SOME CHARACTERISTICS OF LABORATORY MODELING IN PROBLEMS ON THE THEORY OF VISION

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The possibility of taking into account the effect of the glass walls of a cell holding a dispersed medium on the measurements of the characteristics of an optical image in laboratory experiments on the theory of vision is evaluated. The applicability of the principle of similarity to the interpretation of the results of laboratory investigations with a short measuring path is discussed.

The methods of laboratory simulation are now widely employed for solving the problems of radiation transfer in scattering media. In particular, the transfer of an optical image.[1,2] Laboratory simulation is used as an operational tool that makes it possible to investigate under controlled conditions the processes involved In the transfer of an optical image in a turbid medium for a wide range of values of parameters of the medium.[3–7] The results of laboratory investigations can be used to confirm or supplement theoretical results,[8] and in some cases to interpret real situations quite completely.[2]

At the same time the advantages of laboratory simulation, lister above, often cannot be realized completely because of their inherent deficiencies.[9]

In this paper some limits of laboratory simulation and possibilities of taking these limits into account when studying the conditions of observation through a localized layer of a dispersed medium, placed in a cell with glass walls, are examined.

In investigations of the characteristics of image quality, for example, the point-spread function (PSF) or line-spread function (LSF) of viewing systems, measurements are performed by two methods.[10]

A commonly used scheme for determining the PSF (approximate method or method of angular scanning[10]) is shown in Fig. 1.

We shall study a cell filled with distilled water containing suspended particles. One can see from Fig. 1 that when light is incident on the glass surface the angle of incidence θ of a pencil of rays on an elementary scattering volume of the medium decreases and, neglecting the glass-water interface, is equal to the angle of refraction θ_1 at the air-glass interface. If the index of refraction of air, glass, and water are denoted by n_1, n_2, and n_3 respectively, then, neglecting polarization effects, the relation between the angle of incidence of an elementary pencil on the scattering medium and the pencil incident on the surface of the cell is given by the law of refraction $\sin\theta/\sin\theta_1 = n_2/n_1$, where for $n_1 \approx 1$ $\sin\theta_1 = \sin\theta/n_2$.

Since the angle of refraction at the air-glass interface is equal to the angle of incidence at the glass-water interface

$$\sin\theta_2 = n_2\sin\theta_1/n_3 = \sin\theta/n_3,$$

where θ_0 is the true direction of the axis of the incident beam, relative to which the scattered radiation is formed.

It should be noted that in such laboratory experiments the existence of a glass surface imposes fundamental restrictions on measurements of the PSF, since even for angles of incidence $\theta - 90°$ the angle of entry into the medium does not exceed angles of the order of 48.5°. Taking into account the dependence of the reflectance of the glass on the angle of incidence,[11] the real angle of incidence on the scattering volume $\theta < 48.5°$, since for angles of 85° the reflectance exceed 90% and the magnitude of the recorder signal becomes insignificant. The decrease in the illumination of the scattering volume owing to geometric factors of the radiator and the surface effects can be estimated from the following considerations. The intensity of the illumination generated on the surface of the cell by an elementary area of constant brightness, whose dimensions satisfy the criterion of a photometric point and which makes and angle в with the normal to it, can be written as follows, taking into account the reflection from the glass and assuming that absorption is insignificant:

$$E_1 = \frac{I_0}{l^2}[1 - \rho(\theta)]\cos^4\theta,$$

where I_0 is the intensity of the light along the normal; l is the distance to the surface of the cell; and, $\rho(\theta)$ is the reflectance of the glass for the angle of incidence θ.

Since for angles of incidence ~ 40° the reflectance of the glass-water interface does not change significantly and it cam be assumed to be constant and equal to approximately 0.02,[12]

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$$E_2 = 0.98\,\frac{I_0}{l^2}\,[1 - \rho(\theta)]\cos^4\theta.$$

For the same scattering angle, the intensity of illumination neglecting reflection and refraction is equal to

$$E'_2 = \frac{I_0}{l^2}\cos^4\theta.$$

Then the coefficient expressing the difference between the computed and experimental intensities of illumination is equal to

$$K(\theta) = \frac{E'_2}{E_2} = \frac{\cos^4\theta_2}{0.98[1 - \rho(\theta)]\cos^4\theta}$$

Another feature that must be taken into account in order to compare correctly the theoretical and experimental results is as follows. Since the cell is a layered medium for which the index of refraction at the entrance is equal to the index of refraction at the exit, the unscattered (directly transmitted) radiation and the radiation scattered forward at small angles will leave the cell at an angle to the normal to the surface equal to the angle of incidence and it will merely be displaced in space. The scattered radiation leaving the volume is refracted. Thus, in accordance with Fig. 1, the receiving system records the scattered radiation at the angle $\gamma = \psi + \theta$, where ψ is the angle of reception. The true scattering angle γ will be related with the input and output parameters by the following relation:

$$\gamma = \arcsin\left(\frac{\sin\theta}{n_3}\right) + \arcsin\left(\frac{\sin\psi}{n_3}\right). \quad (1)$$

Thus in order to compare correctly the results of experiments and the corresponding calculations a correction must be made for the angles of emission and reception in accordance with Eq. (1), taking into account the index of refraction of the medium containing the suspension.

Of course, such a correction is a quite rough approximation to the real processes occurring in the measurements. It should be noted that the scattered radiation leaving the medium can be completely reflected at the glass-air interfaces for appropriate angles of incidence and can return once again into the medium, reflecting and scattering repeatedly. Processes of this type cannot be taken into account in the experiment. The critical angles at which total interval reflection of the radiation leaving the medium starts can be easily estimated. For example, for index of refraction $n_2 = 1.52$ the limiting angle of incidence ~ 40°. This means that the radiation can leave the scattering medium in an ~ 80° cone, whose axis as also the normal to the surface of the glass and whose apex is located at the center of the elementary scattering volume. In the process some of the emerging radiation will not be detected by the receiving system, if ψ is less than the angle of refraction at the glass-air interface (Fig. 1).

Total interval reflection cannot be achieved for the entering radiation at the glass-water interface and the main restrictions are determined by the reflection and refraction coefficients of the glass.

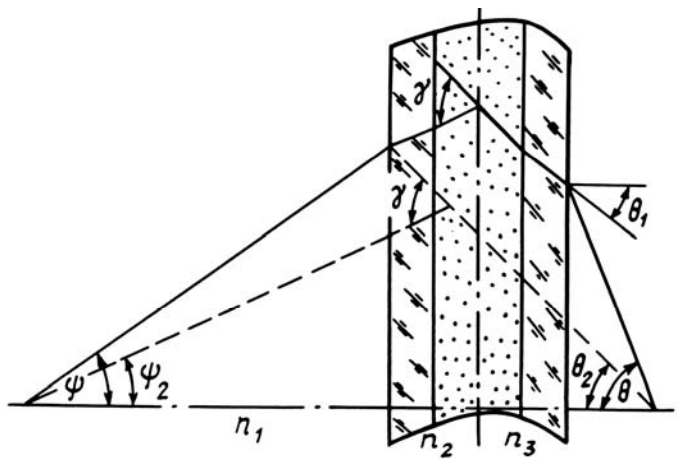

FIG. 1. Geometric diagram of measurements of the PSF in the method of angular scanning. θ *are the angles of emergence from a point object.*

In the scheme for determining the PSF by the method of spatial scanning[10] the scattered radiation emerging from the medium in a direction toward the detector is not refracted and reflected by large angles (Fig. 2).

Neglecting the refraction at the interfaces the receiver records radiation scattered by the angle φ. For an axisymmetric measurement scheme and the center of the cell this corresponds to the situation when the source lies in the object plane

$$r_2 = (\Delta L/2 + d + l)\,\mathrm{tg}\varphi$$

where ΔL is the distance between the walls of the cell (the thickness of the scattering layer), d is the thickness of the glass, and l is the distance from the surface of the cell to the object plane. In a real experiment, for the same scattering angle φ the spatial position of the source is determined by the distance $r = r_1' + r_1$ (Fig. 2). It is not difficult to find from the geometry of Fig. 2 the following relation for the angle of incidence φ_1 of the beam on the surface of the cell:

$$r = l\,\mathrm{tg}\varphi_1 + \frac{\Delta L}{2}\mathrm{tg}\varphi + d\,\mathrm{tg}\varphi_2,$$

where $\varphi_2 = \arcsin\left(\frac{\sin\varphi_1}{n_2}\right)$. Then for the same scattering angle φ the experimental spatial position of the source r differs from the computed value r_2 by the amount

$$\Delta r = l\left(\mathrm{tg}\varphi_1 - \mathrm{tg}\varphi\right) - d\left(\mathrm{tg}\varphi - \mathrm{tg}\varphi_2\right). \quad (2)$$

Thus in comparing the computational results with the PSF measured experimentally by the spatial scanning method the spatial differences in the positions of the source for the same scattering angles must be taken into account. This correction for the spatial position of the source in the calculation can be determined from the experimental value of r and the corresponding angle of incidence φ_1 on the surface of the cell.

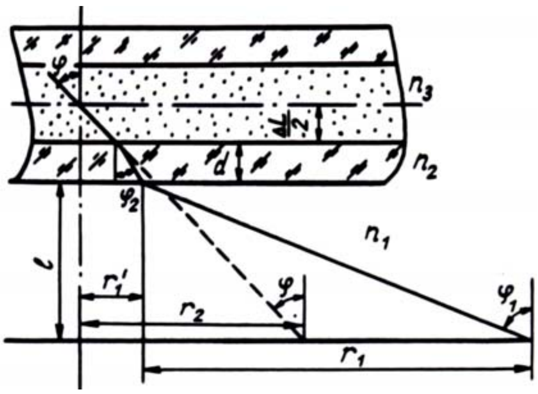

FIG. 2. The geometric diagram of measurements of the PSF in the method of spatial scanning. φ_1 *are the angles of emergence from a point test object.*

Like in the method of angular scanning, because of the geometric factor the difference in the spatial positions of the sources in the calculation and the experiment results in a difference in the intensity of illumination of the scattering volume. In this case the intensity of the illumination generated by a conical radiator on the surface of the cell is determined for a given angle φ_1 by the following relation:

$$E_1 = \frac{I_0}{l^2}\left[1 - \rho(\varphi_1)\right]\cos^4(\varphi_1),$$

where $\rho(\varphi_1)$, like in the first case, is the dependence of the reflective coefficient of the glass on the angle of incidence of the radiation. For the same angle of scattering in the medium, neglecting reflection and refraction, the intensity of illumination at the air-glass boundary will be equal to

$$E_2' = \frac{I_0}{l^2}\cos^4(\varphi),$$

where

$$\varphi = \left[\frac{\pi}{2} - \mathrm{arctg}\,\frac{l}{r_2}\right].$$

Formally the coefficient characterizing the difference in the intensity of illumination of the surface of the scattering medium for one and the same scattering angle φ can be written analogously to the corresponding relation for the method of angular scanning

$$K(\varphi) = \frac{E_2'}{E_2} = \frac{\cos^4\varphi_2}{0.98\left[1 - \rho(\varphi_1)\right]\cos^4(\varphi_1)}.$$

The above-enumerated restrictions and characteristics of laboratory simulation, which are associated with the use of glass cells, distort the processes responsible for the formation of multiple scattering in the model medium and make it impossible to compare with calculations and to make predictions in real media.

Such laboratory experiments all have the deficiency that the measurements are performed on limited paths.

In experiments[13] objectives with a long focal length are often employed to record the scattered radiation. This makes is possible to Increase the dimensions of the slit of the image analyzer without degrading the resolution of the analyzer. The results obtained in measurements of the PSF on short paths using objectives with long focal lengths may be different from the result obtained under natural conditions.

We shall now examine the qualitative aspect of the effect of these factors In an experiment for the example of the angular-scanning method. In Ref. 14 it was shown in a Monte Carlo calculation similar to the experimental scheme that the brightness of the scattered light can be taken into account more accurately by taking into account the size of the scattering spots arising in the image plane from each scattering point. Such a scattering spot arises in a real experiment with a fixed focusing plane (the plane optically conjugate to the image plane). Remaining constant during focusing on the object, for the scattering layer the position of the focusing plane depends on the distance of the layer from the source. Thus when analyzing image in the conjugate plane the intensity of illumination produced by the scattered radiation, aside from everything else, will decrease as the layer of the medium is moved toward the receiver owing to the "diffuse" image of the secondary source (we have in mind the brightness of an elementary scattering volume regarded as a point source).

In geometric optics there exists the concept of the depth of field of the image space, within which the points of the objects are imaged in the form of scattering circles, but because of their smallness they can be perceived as points.[14] The diameter of the scattering circle and hence also the depth of the imaging space are determined by the resolution of the photodetecting apparatus. For example, for the eye the size of the scattering circle should not be greater than one angular minute at the distance of best view. For photographic systems the size of the image of a point is assumed to be equal to 0.01–0.03 mm.[16] It can be shown that for most objectives employed in experiments and when the focusing plane lies at infinity sharp images of objects can be formed from distances of several tens of meters from the receiving system. This mean that for practically all laboratory experiments it is difficult to achieve a focusing plane at infinity without significantly increasing the dimensions of the experimental apparatus.

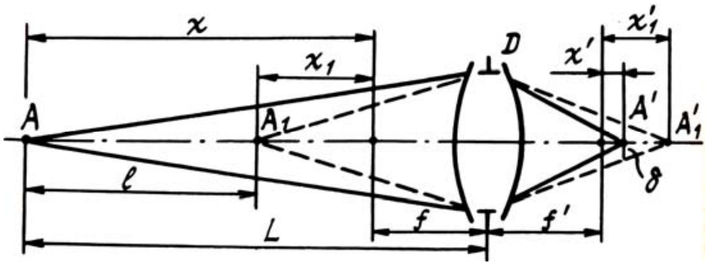

FIG. 3. The geometric scheme for calculating the scattering spot in the method of angular scanning.

For the conjugate points the magnitude of the displacement of the image from the focal plane can be found using the formulas of geometric optics.[15] Thus for equal back and front focal lengths the formula assumes the simple form

$$xx' = -f^2, \tag{3}$$

where x and x' are the distance from the object to the front focal point and from the image to the back focal point, respectively. For example, for an objective with a focal length of 1 m and a baseline of 10 m the deviation from the focal plane is equal to 0.1 m while for an objective with $f = 0.3$ m and $L = 4$ m it is equal to $2.4 \cdot 10^{-3}$ m; in addition, displacement of the layer by 1 m will cause the image of the secondary source to be displaced from the image plane by approximately $9 \cdot 10^{-3}$ m. In laboratory experiments, displacement of the layer of scattering medium away from the source toward the detector will always result in defocusing of the image of secondary source. The change brought about in the geometric size of the scattering spot by the displacement of the layer can be estimated by using Eq. (3). From Fig. 3 we find

$$x' = \frac{f^2}{(L - f)}; \quad x'_1 = \frac{f^2}{(L - l - f)},$$

where x' and x'_1 are, respectively, the deviation of the image of the object and secondary radiation from the focal plane. From the similarity of the triangles resting on D and δ with vertex at the point A' we obtain, after some transformations, the diameter of the scattering spot δ owing to defocusing of the image of secondary sources of the layer of scattering medium in terms of the optical and geometrical parameters of the experimental apparatus:

$$\delta = \frac{fDl}{(L - l)(L - f)}. \tag{4}$$

It is obvious from Eq. (4) that for given optical characteristics of the receiving objective and the baseline, δ will be a function of the distance l from the object or the relative distance $t = l/L$:

$$\delta = \frac{Df}{(L - f)} \cdot \frac{l}{(L - l)} = k\, \frac{t}{1 - t}, \tag{5}$$

where $k = \dfrac{fD}{(L - f)}$ is the optical-geometric constant of the given experimental apparatus.

The value of δ is minimum for t close to zero, i.e., when the scattering layer lies near the object. For long baselines L the focusing plane lies at infinity and the conjugate plane lies in the focal plane. By analogy to optical system, we shall give the size of the scattering spot arising as a results of the displacement of the layer of medium, such that in the viewing system for a given range of t it can be assumed to be a point. For example, for photographic systems we shall choose the size of the scattering spot to be ~ 0.01 mm, i.e, close to the minimum scattering circle, usedin such systems. For a given baseline L the value of t can be found from Eq. (5).

For example, for $L = 10.000$ m, $f = 0.3$ m, $D = 0.15$ m, and $\delta = 1 \cdot 10^{-5}$ m t and l assume the values 0.69 and 6896 m, respectively. From this estimate it follows that for positions of the layer $0 \le t \le 0.69$ the changes in the geometric sizes of the scattering spots will be less than the probing slit of the image analyzer, lying in the focal plane. For the directly transmitted unscattered radiation the zone of the sharply imaged space with a fixed scattering circle starts at several tens of meters from the objective (this is the distance from which the object can be assumed to be located in the focal plane). In a laboratory experiment, for the directly transmitted radiation this condition is not satisfied and the object is viewed in the image plane. If the baseline distance is changed, for example, $L = 4$ m is used, while δ, D, and f remain unchanged, then it is found that $t \approx 8 \cdot 10^{-4}$, and for $t = 0.69$ the scattering spot $\delta = 2.7 \cdot 10^{-2}$ m.

From the foregoing discussion and elementary calculations it can be concluded, first of all, that in this case the principle of similarity does not hold for the viewing system widely employed for simulation in scattering media. Thus when comparing the results of a laboratory experiment for a relative distance, for example, t_1, with the results of measurements on long path with the same value of t_1, significant discrepancies should be expected, since the distribution of the scattered light on a short baseline does not adequately describe the distribution on a long baseline. This result also pertains to comparing results of a model experiment with calculations for scattering layers.

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