{opt} of the filter at the output of the photomultiplier.

We shall now determine the threshold which gives a fixed false-alarm probability. The threshold signal power at the input of the photodetector is equal to $P_{th} = P_{s0} \cdot f_0$, where the relative threshold, defined by Eq. (11), substituting Eq. (13), is equal to

$$f_0 = M\sqrt{2\pi/z_{opt}}\ . \tag{18}$$

The threshold number of photoelectrons N_{th} in the observation interval z_{opt} is determined in the form $N_{th} = N_n + N_c f_1$, where

$$f_1 = M\sqrt{z_{opt}}/[\Phi(z_{opt}) - 0.5]\ . \tag{19}$$

We studied above the characteristics of detection of the signal formed when the underlying surface is oriented perpendicular to the direction of sounding. As the angle of inclination of the underlying surface increases the probability of correct detection of the signal will decrease, since the energy of the received signal will decrease in proportion to the cosine of the angle of inclination. The ratio of the powers of the received signals from the normal P_{s0} and the inclined $P_{s\alpha}$ surfaces is equal to

$$P_{s0}/P_{c\alpha} = m/\cos\alpha, \tag{20}$$

where the factor $m = \tau_\alpha/\tau_0$ expresses the increase in the duration of the signal received from the inclined surface.

The value of the critical angle of inclination of the underlying surface for which the amplitude of the power of the received signal at the input of the photodetector is equal to the threshold power, i.e., $P_{s\alpha} = P_{th}$, is determined by the expression

$$\alpha_{cr} = \arcsin\sqrt{\left[1 - f_0^2\right]/\left[1 + f_0^2G_1^2\right]}\ , \tag{21}$$

where $G_1 = (2 \cdot \sqrt{2} \cdot H \cdot \Theta_0)/c \cdot \tau_1$, H is the altitude of the rangemeter above the underlying surface, Θ_0 is is the plane angle of divergence of the radiation of the laser transmitted at half power, and τ_1 is the width of the transmitted pulse at half power. The critical value of the signal duration enhancement factor cam be represented as

$$m_{cr} = (\cos\alpha_{cr})/f_0. \tag{22}$$

We shall now estimate the accuracy of the range measurement. The fluctuation component of the rms error σ_τ in the range measurement for the case when the measurements are performed between the leading edges of the sounding and reflected pulses and for the maximum power of the received signal is equal to

$$\sigma_\tau = \frac{0.36\tau_{s0}}{f_0\sqrt{(N_s/F)\ln(1/f_0)}}\ . \tag{23}$$

The results of comparison of the potential accuracy of the range measurements for different pulse detector processing schemes with $\tau_p = 10$ ns, $N_s = 16$, $f_0 = 0.607$, and $F = 1$ are presented in Table I, where τ_p is the width of the received signal pulse at half power.

The nonoptimal temporal processing, i.e., the bandwidth of the pulse detector filter, also affects the probability of correct detection of a signal. For relative detuning $\Delta = (z - z_{opt})/z_{opt}$ y decreases (see Fig. 2) and therefore the probability of correct detection of a signal also decreases. We shall estimate the relative decrease in y with the help of the coefficient $\delta = (y_{max} - y)/y_{max}$ and we shall compare the maximum probability of correct detection of a signal P_{cdm}, calculated for $z = z_{opt}$, with the probability of correct detection P_{cd} with detuning Δ and different values of M. The computational results are summarized in Table II. It was assumed that $P_{fa} = 10^{-6}$, $q^2 = 100$, $\Delta = 1.5$.

The obtained results show that a relative detuning of the transmission band of the pulse detector filter by a factor of 1.5 does not lead to significant changes in the probability of correct detection of the signal. In addition, as M increases, which is equivalent to a decrease of the energy of the input signal, the effect of detuning on the probability of correct detection of a signal increases.

The proposed method could be useful for calculating pulsed laser altimeters. It permits determining the signal-to-noise ratio at the input of the photodetector and to calculate the probability of correct detection of a signal from the given values of the false-alarm probability and the tactical characteristics and specifications of the laser altimeter being designed. To maximize the probability of correct detection of a signal the observation interval is optimized and the threshold is selected. Comparison of the potential accuracy of range measurements for different processing schemes showed that they do not differ significantly.

TABLE I.

Gausian signal. Photon counting mode[8]	The signal envelope is of the form sinx/x. Envelope reconstruction mode	Gausian signal. Envelope reconstruction mode
$\sigma_\tau = \dfrac{\tau_p}{2.35\sqrt{N_s}}$	$\sigma_\tau = \dfrac{\tau_p}{1.8\sqrt{N_s}}$	$\sigma_\tau = \dfrac{0.3\tau_p}{f_0\sqrt{(N_s/F)\ln(1/f_0)}}$
1.06 ns	1.38 ns	1.75 ns

TABLE II

y_{max}	M	δ	P_{cd}	P_{cdm}
0.100	0.30	$4\cdot10^{-1}$	0.548	0.579
0.211	0.24	$1.61\cdot10^{-1}$	0.671	0.700
0.407	0.14	$7.13\cdot10^{-2}$	0.946	0.958
0.484	0.10	$5.7\cdot10^{-2}$	0.997	0.998

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