MESOSCALE MODEL OF THE EVOLUTION OF CLOUDS WITH A MIXED PHASE COMPOSITION TAKING INTO ACCOUNT THE INTERACTION OF OPTICAL, RADIATION, AND METEOROLOGICAL PROCESSES

K.A. Kondrat'ev, M.V. Ovchinnikov, and V.I. Khvorost'yanov

Institute of Limnology, Leningrad and
Central Aerological Observatory, Dolgoprudny
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A two-dimensional mesoscale numerical model of the evolution of clouds with a mixed phase, composition is formulated. In this model the microphysical processes are taken into account in detail in the calculation of the optical and radiation characteristics. This make it possible to investigate their change and interaction with the meteorological parameters in cloud formation, development, and dispersal. Equations describing the basic physical processes in a cloudy atmosphere and an algorithm for solving these equations numerically based on the method of splitting are presented. The results of application of this model for calculation of the evolution of the optical, radiation, and microphysical characteristics of clouds will be presented in a separate paper.

INTRODUCTION

The optical, meteorological and radiation characteristics of the atmosphere are closely related with one another, and in addition their interaction with one another is especially strong during the daytime. The term "optical weather" which appeared at the beginning of the 1980s should have, in our opinion, underscored the unity of the indicated characteristics. In Ref. 1 it is correctly pointed out that in application to optical weather seemingly strictly meteorological phenomena, such as precipitation, not to mention clouds, should also be regarded as an optical phenomenon.

At the present time our knowledge of the characteristics of optical weather and the principles governing the change in optical weather is very limited. For this reason it is for the time being impossible to proceed directly to optical forecasting, in spite of its great practical value.[1] In this connection it becomes very important to simulate numerically the evolution of optical weather. This can be done either by including in the optical-radiation models that have already been developed parameterizations and computing blocks that describe the meteorological processes or by adding to the characteristics studied in the models of cloud formation and precipitation the basic optical and radiation quantities. This paper is an example of the second approach.

The described two-dimensional nonstationary mesoscale numerical model, in which the microstructure of the clouds and precipitation and the dynamics of the atmosphere are taken into account in detail, makes it possible to calculate the fluxes and influxes of both long-wavelength and solar radiation, as well as some optical characteristics, for example, the meteorological visibility range. This model makes it possible to follow the evolution of the optical and radiation characteristics during cloud formation, development, and dispersal and the relationship between the microphysical, radiation, and dynamical processes occurring in the atmosphere.

SYSTEM OF EQUATIONS FOR CALCULATING THE HYDROTHERMODYNAMICS AND MICROPHYSICS OF CLOUDS

The microstructure and phase state of clouds under conditions of natural evolution and in the process of seeding of the clouds is calculated by solving the kinetic equations for the size distribution functions of the drops $f_1(x, z, r_1, t)$ and crystals $f_2(x, z, r_2, t)$ together with the equations for the temperature T and the moisture content q:

$$\frac{\partial f_1}{\partial t} + \frac{\partial (u f_1)}{\partial x} + \frac{\partial [(\omega - v_1(r_1)) f_1]}{\partial z} + \frac{\partial}{\partial r_1}(\dot{r}_1 f_1) = \frac{\partial}{\partial z} k_z \frac{\partial f_1}{\partial z} + \frac{\partial}{\partial x} k_x \frac{\partial f_1}{\partial x} + \left[\frac{\partial f_1}{\partial t}\right]_{col} + J_1 + J_{1a}; \qquad (1)$$

$$\frac{\partial f_2}{\partial t} + \frac{\partial (u f_2)}{\partial x} + \frac{\partial [(\omega - v_2(r_2)) f_2]}{\partial z} + \frac{\partial}{\partial r_2}(\dot{r}_2 f_2) = \frac{\partial}{\partial z} k_z \frac{\partial f_2}{\partial z} + \frac{\partial}{\partial x} k_x \frac{\partial f_2}{\partial x} + \left[\frac{\partial f_2}{\partial t}\right]_{col} + J_2; \qquad (2)$$

$$\frac{\partial T}{\partial t}+\frac{\partial}{\partial x}(uT)+\frac{\partial}{\partial z}[w(T+\gamma_a z)]=$$
$$=\frac{\partial}{\partial z}k_z\left(\frac{\partial T}{\partial z}+\gamma_a\right)+\frac{\partial}{\partial x}k_x\frac{\partial T}{\partial x}+$$
$$+\frac{L_F}{C_p}E_F+\frac{L_1}{C_p}E_{c1}+\frac{L_2}{C_p}E_{c2}+\left(\frac{\partial T}{\partial t}\right)_{\text{rad}}; \tag{3}$$

$$\frac{\partial q}{\partial t}+\frac{\partial}{\partial x}(uq)+\frac{\partial}{\partial z}(wq)=$$
$$=\frac{\partial}{\partial z}k_z\frac{\partial q}{\partial z}+\frac{\partial}{\partial x}k_x\frac{\partial q}{\partial x}-E_{c1}-E_{c2}. \tag{4}$$

The indices $i = 1$ and 2 denote drops and crystals, respectively; t is the time; x and z are the horizontal and vertical coordinates, respectively, and u and w are the corresponding components of the wind velocity; k_x and k_z are the coefficients of horizontal and vertical turbulent diffusion; $v_1(r_1)$ are the fall velocities of the particles[2]; γ_a is the adiabatic lapse rate; L_1 is the specific heat of condensation and sublimation, L_F and E_F are the specific heat and rate of freezing of the drops; $(\partial f_1/\partial t)_{\text{col}}$ is the rate of change of the spectra of the drops and crystals owing to coagulation and accretion;[3,4] and, $(\partial T/\partial t)_{rad}$ is the rate of radiation-induced change in the temperature.

The rate of growth of individual drops r_1 and of a crystal r_2 as well as the local rates of condensation E_{c1} and sublimation E_{c2} are calculated from the formulas

$$\dot{r}_1=\frac{D\Delta_1\rho_a k_{f1}}{\rho_1 Q_1 r_1 \xi_1^2},$$
$$E_{c1}=4\pi\rho_1\int_0^\infty \dot{r}_1 r_1^2 f_1\,dr_1, \quad Q_1=1+\frac{L_1}{C_p}\frac{\partial q_{s1}}{\partial T}. \tag{5}$$

where Δ_1 is the supersaturation above water and ice ($i = 1$ and 2, respectively); k_{f1} and ξ_1 are the form factor and the ratio of the characteristic sizes of a nonspherical particle (for example, for a prolate ellipsoid of revolution with semiaxes $a > b$, approximating a columnar crystal, $\xi_2=\dfrac{b}{a}$, $k_{f2}=2\eta/\ln\dfrac{1+\eta}{1-\eta}$, $\eta=(1-\xi_2^2)^{1/2}$ (Ref. 3)); ρ_a and ρ_1 are the density of air, water, and ice; D is the diffusion coefficient of vapor; and, q_{s1} is the saturating moisture content above water and ice.

The expressions J_1 and J_{1a} describe the freezing of drops and the nucleation of crystals under natural conditions, and they can be parameterized In the form[2,3]

$$J_1=-a_F r_1^3\exp[b_F(T_F-T)]\,f_1(r_1); \tag{6}$$

$$J_2=a_s\exp[b_s(T_s-T)]\,S(r_2-r_{2a})\times$$
$$\times\left[-\frac{dT}{dt}\right]\theta\left[-\frac{dT}{dt}\right]\theta(\Delta_2)-J_1; \tag{7}$$

$$J_{1a}=N_0 e^{-z/l_0}\Delta_1^\beta\delta(r_1-r_{1a})\theta(\Delta_1)\times$$
$$[\theta(T-T_c)+u\delta(u)\theta(u)+w\delta(w)\theta(w)], \tag{8}$$

where r_{1a} are the radii of the activated of condensation and sublimation nuclei; $\delta(x)$ is the Dirac δ function; N_0e^{-z/l_0} describes the vertical distribution of condensation nuclei; and, Δ_1^β is the distribution of the condensation nuclei over the supersaturations analogously to the distribution proposed by Tumy.[2,3]

SYSTEM OF DYNAMICAL EQUATIONS

The system of dynamical equations includes the equations of motion for the horizontal components of the wind velocity u and v along the x and y axes, respectively, the equation of continuity for determining the vertical wind velocity w, and the equation of state

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho_a}\frac{\partial p}{\partial x}+f_c v+\frac{\partial}{\partial z}k_z\frac{\partial u}{\partial z}; \tag{9}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho_a}\frac{\partial p}{\partial y}-f_c u+\frac{\partial}{\partial z}k_z\frac{\partial v}{\partial z}; \tag{10}$$

$$\frac{\partial \rho_a}{\partial t}+\frac{\partial(\rho_a u)}{\partial x}+\frac{\partial(\rho_a w)}{\partial z}=0; \tag{11}$$

$$p=\rho_a RT, \tag{12}$$

where p is the pressure, R is the gas constant, f_c is the Corioll s parameter. Introducing the pressure function

$$\pi=\frac{C_p\theta}{A}\left[\frac{p}{p_0}\right]^{AR/C_p}=\frac{C_p\bar{\theta}}{A\theta}T, \tag{13}$$

where θ is the potential temperature, $\bar{\theta}$ is the average value of the potential temperature, p_0 is the standard pressure on the underlying surface, and A is the heat equivalent of work, the following substitutions can be made on the right sides of Eqs. (9) and (10):

$$\frac{1}{\rho_a}\frac{\partial p}{\partial x}=\frac{\theta}{\bar{\theta}}\frac{\partial \pi}{\partial x}, \quad \frac{1}{\rho_a}\frac{\partial p}{\partial y}=\frac{\theta}{\bar{\theta}}\frac{\partial \pi}{\partial y}. \tag{14}$$

We shall write π in the form $\pi = \pi_0 + \pi'$, where π_0 is the large-scale component of the pressure giving rise to the geostrophic wind:

$$U_g(z) = -\frac{1}{f_c}\frac{\partial\pi_0}{\partial y}, \quad V_g(z) = -\frac{1}{f_c}\frac{\partial\pi_0}{\partial x}, \tag{15}$$

and π' describes the pressure perturbation owing to the thermal and orographic nonuniformities. If we study processes with horizontal scale $L_x \sim 10^1$–10^2 km and vertical scale $L_z \sim 0.5$–1.5 km, then for $L_z \ll L_x$ we can write in the quasistatics approximation (assuming that $\theta = \theta + \theta'$, $\theta' \ll \theta$)

$$\frac{\theta}{\bar{\theta}}\frac{\partial(\pi_0 + \pi')}{\partial z} = -g; \quad \frac{\partial\pi_0}{\partial z} = -g;$$

$$\frac{\partial\pi'}{\partial z} = \lambda_s\theta'; \quad \lambda_s = \frac{g}{\bar{\theta}}. \tag{16}$$

The second relation is the equation of hydrostatics and the third equation relates the temperature and pressure perturbations. This relationship depends on the scale ($L_x \sim 10^2$–10^3 km). Expressing π' in terms of θ' with the help of the equations for the thermal wind,[5] it is possible to represent the terms with the pressure gradient in Eqs. (9) and (10) in the form of am effective geostrophic wind:

$$U_g^{ef}(z) = U_{g0} - \frac{\lambda_s}{f_c}\int_{z_0}^{z}\frac{\partial\theta'_\alpha}{\partial y}dz' + \frac{\lambda_s}{f_c}\int_{z}^{H_0}\frac{\partial\theta'_\gamma}{\partial y}dz'; \tag{17}$$

$$V_g^{ef}(z) = V_{g0} + \frac{\lambda_s}{f_c}\int_{z_0}^{z}\frac{\partial\theta'_\alpha}{\partial x}dz' - \frac{\lambda_s}{f_c}\int_{z}^{H_0}\frac{\partial\theta'_\gamma}{\partial x}dz'; \tag{18}$$

where $\theta'_\alpha = \theta - \langle\theta\rangle_\alpha$; $\theta'_\gamma = \theta - \langle\theta\rangle_\gamma$; and, U_{g0} and V_{g0} are the components of the geostrophic wind in the absence of baroclinicity. These expressions were used in the equations of motion (9) and (10), which were solved simultaneously with the equation of balance of the turbulent energy b, and the similarity and dimensional relations for b, its rate of dissipation E, the turbulence coefficient k_z and the mixing length l:

$$\frac{\partial b}{\partial t} + u\frac{\partial b}{\partial x} + w\frac{\partial b}{\partial z} = k\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 - \frac{g}{T}\frac{\partial\theta}{\partial z}\right] - E + \frac{\partial}{\partial z}k_z\frac{\partial b}{\partial z}; \tag{19}$$

$$k_z = C_0 l\sqrt{b}; \quad E = C_1 b^{3/2}/l; \quad l = -\psi\Big/\frac{d\psi}{dz};$$

$$\psi = b^{1/2}/l. \tag{20}$$

This approach makes it possible, in principle, to calculate both the dynamics of mesoscale processes in the atmospheric boundary layer and the dynamics on warm fronts in the approximation of quasistatics.

METHODS FOR CALCULATING THE SOLAR AND LONG-WAVELENGTH RADIATION

The total rate of radiation-induced change in the temperature $(dT/dt)_{rad}$ is defined as the sum of the rate of long-wavelength cooling $(dT/dt)_1$ and the rate owing to the influx of solar radiation $(dT/dt)_s$. These and other characteristics of the short- and long-wavelength radiation (SWR and LWR) were calculated by the two-flux method taking into account in detail the microstructure of the droplet and crystalline phases in the clouds. Thus the upward $F^{\uparrow}_{s\lambda}$ and downward $F^{\downarrow}_{s\lambda}$ spectral fluxes of SWR are determined from the equations

$$\pm\frac{1}{\sqrt{3}}\frac{dF^{\uparrow,\downarrow}_{s\lambda}}{d\tau_\lambda} = F^{\uparrow,\downarrow}_{s\lambda} - \frac{\omega_\lambda}{2}\left(F^{\uparrow}_{s\lambda} + F^{\downarrow}_{s\lambda}\right) \mp \frac{\Omega_\lambda}{2}\left(F^{\uparrow}_{s\lambda} + F^{\downarrow}_{s\lambda}\right); \tag{21}$$

$$\omega_\lambda = \sum_i \sigma^s_{\lambda Li}q_{Li}/D; \quad \Omega_\lambda = \sum_i \sigma^s_{\lambda Li}q_{Li}\langle\cos\theta\rangle_i/D;$$

$$D = \sum_i\left(\sigma^s_{\lambda Li} + \lambda^s_{\lambda Li}\right)q_{Li} + \alpha_{\lambda v}q; \tag{22}$$

$$\bar{\sigma}^s_{\lambda i} = \frac{3}{2p_i\bar{r}_i}\frac{p_i+1}{p_i+3}\times\left[1 + \frac{p_i+1}{p_i+2}\frac{\lambda^2}{8\pi^2\cdot\bar{r}^2}\frac{\left(m_{\lambda i}-1\right)^2 - \kappa^2_{\lambda i}}{\left[\left(m_{\lambda i}-1\right)^2 + \kappa_{\lambda i}\right]^2}\right]; \tag{23}$$

$$\alpha^s_{\lambda i} = \frac{3}{4p_i\bar{r}_i}\frac{p_i+1}{p_i+3}\left[1 - \left[1 + \frac{8\pi\kappa_{\lambda i}\bar{r}_i}{\lambda(p_i+1)}\right]^{-(p_i+3)}\right];$$

$$\sigma^s_{\lambda i} = \bar{\sigma}^s_{\lambda i} - \alpha^s_{\lambda i}. \tag{24}$$

Here and in Eq. (5) the upper signs correspond to $F^{\uparrow}_{s\lambda}$ and the lower signs correspond to $F^{\downarrow}_{s\lambda}$; ω_λ is the single-scattering albedo; τ_λ is the optical thickness; $\sigma^s_{\lambda 1}$, $\alpha^s_{\lambda 1}$, and $\alpha_{\lambda v}$ are the average extinction, scattering, and absorption coefficients of the drops ($i = 1$) and crystals ($i = 2$) and the absorption coefficient of water vapor; $m_{\lambda 1}$ and $\kappa_{\lambda 1}$ are the real and imaginary parts of the refractive indices of water and ice; $\langle\cos\theta\rangle_1$ is the asymmetry factor of the scattering phase function, and, $\bar{r}_1$ and p_1 are the average effective radii and indices of the γ distributions, which are used to approximate the size spectra of the drops and crystals. This method and the computational algorithm are described in detail in Refs. 6–9.

To calculate the rate of radiation cooling in a cloud layer R_1 and the effective radiation of the surface R_0 we used K.Ya. Kondrat'ev's idea for describing the spectra[10] and we solved the transfer equation for LWR in the two-flux approximation. In Ref. 11, it was shown that the central part of the transmission window (8–13 μm) makes 90 to 95% of the contribution to in the bottom 1–2 km and to R_0. As a result, the entire spectrum of the long-wavelength radiation for modeling low clouds and fogs can be divided roughly into only two sections: 1) the region of the transmission window and 2) the region outside the window, where the fluxes are equal to the fluxes of black body radiation. The upward $F_1^{\uparrow}$ and downward $F_1^{\downarrow}$ fluxes and the influx $R_1 = C_p\rho_a(\partial T/\partial t)_1$ of LWR can be calculated in the two-flux approximation:

$$\frac{dF_w^{\uparrow}}{dz} = \beta_1\rho_a\left[\alpha_v q + \sum_{l=1}^{N}\alpha_{Ll}q_{Ll}\right](p_w B - F_w^{\uparrow}); \tag{25}$$

$$\frac{dF_w^{\uparrow}}{dz} = \beta_1\rho_a\left[\alpha_v q + \sum_{l=1}^{N}\alpha_{Ll}q_{Ll}\right](F_w^{\downarrow} - p_w B); \tag{26}$$

$$F_l^{\uparrow,\downarrow} = F_w^{\uparrow,\downarrow} + (1 - p_w)B;\quad R_l = \beta_l\rho_a(F_l^{\uparrow} + F_l^{\downarrow} - 2B); \tag{27}$$

$$\alpha_{Li} = \alpha_0\left[1 - \frac{p_i + 4}{p_i + 1}\bar{r}_i c_1 + \frac{(p_i + 4)(p_i + 5)}{(p_i + 1)^2}\bar{r}_i^2 c_2\right], \tag{28}$$

where $F_w^{\uparrow,\downarrow}$ are the fluxes in the 8–13 μm window; $F_1^{\uparrow,\downarrow}$ are the integral radiation fluxes; α_v and α_{Li} are the average absorption coefficients of the vapor, drops, crystals, and aerosol particles; the number of substances (in addition to the vapor) $N = 2$; the index $i = 1$ corresponds to the drops; $i = 2$ corresponds to crystals; $\alpha_0 = 550$ cm^2/g, $c_1 = 2.26\cdot 10^{-2}$ μm^{-1}, and $c_2 = 8.44\cdot 10$ μm^{-2}. These coefficients were determined by comparing with data from spectral calculations.[11]

INITIAL AND BOUNDARY CONDITIONS AND THE SOLUTION ALGORITHMS

The initial fields T and q are given in the form

$$T(x, z) = T_0(x) - \gamma z;$$

$$q(x, z) = q_s(T)\left[1 - (1 - q_{r0}(x))\exp\left(\frac{z}{A_D}\right)\right]. \tag{29}$$

As the altitude increases the temperature decreases linearly from the value $T_0(x)$ at the surface, the relative moisture content at the surface is equal to $q_{r0}(x)$, and the undersaturation increases by a factor of e at the altitude A_D.

The boundary condition for T at the surface is the equation of heat balance while the boundary condition for q is the condition that the vapor flux be continuous:

$$-\rho_a C_p k_0\left[\frac{\partial T}{\partial z} + \gamma_a\right] - L\rho_a k_0\frac{\partial q}{\partial z} - c_s\rho_s k_s\frac{\partial\tau}{\partial z} - \left[F_1^{\uparrow} - F_1^{\downarrow}\right] - \left[F_s^{\downarrow} - F_s^{\uparrow}\right] = 0; \tag{30}$$

$$-k_0\frac{\partial q}{\partial z} = \alpha_{ef}\frac{\bar{V}_w}{4}(q_* - q_0). \tag{31}$$

where α_{ef} is the effective condensation coefficient, which describes the moistening of the soil; $\bar{V}_v$ is the velocity of the vapor molecules; $F_1^{\uparrow,\downarrow}$ and $F_s^{\uparrow,\downarrow}$ are the upward and downward fluxes of long-wavelength and solar radiation; and, c_s, p_s, k_s, and τ are the heat capacity, the density, the thermal dlffusivity and the temperature of the soil. The value of τ was determined by solving the equation of heat conduction in the soil $\frac{\partial\tau}{\partial t} = k_s\frac{\partial^2\tau}{\partial z^2}$ with the boundary condition $\tau(z)_\infty = \text{const}$. The remaining boundary conditions on the underlying surface (at the level of the roughness z_0) are $\frac{\partial f_1}{\partial z} = \frac{\partial f_2}{\partial z} = u\frac{\partial b}{\partial z} = v = \frac{\partial b}{\partial z} = 0$. At the top boundary, $b = \frac{\partial q}{\partial z} = 0$, $\frac{\partial T}{\partial z} = -\gamma$, $u = G$, $v = 0$.

The boundary conditions in the horizontal plane for Eqs. (1)–(4) are $\frac{\partial\phi}{\partial x} = 0$ on both boundaries, where φ is any of the quantities f_1, f_2, T, and q.

The system of equations (1)–(8) was solved by the method of componentwise splitting analogously to Ref. 12. At the first two stages the horizontal and vertical transfer were calculated:

$$\frac{\partial\varphi^{(1)}}{\partial t} + u\frac{\partial\varphi^{(1)}}{\partial x} = \frac{\partial}{\partial x}k_x\frac{\partial\varphi^{(1)}}{\partial x};$$

$$\frac{\partial\varphi^{(2)}}{\partial t} + w\frac{\partial\varphi^{(2)}}{\partial z} = \frac{\partial}{\partial z}k_z\frac{\partial\varphi^{(2)}}{\partial z}; \tag{32}$$

where $\varphi^{(1,2)}$ is any of the quantities f_1, f_2, θ, and q at the corresponding stage of the calculations. In calculating the vertical transfer the quantities $w - v_1(r_1)$ and $w - v_2(r_2)$ should be chosen for f_1 and f_2, respectively, instead of w.

At the third stage, when condensation, sublimation, and transfer of vapor from drops to the crystals are calculated, we transferred to the equations for the supersaturation Δ_1, which is a small difference of large quantities q and q_{s1}. The following system of equations was solved:

$$\frac{\partial\Delta_1}{\partial t} = -\left[\tau_{f_1}^{-1} + \frac{k_{f_2}}{\xi_2^2}\frac{Q_{21}}{Q_2}\tau_{f_2}^{-1}\right]\Delta_1 - \frac{k_{f_2}}{\xi_2^2}\frac{Q_{21}}{Q_2}\tau_{f_2}^{-1}\left(q_{s1} - q_{s2}\right) + \left[\gamma_a w - \left(\frac{\partial T}{\partial t}\right)_{rad}\right]\cdot\frac{\partial q_{s1}}{\partial T}; \tag{33}$$

$$\frac{\partial T}{\partial t} = \left[\frac{L_1}{C_p Q_1}\tau_{f_1}^{-1} + \frac{L_2}{C_p Q_2}\frac{k_{f_2}}{\xi_2^2}\tau_{f_2}^{-1}\right]\Delta_1 + \frac{L_2}{C_p Q_2} - \frac{k_{f_2}}{\xi_2^2}\tau_{f_2}^{-1}\left(q_{s1} - q_{s2}\right) - \gamma_a w - \left(\frac{\partial T}{\partial t}\right)_{rad}; \tag{34}$$

$$\frac{\partial y_1}{\partial t} = \frac{2D\rho_a}{\rho_1 Q_1}\Delta_1; \quad \frac{\partial y_2}{\partial t} = \frac{2D\rho_a k_{f_2}}{\rho_2 Q_2 \xi_2^2}\left(\Delta_1 + q_{s1} - q_{s2}\right); \tag{35}$$

$$\tau_{f1}^{-1} = 4\pi D\rho_a k_{f1}\int_0^\infty f_1(r_1)\sqrt{r_1^3 + y_1}\,dr_1. \tag{36}$$

where τ_{f1} and τ_{f2} are the phase-relaxation times for drops and crystals, Δ_1 and Δ_2 are the supersaturation above water and ice; and, y_1 and y_2 are the correspond integral supersaturations.

In Eq. (33) the first term describes the relaxation of the supersaturation owing to the efflux of vapor onto the drops and crystals, the second term describes the relaxation owing to transfer of vapor from drops to the crystals (the rate of these processes decreases as the phase-relaxation time increases), and the third term describes the generation of supersaturation owing to vertical motions and radiat ion-induced ranges in the temperature. In Eq. (34) the corresponding terms describe the change in the temperature owing to these processes.

The equation (32) were solved by the difference factorization method[12] and Eqs. (33)–(36) were solved by the Runge-Kutta method.

The system of dynamical equations was solved by the method of matrix factorization for the components of the wind velocity and simple factorization using an iterative procedure for the turbulent energy.

A difference grid that included 31 levels in the vertical direction with a step $\Delta z = 40$ m and 61 levels in the horizontal direction with a step $\Delta x = 1$ km was introduced in the x–z plane. The radial steps in the drops $\Delta r_1 = 2$ μm in the interval 0–20 μm and the radial steps in crystals $\Delta r_2 = 20$ μm in the interval 0–200 μm. In solving Eq. (31) the time step $\Delta t = 150$ sec and in solving Eqs. (33)–(36) the step size was chosen based on considerations of stability ($\Delta t' \le \min\tau_{f1}$). and varied from several to tens of seconds.

The difference grid chosen makes it possible to obtain a quite complete picture of the mesoscale processes studied. The range of sizes of drops and crystals studied in the model was chosen taking into account the data from natural measurements and encompassed most of the spectrum of the cloud particles.

The following characteristics of the clouds and precipitations were determined from the computed distribution functions of the drops f_1 and crystals f_2: the liquid water content q_{L1}, the ice content q_{L2}, the average radii and concentrations of the drops $\bar{r}_1$, N_1 and crystals r_2, N_2, their radar reflectance Z, the horizontal visibility range L, and the intensity I and sum S of the precipitation:

$$q_{L1} = 4/3\pi\rho_1\int_0^\infty r_1^3 f_1 dr_1; \quad N_1 = \int_0^\infty f_1 dr_1 \tag{37}$$

$$\bar{r}_1 = \int_0^\infty r_1 f_1 dr_1/N_1; \quad Z = \sum_{i=1}^{2} 64\int_0^\infty r_1^6 f_1 dr_1; \tag{38}$$

$$L = (3.91/2\pi)\left[\int_0^\infty r_1^2 f_1 dr_1 + 2\left[\int_0^\infty r_2^2 f_2 dr_2\right] \Big/ \left[\frac{2.5}{\bar{r}_2}\left[\frac{6q_{L2}}{\pi\rho_2 N_2}\right]^{1/3} - 3\right]\right]^{-1}; \tag{39}$$

$$I(x, t) = \frac{4\pi}{3}\sum_{i=1}^{2}\rho_1\int_0^\infty r_1^3 v_1(r_1) f_1 dr_1; \tag{40}$$

$$S(x, t) = \int_{t_0}^{t} I(x, t')dt', \tag{41}$$

where t_0 is the time at which the precipitation starts.

We used the model described above to perform a series of numerical experiments in order to study in detail the mutual effect of optical, radiation, and microphysical processes occurring in clouds on the crystallization of the clouds. The results obtained are published in a separate paper (see Russian pages 647–661 of this issue).

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