AN ALGORITHM FOR DETERMINATION OF THE REFRACTIVE INDEX AND ORIENTATION OF THE ICE PLATES FROM THE POLARIZATION LASER SENSING DATA

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Within the framework of the optical model of a cloud in the form of an ensemble of oriented plates and on the basis of the two-angle scheme of sounding, a system of two nonlinear equations is obtained for determining the refractive index and one of the angles of orientation of the plates. The procedure of solving this system is reduced to a simple iterative algorithm. The algorithm is illustrated by a two-dimensional nomogram, whose output parameters are the refractive index and the angle orientation of the plates. Another angle of plate orientation is determined from the properties of the depolarization ratio.

The angles of orientation and the refractive index of the ice plates are classified among those parameters of crystalline clouds which are subject to the minimum variations compared to the rest parameters. In addition, within the narrow ranges of variations in these parameters it is always possible to indicate one *a priori* known value for each of them. In fact, for low horizontal wind velocity, the ice plates are oriented in the horizontal plane due to their movement in the field of gravitational forces. As to the refractive index, its value measured for a pure ice at a given wavelength in laboratory conditions may be taken as the *a priori* quantity. However, the values of the refractive index of different atmospheric crystals even at a fixed wavelength are spread in the intervals whose end points differ from each other by tens of per cent. This is apparently caused by the presence of various microimpurities in natural crystals. In analogy with the case of the refractive index, the ice plate orientation cannot be considered to be *a priori* known. In fact, the horizontal component of wind velocity shifts the direction of air flow around every particle of the polydisperse medium from the vertical direction. This, in its turn, alters the plate orientation.

We have studied in Ref. 1 the backscattering properties of a system of oriented plates with different diameters and thicknesses. It is well known that clouds consisting mainly of oriented ice plates are most often encountered in the atmosphere.[2,3] Therefore, the model of a polydisperse medium proposed in Ref. 1 describes adequately quite the real atmospheric formations. Based on this model it was shown that the ratios of the parameters of the Stokes vector, in the first approximation, depend solely on the refractive index and plate orientation. This permits one to monitor the parameters of the polydisperse medium with the help of a polarization lidar without using additional information. In this paper we propose an algorithm for determining the refractive index and angles of orientation of the ice plates from the polarization laser sounding data.

Assume that in addition to the measurements of lidar return intensity I_{π_1} the lidar configuration allows one to determine all three other parameters of the Stokes vector, i.e., I_{π_2}, I_{π_3}, and I_{π_4}. Let us introduce into consideration the following ratios: I_{π_2}/I_{π_1}, I_{π_3}/I_{π_1}, and I_{π_4}/I_{π_1}. As shown in Ref. 1, if the scattering volume contains oriented ice plates, the following formula is valid for these ratios:

$$\frac{I_{\pi_j}}{I_{\pi_1}} = \frac{A_j}{A_1}, \quad j = 2, 3, 4 , \tag{1}$$

with an error of not more than 2%. Here A_j are certain coefficients dependent on the angles of orientation and refractive index of plates as well as on the polarisation state of incident radiation. Presented in Ref. 1 are the relations for the coefficients A_j for an arbitrary polarization state of an incident field. In our paper we consider only the cases of linear and circular polarizations which are most often used in practice. It should be noted that in the case of linearly polarized incident light the ratio A_2/A_1 is more informative while for circular polarization – A_4/A_1. The ratios A_2/A_1 and A_4/A_1 determined for linear and circular polarizations, respectively, we denote by P_l and P_c. The coefficients P_l and P_c were derived in Ref. 1 in the form

$$P_l = \frac{\left(|R_{||}|^2\cos^2\gamma - |R_\perp|^2\sin^2\gamma\right)\cos 2\gamma - \mathrm{Re}\left(R_{||}R_\perp^*\right)\sin^2 2\gamma}{|R_{||}|^2\cos^2\gamma + |R_\perp|^2\sin^2\gamma}, \tag{2}$$

$$P_c = -\frac{2\mathrm{Re}\left(R_{||}R_\perp^*\right)}{|R_{||}|^2 + |R_\perp|^2}, \tag{3}$$

where $R_{||}$ and $R_\perp$ are Fresnel's coefficients for parallel and perpendicular polarized waves and γ is the angle between the electric vector of the electromagnetic wave and the plane of incidence. For ice crystals the refractive index n and the absorption coefficient κ are related by the inequality $n - 1 \gg \kappa$. This implies that in the expression for Fresnel's coefficients $R_{||}$ and $R_\perp$ the imaginary part of the complex refractive index $\tilde{n} = n + i\cdot\kappa$ can be neglected. In this case Fresnel's coefficients become real that allow us to simplify Eqs. (2) and (3). As a result, we have

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$$P_l = \frac{(R_{||}{}^2\cos^2\gamma - R_\perp{}^2\sin^2\gamma)\cos2\gamma - R_{||}R_\perp^*\sin^2 2\gamma}{R_{||}{}^2\cos^2\gamma + R_\perp{}^2\sin^2\gamma}, \quad (4)$$

$$P_c = -\frac{2R_{||}R_\perp}{R_{||}{}^2 + R_\perp{}^2}, \quad (5)$$

where $R_{||}$ and $R_\perp$ are real Fresnel's coefficients determined from the following formulas:

$$R_{||} = \frac{n^2\cos\beta - s}{n^2\cos\beta + s},\ R_\perp = \frac{\cos\beta - s}{\cos\beta + s},\ s = \sqrt{n^2 - \sin^2\beta}\,. \quad (6)$$

Here β is the acute angle between the direction of propagation of incident wave and the normal to the base of any ice plate.

Let P_l and P_c be the measurable quantities. It can be easy to seen that each of these quantities is related, at least, to three (see Eqs. (2) and (3)) or two (see Eqs. (4) and (5)) unknowns, i.e., the problem of determining the parameters n, κ, and β is ambiguous. In this connection, one possible way of eliminating this ambiguity, which is used in solving this kind of problems, should be mentioned. Equations (2) and (4) have a free parameter, namely, the angle γ, which can be varied by rotating the lidar about its axis. As a result, one can experimentally measure P_l and P_c as functions of γ and then using the least–squares technique to fit the values of n, κ, and β using Eq. (2) or n and β using Eq. (4). However, in this case such an approach makes it impossible to adequately resolve the ambiguity of the problem. Actually, the function P_l determined from Eq. (4) is uniform in Fresnel's coefficients $R_{||}$ and $R_\perp$. Hence, only the ratio of Fresnel's coefficients can be determined but it incorporates two unknown parameters n and β. Even though the introduction of small attenuation κ formally removes the ambiguity, it calls for the solution of an ill–posed problem of determining the parameters n, κ, and β from Eq. (2). Therefore, we futher deal with different approach to remove the ambiguity in the determination of angles of orientation and refractive index of the ice plates.

Let Eqs. (4) and (5) be reduced to a form

$$p_l = \frac{(p^2\cos^2\gamma - \sin^2\gamma)\cos2\gamma - p\,\sin^2 2\gamma}{p^2\cos^2\gamma + \sin^2\gamma}; \quad (7)$$

$$p_c = -\frac{2p}{p^2 + 1}, \quad (8)$$

where p is the ratio of Fresnel's coefficients, i.e., $p = R_{||}/R_\perp$. It is obvious that one can easily express the parameter p in terms of the measured value P_c using formula (8) or to adjust its value to fit the experimental curve $P_c(\gamma)$ with the help of Eq. (7). Thus, the parameter p could be regarded as known and we may proceed to calculating the unknown parameters n and β on which it depends. However, it should be noted here that the angle β alone does not specify the plate orientation. In fact, the angle β determines only the possible directions of normals to the base of one or other plate which form a cone around the direction of the incident wave propagation. But from the set of normals corresponding to the angle β, we may always choose the single one if the plane of wave incidence is prescribed. It should be noted in this connection that if the electric vector of linearly polarized wave lies in the plane of incidence (γ = 0°) or is perpendicular to it (γ = 90°), the measured quantity P_l reaches a maximum equal to unity. Moreover, it is known[1] that the curve $P_l(\gamma)$ on a segment [0°, 90°] is asymmetric, i.e., its minimum is shifted with respect to the center toward the left end γ = 0°. Thus, the analysis of the change of the parameter P_l due to rotating the lidar around its axis enables one to determine the plane of wave incidence.

Let the parameters P_l (j = 1, 2) be determined for two sounding directions with the angle Δ between them. Let also the vectors of these directions lie in the same plane of incidence. Then the unknown parameters n and β can be found from the system of equations

$$\begin{aligned} p_1 &= R_{||}(\beta, n)/R_\perp(\beta, n)\,; \\ p_2 &= R_{||}(\beta + \Delta, n)/R_\perp(\beta + \Delta, n)\,. \end{aligned} \quad (9)$$

The system of equations (9) can be most conveniently solved by using the iterative algorithm described below. Let n_0 be the initial approximation of the refractive index n. Then the angles β_{01} and β_{02} corresponding to this refractive index n_0 are calculated. These angles determine the deflaction of the normal to the plate base from the two sounding directions. To do this, we shall make use of the following chain of formulas:

$$a_j = 0.5\left(1 - \frac{1}{p_j}\right)\frac{n_0^2 + 1}{n_0^2 - 1};\ \ b_j = 1/p_j\left(\sqrt{a_j^2 + 1/p_j} + a_j\right);$$

$$c_j = \sqrt{(n_0^2 - 1)/[((1 - b_j)/(1 + b_j))^2 - 1]}\,; \quad (10)$$

$$\beta_{0j} = \arccos(c_j)\,.$$

(The index j takes here the values 1 and 2). After β_{01} and β_{02} have been calculated, the condition $|\beta_{01} - \beta_{02}| = \Delta$ is verified. If the condition holds, the iteration stops and β_{01} and n_0 are considered to be determined. If this condition does not hold we must change the value of the refractive index n. In the next step of the algorithm n should be increased if $|\beta_{01} - \beta_{02}| < \Delta$ and decreased if $|\beta_{01} - \beta_{02}| > \Delta$. The performance of the algorithm can also be illustrated by a nomogram depicted in Fig. 1.

Let two values of the parameter p be known, namely, $p_1 = -0.612$ and $p_2 = -0.462$ and the latter ($p = p_2$) be obtained for the sounding direction being at the angle Δ = 6° with respect to the initial one. The parameters p_1 and p_2 determine two ratios of Fresnel's coefficients, i.e., $R_{||} = p_1R_\perp$ and $R_{||} = p_2R_\perp$. These relations describe two straight lines which cross the coordinate origin. Variations in the refractive index in the iterative process are equivalent to movement along these straight lines. In each step of the algorithm we can always determine two points where these straight lines ($R_{||} = p_1R_\perp$ and $R_{||} = p_2R_\perp$) cross the line of constant $n = n_j$ corresponding to jth iteration of the refractive index. One line of constant β (β_{j1} and β_{j2}) crosses each of these intersection points. If the angle between these lines is 6° then the algorithm stops. The sought–after points on the nomogram with the angle $\Delta = |\beta_{j1} - \beta_{j2}| = 6°$ between them are denoted by A and B. The two lines of the constant parameters n and β cross each of these points: the lines n = 1.30 and β = 30° cross the point A, and the lines n = 1.30 and β = 36° cross the point B. It is evident that in this case n = 1.30 and β = 30° are the sought–after quantities. The similarly lines for parameters n and β in the nomogram do not intercross what ensures that the refractive index n and the angle of orientation β are determined unambiguously.

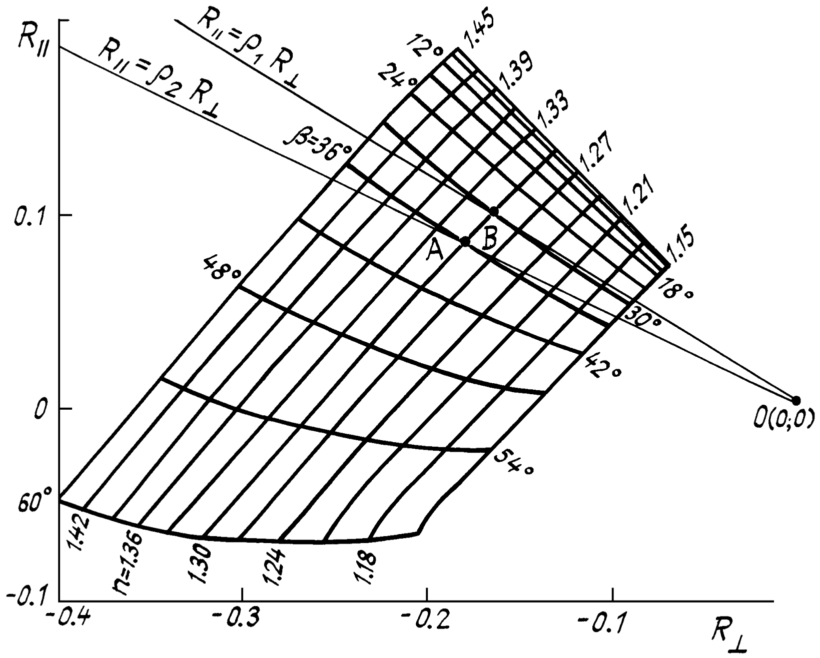

FIG. 1. Nomogram for determining the refractive index n and the angle of orientation β *of the ice plates: the dimensionless parameters* $p_1 = -0.612$ *and* $p_2 = -0.462$ *are uniquely related to the angles* $\beta_0 = 30°$ *and* $\beta_2 = 36°$ *and the refractive index* $n = 1.30$ *for* $\Delta = |\beta_1 - \beta_2| = 6°$.

The proposed algorithm is valid if the condition $n - 1 \gg \kappa$ is satisfied and permits one to determine solely the unknown parameters n and β, i.e., this condition *a priori* excludes the parameter κ from the algorithm. Moreover, any attempt to develop the algorithm which allows one to determine all three parameters n, κ, and β will fail, except for the case, in which $n - 1 \approx \kappa$. As was mentioned above, in this case the parameters n, κ, and β are fitted to the experimental curve $P_l(\gamma)$ by means of Eq. (2).

In developing the algorithm we assumed that all ice plates are in fixed positions, while in fact the plates have flutter, i.e., they oscillate near a certain plane which can be conditionally taken as an orientation plane. The amplitude of these oscillations is, as a rule, insignificant. In particular, it was shown in Ref. 4 that the angle of flutter for the plates is somewhat larger than 0.5°. Moreover, the curves of constants n and β on the nomogram are monotonic and on small intervals they are practically linear. As a result, by averaging the characteristics P_l and P_c over a narrow interval of angles $[\beta - \Delta\beta, \beta + \Delta\beta]$ the obtained characteristics $\overline{P_l}$ and $\overline{P_c}$ can be expected to be very close to these for the plates located in the plane of orientation.

In conclusion it can be summarized that in this paper we have constructed the algorithm which permits one to determine the orientation and refractive index of the ice plates from the data of single–frequency polarization laser sounding without using *a priori* information.

REFERENCES

1. A.A. Popov and O.V. Shefer, Opt. Atm. **3**, No. 1, 36–41 (1990).
2. A. Ono, J. Atm. Sci. **27**, No. 4, 649–658 (1970).
3. C.M.R. Platt, J. Scott, and A.C. Dilley, J. Atm. Sci. **44**, No. 4, 729–747 (1987).
4. C.M.R. Platt, J. Appl. Meteor. **17**, 1220–1224 (1978).