EFFECT OF THE SPATIAL NONUNIFORMITY OF THE ABSORPTION COEFFICIENT AND THE INDEX OF REFRACTION OF A MEDIUM ON THE CURVATURE OF THE WAVEFRONT OF THE SOUNDING RADIATION

I.P. Lukin

Institute of Atmospheric Optics,
Siberian Branch of the Academy of Sciences of the USSR, Tomsk
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The characteristics of a narrow optical sounding beam, propagating in a medium with weak saturation of resonance absorption, are investigated theoretically. An analytical solution is constructed, based on an aberration-free approximation, for the second-order mutual coherence function of the sounding radiation. The simultaneous effect of spatial nonuniformities of the absorption coefficient and the index of refraction of the medium on the characteristics of an optical sounding beam, which is narrow compared with the transverse linear size of the beam of intense laser radiation, is analyzed. The conditions under which the parameters of the absorbing medium can be measured by recording the displacement of the image of the sounding laser beam after the focusing lens are determined.

When a high—power laser beam propagates in a nonlinear medium a spatial nonuniformity of the index of refraction of the medium is created.[1,2] This nonuniformity of the index of refraction is manifested in both the self-action of the high-power laser beam and propagation of sounding radiation in the zone of action.[1–4] Under the conditions of thermal self-action the real part of the index of refraction changes, i. e., the refraction properties of the medium change.[1,2] In Refs. 3 and 4 it is shown that information about the change in the parameters of the medium accompanying thermal self-action can be obtained by measuring the displacement of the image of the sounding laser beam after the focusing lens. It is well known[5,6] that under the conditions of resonance self-action of the radiation on the medium both the refraction part of the index of refraction and the absorption coefficient of the medium change. An analogous picture is also observed in the case when a high-power laser beam propagates in an aerosol medium.[2] In this paper the characteristics of a narrow sounding beam, propagating in a medium with weak saturation of resonance absorption, are studied theoretically. The conditions under which the parameters of the absorbing medium can be measured by recording the displacement of an Image of the sounding laser beam after the focusing lens are determined.

We shall study a gas of two-level atoms in the field of two oppositely propagating waves with a Gaussian distribution of the intensity and frequency ω. One of the waves saturates the medium while the other is the sounding wave. For small saturations the sounding wave will propagate in the medium with dielectric susceptibility[7,8]

$$X = x' + iX'', \tag{1}$$

where

$$X' = \frac{c\kappa_0}{4\pi\omega}\,\frac{I}{I_s}\,\exp\left[-\frac{2\rho^2}{a_0^2}\right]\frac{\delta\Gamma}{\Gamma^2+\delta^2},$$

$$X'' = \frac{c\kappa_0}{2\pi\omega}\left\{1-\frac{1}{2}\,\frac{I}{I_s}\,\exp\left[-\frac{2\rho^2}{a_0^2}\right]\frac{\delta\Gamma}{\Gamma^2+\delta^2}\right\},$$

c is the velocity of light; κ_0 is the linear absorption coefficient; I is the intensity of the saturating wave on the axis of the beam; I_s is the intensity of saturation; $\rho = \sqrt{y^2+z^2}$ is the distance to the axis of the saturating beam; a_0 is the radius of the saturating beam; Γ is the homogeneous linewidth; and, δ is the detuning of the frequency from the center of the line. In the derivation of the expressions (1) for the dielectric susceptibility of the medium the standard formulas from the third-order perturbation theory in the electric field strength ($I \ll I_s$), in which the averaging over the velocities is performed in the Doppler limit. The ratio of the gas pressure and the radii of the light beams is such that impact broadening is much smaller than Doppler broadening, but much larger than transit broadening.

The propagation of the optical sounding wave in a saturated medium is described by a parabolic equation:[1,2,7,8]

$$\left\{2ik\,\frac{\partial}{\partial x} + \Delta_\perp + 4\pi k^2\left[X'(x,\,\rho) + X''(x,\,\rho)\right]\right\}\times E(x,\,\rho) = 0 \tag{2}$$

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where $k = \omega/c = 2\pi/\lambda$; λ is the wavelength of the optical radiation; and, $\Delta_\perp = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the two-dimensional Laplacian operator. The corresponding equation for the second-order mutual coherence function

$$\Gamma_2(x, \rho_1, \rho_2) = E(x, \rho_1)E^*(x, \rho_0)$$

has the form

$$\left\{2ik\frac{\partial}{\partial x} + \left[\Delta_{\perp_1} - \Delta_{\perp_2}\right] + 4\pi k^2\left[X'(x, \rho_1) - X'(x, \rho_2) + iX''(x, \rho_1) - iX''(x, \rho_2)\right]\right\}\Gamma_2(x, \rho_1, \rho_2) = 0, \tag{3}$$

$$\Gamma_2(0, \rho_1, \rho_2) = E_0(\rho_1)E_0^*(\rho_2).$$

We shall transform in Eq. (3) to the near-axis approximation, i.e., for ρ_1, $\rho 2 \ll a_0$ we obtain from Eqs. (1) and (3)

$$\left\{2ik\frac{\partial}{\partial x} + \left[\Delta_{\perp_1} - \Delta_{\perp_2}\right] + 4ik\kappa + k^2\frac{\rho_2^2 - \rho_1^2}{F_r^2} + ik^2\frac{\rho_1^2 + \rho_2^2}{F_D^2}\right\}\Gamma_2(x, \rho_1, \rho_2) = 0, \tag{4}$$

where

$$F_L^2 = \frac{1}{2}\frac{k}{\kappa_0}\frac{I_s}{I}\frac{\Gamma^2 + \delta^2}{\Gamma\delta}a_0^2$$

Is the squared focal length of the induced lens-like medium (refraction channel);[3,4]

$$k = K_0\left(I - \frac{I}{2}\frac{I}{I_s}\frac{r^2}{r^2 + \delta^2}\right)$$

Is the nonlinear (saturated) absorption coefficient;

$$F_d = \sqrt{\frac{1}{2}\frac{k}{\kappa_0}\frac{I_s}{I}}\frac{\sqrt{\Gamma^2 + \delta^2}}{\Gamma}a_0$$

is the linear characteristic length of the problem, characterizing the influence of the diaphragming effect on the propagation of optical radiation ($F_L^2 / F_d^2 = \Gamma / \delta$).

We shall study the propagation of a Gaussian sounding beam with an initial distribution of the form

$$E_0(\rho) = E_0\exp\left\{-\frac{\rho^2}{2a^2} - \frac{ik}{2R_0}\rho^2\right\}, \tag{5}$$

where E_0 is the amplitude of the sounding beam; a is the initial radius of the sounding beam; and, R_0 is the initial curvature of the wavefront of the sounding beam.

We shall seek in the parametric form the solution of Eq. (4) with the boundary condition (5):

$$\Gamma_2(x, \rho_1, \rho_2) = E_0^2 f(x)\exp\left\{-g(x)\frac{\rho_1^2 + \rho_2^2}{2a^2} - i\frac{S(x)}{2}\left[\rho_1^2 - \rho_2^2\right] - \varphi(x)\left[\rho_1 - \rho_2\right]^2\right\}, \tag{6}$$

where $f(x)$, $g(x)$, $S(x)$, and $\varphi(x)$ are unknown functions.

Substituting Eq. (6) into Eq. (4) we obtain the following system of equations:

$$\begin{cases} \frac{f'(x)}{f(x)} - \frac{2}{k}S(x) + 2\kappa = 0, \\ S'(x) - \frac{1}{k}S^2(x) + \frac{1}{ka^4}g^2(x) + \frac{4}{ka^2}\varphi(x)g(x) - \frac{k}{F_L^2} = 0, \\ g'(x) - \frac{2}{k}S(x)g(x) - \frac{ka^2}{F_d^2} = 0, \\ \varphi'(x) - \frac{2}{k}S(x)\varphi(x) = 0 \end{cases} \tag{7}$$

with the boundary conditions

$$f(x)\big|_{x=0} = 1, \quad g(x)\big|_{x=0} = 0,$$

$$S(x)\big|_{x=0} = \frac{k}{R_0}, \quad \varphi(x)\big|_{x=0} = \text{const}.$$

When coherent optical radiation ($\varphi(x)\big|_{x=0} = 0$) propagates in the medium the solution of the fourth equation of the system Eqs. (7) is trivial: $\varphi(\boldsymbol{x}) = 0$. Under this condition the system of equations (7) will assume the form

$$\begin{cases} \frac{f'(x)}{f(x)} - \frac{2}{k}S(x) + 2\kappa = 0, \\ S'(x) - \frac{1}{k}S^2(x) + \frac{1}{ka^4}g^2(x) - \frac{k}{F_L^2} = 0 \\ g'(x) - \frac{2}{k}S(x)g(x) - \frac{ka^2}{F_d^2} = 0. \end{cases} \tag{8}$$

The system of equations (8) does not have an exact analytical solution. We shall seek the solution of the system of Eqs. (8) by expanding the functions

$g(x)$ and $S(x)$ in a series in powers of x. This solution is applicable for sounding paths much shorter than the diffraction length ($x < ka^2$), the initial curvature of the wavefront of the sounding beam ($x < R_0$), and the characteristic length of the problem, characterizing the influence of the diaphragming effect on the propagation of optical radiation ($x < F_q < F_1$). Then, taking into account the boundary conditions, the solution of the system of equations (8) can be written as follows:

$$\begin{cases} f(x) = \exp\left\{-2\kappa x + \dfrac{2}{k}\displaystyle\int_0^x dx' S(x')\right\}, \\ g(x) \simeq 1 + 2\dfrac{x}{R_0} - \dfrac{x^2}{k^2a^2} + \dfrac{x^2}{F_L^2} + \dfrac{ka^2}{F_D^2}x, \\ S(x) \simeq \dfrac{k}{R_0} - \dfrac{x}{ka^4} + \dfrac{kx}{F_L^2} + \dfrac{x^2}{a^2F_D^2}. \end{cases} \tag{9}$$

Analysis of the solution (6) and (9) obtained above shows that under the conditions of propagation of the sounding beam in the zone of action on the medium and saturation of absorption, because of the diaphragming effect the radius of the sounding beam decreases and additional curvature of the wavefront of the beam appears. In addition, because of the diaphragming effect the correction to $S(x)$ is negative, i.e., the additional focusing of the beam by the extended soft diaphragm results in the appearance of additional divergence of the optical radiation. For the case of a collimated ($R_0 = \infty$) sounding beam with $ka^2/x > 1$ the radius $a(x)$ and the curvature of the wavefront $R(x) = k/S(x)$ are determined by the following relations:

$$a(x) = \frac{a}{\sqrt{g(x)}} \simeq a\left\{1 - \frac{ka^2}{2F_d^2}x - \frac{1}{2}\frac{x^2}{F_L^2}\right\}, \tag{10}$$

$$R(x) = \frac{k}{S(x)} \simeq \left\{\frac{x}{F_L^2} - \frac{x}{k^2a^4} - \frac{x^2}{ka^2F_D^2}\right\}^{-1}. \tag{11}$$

It follows from the formulas (10)–(11) that the effect of the additional focusing of the sounding beam by the extended diaphragm can be neglected if

$$\begin{cases} a \ll a_0, \quad x < F_d, \quad \Omega > \Gamma/\delta & \delta \neq 0, \\ a \ll a_0, \quad x < F_d, \quad 1 < \Omega < (F_d/x)^2 & \delta = 0, \end{cases} \tag{12}$$

where $\Omega = ka^2/x$ is the Fresnel number of the transmitting aperture of the sounding beam.

Thus, when the conditions (12) hold the measurements of the wavefront curvature of a narrow sounding beam (for example, based on refocusing of its image after the focusing lens[3,4]) make it possible to determine the focal length of the induced lens-like medium F_1. If the focal length of the induced lens-like medium F_1 and the coefficient of nonlinear saturated absorption κ (measured based on the total extinction of the sounding beam (9)) are known, the linear absorption coefficient κ_0 and the saturation intensity I_s can be determined.

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REFERENCES

1. D. Stroben, ed., *The Laser Beam Propagation Through the Atmosphere* [Russian translation] (Mir, Moscow, 1981).
2. V.E. Zuev, A.A. Zemlyanov, Yu.E. Kopytin, et al., eds., *High-Power Laser Radiation in Atmospheric Aerosol* [in Russian] (Nauka, Novosibirsk, 1984).
3. M.S. Belen'kii, I.P. Lukin, and V.L. Mironov, "*Possibilities of optical sounding of atmospheric refraction channels*", Preprint No. 5, Institute of Atmospheric Optics, Siberian Branch of the Academy of Sciences of the USSR, Tomsk (1984).
4. M.S. Belen'kii, I.P. Lukin, and V.L. Mironov, Opt. Spektrosk. **60**, No. 2, 388 (1986).
5. E.V. Lugin and Yu.N. Ponomarev, Izv. Vyssh. Uchebn. Zaved. SSSR, Ser. Fiz., No. 3, 58 (1980).
6. B.G. Ageev, Yu.N. Ponomarev, and B.A. Tikhomirov, eds., *Nonlinear Opto-acoustic Spectroscopy of Molecular Cases* [in Russian] (Nauka, Novosibirsk, 1987).
7. A.D. Novikov, V.L. Derbov, and L.A. Mel'nikov, in: *Some Problems in the Applied Physics* [in Russian] (Saratov State University Press, Saratov, 1985), Pt. 2, pp. 15–20.
8. V.L. Derbov, L.A. Mel'nikov, and A.D. Novikov, ''*Theory of narrow resonances of saturated absorption of gaussian beams taking into account induced nonuniformity of the medium*", Saratov (1986); VINITI, No. 5315-V86, June 6, 1986.