FLUCTUATIONS OF FREQUENCY–DIVERSITY WAVES IN A LENS–LIKE MEDIUM

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Received February 7, 1991

The variance and the frequency correlation function of intensity fluctuations of optical waves are theoretically studied in the case of wave propagation through an aberration-free defocusing lens-like medium (a refraction channel) with either discrete or continuous random inhomogeneities of the dielectric permittivity. The statistical characteristics of the Gaussian beam intensity fluctuations are calculated employing the Bom approximation for the solution of the equation describing the fourth-order coherence function of the frequency-diversity monochromatic waves. It is demonstrated that the intensity fluctuations of optical wave in such a lens-like medium with either discrete or continuous random inhomogeneities, are weaker than in a regularly .homogeneous medium. The intensity fluctuations become the weaker, the larger is the initial beam divergence. The intensity fluctuation frequency correlation for optical wave propagating through the tens-like medium with continuous random inhomogeneities coincides with that for the regularly homogeneous medium. The existence of the regularly refractive inhomogeneity in the discrete scattering medium results in a larger scale of the intensity fluctuation frequency correlation.

At present the techniques developed for the optical refraction spectroscopy of super-high resolution[1–7] are based on recording the intensity of a sensing laser beam wtiich propagates through the region exposed to the high-power optical radiation. Systematic errors of such techniques, resulting from the aberrations produced by the lens-like medium, have been considered elsewhere.[8–9] The present paper is devoted to the study of the random errors of the techniques of optical refraction spectroscopy, namely, the variance and the frequency correlation function for the intensity fluctuations of the optical sensing beam propagating through the aberration-free defocusing lens-Like medium (the refraction channel) with either discrete or continuous random inhomogeneities of the dielectric permittivity.

Using the parabolic equation

$$\left\{2ik\frac{\partial}{\partial x}+\Delta_\perp+k^2\left[\frac{q^2}{F^2(x;k)}+\varepsilon(x,\mathbf{q};k)\right]\right\}E(x,\mathbf{q};\kappa)=0,\quad(1)$$

$$E(0,\mathbf{q};\kappa)=E_0(\mathbf{q};\kappa)$$

for the lens-like medium fthe refraction channel), when its optical axis coincides with the OX axis,[4] we may obtain the equation for the fourth–order coherence function of frequency–diversity monochromatic waves, if the dielectric permittivity fluctuations in the medium $\varepsilon(x,\mathbf{q};\kappa)$ are assumed to be Markovtan[4,5,10,11]:

$$\Gamma_4(x,\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3,\mathbf{q}_4;k_1,k_2)=$$

$$=\left\langle E(x,\mathbf{q}_1;k_1)\,E^*(x,\mathbf{q}_2;k_1)\,E(x,\mathbf{q}_3;k_2)\,E^*(x,\mathbf{q}_4;k_2)\right\rangle$$

$$\left\{\frac{\partial}{\partial x}-i\left[\frac{1}{k_1}\frac{\partial^2}{\partial\mathbf{R}_1\partial\boldsymbol{\eta}_1}+\frac{1}{k_2}\frac{\partial^2}{\partial\mathbf{R}_2\partial\boldsymbol{\eta}_2}\right]-i\left[k_1\frac{\mathbf{R}_1\boldsymbol{\eta}_1}{F^2(x;k_1)}+\right.\right.$$

$$\left.\left.+k_2\frac{\mathbf{R}_2\boldsymbol{\eta}_2}{F^2(x;k_2)}\right]+\frac{1}{4}H(\mathbf{R}_1,\mathbf{R}_2,\boldsymbol{\eta}_1,\boldsymbol{\eta}_2;k_1,k_2)\right\}\times$$

$$\times\Gamma_4(x,\mathbf{R}_1,\boldsymbol{\eta}_1,\mathbf{R}_2,\boldsymbol{\eta}_2;k_1,k_2)=0,\qquad(2)$$

$$\Gamma_4(0,\mathbf{R}_1,\boldsymbol{\eta}_1,\mathbf{R}_2,\boldsymbol{\eta}_2;k_1,k_2)=E_0(\mathbf{R}_1+\boldsymbol{\eta}_1/2;k_1)\times$$

$$\times E_0^*(\mathbf{R}_1-\boldsymbol{\eta}_1/2;k_1)\,E_0(\mathbf{R}_2+\boldsymbol{\eta}_2/2;k_2)\,E_0^*(\mathbf{R}_2-\boldsymbol{\eta}_2/2;k_2),$$

where $E(x,\mathbf{q};\kappa)$ is the parabolic amplitude of the optical field at the point $(x,\mathbf{q})$ at the wavelength $\lambda(k=2\pi/\lambda$; for definiteness, we assume $\lambda_2\ge\lambda_1)$. $\Delta_\perp=\partial^2/\partial y^2+\partial^2/\partial z^2$ is the transverse Laplacian operator; $F(x,k)$ is the profile of the local focal distance of the lens–like medium for the optical wave with the wavelength λ; $\varepsilon(x,\mathbf{q};\kappa)$ are the dielectric permittivity fluctuations in the medium; $R_1=(q_1+q_2)/2$;

$$\mathbf{R}_2=(\mathbf{q}_3+\mathbf{q}_4)/2;\quad\boldsymbol{\eta}_1=\mathbf{q}_1-\mathbf{q}_2;\quad\boldsymbol{\eta}_2=\mathbf{q}_3-\mathbf{q}_4;$$

$$H(\mathbf{R}_1,\mathbf{R}_2,\boldsymbol{\eta}_1,\boldsymbol{\eta}_2;k_1,k_2)=k_1^2[A(0;k_1,k_1)-A(\boldsymbol{\eta}_1;k_1,k_1)]+$$

$$+k_2^2[A(0;k_2,k_2)-A(\boldsymbol{\eta}_2;k_2,k_2)]+k_1k_2G(\mathbf{R}_1,\mathbf{R}_2,\boldsymbol{\eta}_1,\boldsymbol{\eta}_2;k_1,k_2);$$

$$G(\mathbf{R}_1,\mathbf{R}_2,\boldsymbol{\eta}_1,\boldsymbol{\eta}_2;k_1,k_2)=A(\mathbf{R}_1-\mathbf{R}_2+(\boldsymbol{\eta}_1-\boldsymbol{\eta}_2)/2;k_1,k_2)+$$

$$+A(\mathbf{R}_1-\mathbf{R}_2-(\boldsymbol{\eta}_1-\boldsymbol{\eta}_2)/2;k_1,k_2)-\hat{A}(\mathbf{R}_1-\mathbf{R}_2+$$

$$+(\boldsymbol{\eta}_1+\boldsymbol{\eta}_2)/2;k_1,k_2)-A(\mathbf{R}_1-\mathbf{R}_2-(\boldsymbol{\eta}_1+\boldsymbol{\eta}_2)/2;k_1,k_2);$$

0235-6880/91/06 446-04 $02.00

$$A(\mathbf{q}; k_l, k_{l'}) = 2\pi \int\int_{-\infty}^{\infty} \mathrm{d}\kappa\, \Phi_\varepsilon(\kappa; k_l, k_{l'}) \exp\{i\kappa\mathbf{q}\} ;$$

$$\hat{A}(\mathbf{q}; k_l, k_{l'}) = 2\pi \int\int_{-\infty}^{\infty} \mathrm{d}\kappa\, \hat{\Phi}_\varepsilon(\kappa; k_l, k_{l'}) \exp\{i\kappa\mathbf{q}\} ;$$

$\Phi_\varepsilon(\kappa, \kappa_l, \kappa_l')$ and $\hat{\Phi}_\varepsilon(\kappa, \kappa_l, \kappa_l')$ arc the fluctuation spectra of the momenta $<\varepsilon(x, \mathbf{q}_1; \kappa_l)\, \varepsilon^*(x, \mathbf{q}_2; \kappa_l')>$ of the dielectric permittivity in the medium and $<\varepsilon(x, \mathbf{q}_1; \kappa_l)\, \varepsilon(x, \mathbf{q}_2; \kappa_l')> (l, l' = 1,2)$.

We restrict ourselves to the case in which the focal distance in such a lens-like medium is independent of the wavelength, i.e., $F(x,\kappa_1) = F(x,\kappa_2) = F(x)$. Such a restriction is quite acceptable because the estimates show that $F(x,\kappa)$ varies only slightly in gaseous media with the wave number κ being outside the absorption lines.[4] According for this and assuming thai the "centered" fourth–order moment of the field

$$W(x, \mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \mathbf{q}_4; k_1, k_2) = \Gamma_4(x, \mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \mathbf{q}_4; k_1, k_2) - \Gamma_2(x, \mathbf{q}_1, \mathbf{q}_2; k_1)\, \Gamma_2(x, \mathbf{q}_3, \mathbf{q}_4; k_2)$$

is small compared to $\Gamma_4^{(0)}(x, \mathrm{q}_1, \mathrm{q}_2, \mathrm{q}_3, \mathrm{q}_4; \kappa_1, \kappa_2)$, i.e., to the fourth–order coherence function of the frequency–diversity monochromatic waves propagating through a lens–like medium without random inhomogeneities, we obtain from Eq. (2) the following equation for $W(x, \mathrm{q}_1, \mathrm{q}_2, \mathrm{q}_3, \mathrm{q}_4; \kappa_1, \kappa_2)$:

$$\left\{\frac{\partial}{\partial x} - i\left[\frac{1}{k_1}\frac{\partial^2}{\partial \mathbf{R}_1 \partial \boldsymbol{\eta}_1} + \frac{1}{k_2}\frac{\partial^2}{\partial \mathbf{R}_2 \partial \boldsymbol{\eta}_2}\right] - \frac{i}{F^2(x)}[k_1\mathbf{R}_1\boldsymbol{\eta}_1 + k_2\mathbf{R}_2\boldsymbol{\eta}_2]\right\} \times$$
$$\times W(x, \mathbf{R}_1, \boldsymbol{\eta}_1, \mathbf{R}_2, \boldsymbol{\eta}_2; k_1, k_2) = -\frac{k_1 k_2}{4} \times$$
$$\times G(\mathbf{R}_1, \mathbf{R}_2, \boldsymbol{\eta}_1, \boldsymbol{\eta}_2; k_1, k_2) \times \Gamma_4^{(0)}(x, \mathbf{R}_1, \boldsymbol{\eta}_1, \mathbf{R}_2, \boldsymbol{\eta}_2; k_1, k_2), \quad (3)$$
$$W(0, \mathbf{R}_1, \boldsymbol{\eta}_1, \mathbf{R}_2, \boldsymbol{\eta}_2; k_1, k_2) = 0 ,$$

where $\Gamma_2(x, \mathrm{q}_1\, \mathrm{q}_2; \kappa_1)$ and $\Gamma_2(x, \mathrm{q}_3, \mathrm{q}_4; \kappa_2)$ are the second–order coherence functions for the monochromatic waves propagating in the lens–like medium.[4–5] Equation (3) can be solved by the method of characteristics after the Fourier transform over the coordinates R_1 and R_2. This solution has the following form:

$$W(x, \mathbf{R}_1, \boldsymbol{\eta}_1, \mathbf{R}_2, \boldsymbol{\eta}_2; k_1, k_2) = -\frac{k_1 k_2}{16\pi^3}\int_0^x \mathrm{d}\zeta \int\int_{-\infty}^{\infty} \mathrm{d}\kappa_1 \times$$
$$\times \int\int\int\int_{-\infty}^{\infty} \mathrm{d}\kappa\, \mathrm{d}\mathbf{q} \exp\left(i\kappa\mathbf{R}_1 + i\mathbf{q}\mathbf{R}_2\right) \int\int\int\int_{-\infty}^{\infty} \mathrm{d}\mathbf{R}_1'\, \mathrm{d}\mathbf{R}_2' \times$$
$$\times \Gamma_4^{(0)}\left(\zeta, \mathbf{R}_1', \boldsymbol{\eta}_1(x, \zeta), \mathbf{R}_2', \boldsymbol{\eta}_2(x, \zeta); k_1, k_1\right) \times$$
$$\times \exp\left\{i\kappa_1(\mathbf{R}_1' - \mathbf{R}_2') + i\left[\kappa(x, \zeta)\, \mathbf{R}_1' + \mathbf{q}(x, \zeta)\, \mathbf{R}_2'\right]\right\} \times$$
$$\times \left\{\cos\left[\frac{\kappa_1}{2}\left(\boldsymbol{\eta}_1(x, \zeta) - \boldsymbol{\eta}_2(x, \zeta)\right)\right] \Phi_\varepsilon(\kappa_1; k_1, k_2) - \right.$$
$$\left. - \cos\left[\frac{\kappa_1}{2}\left(\boldsymbol{\eta}_1(x, \zeta) + \boldsymbol{\eta}_2(x, \zeta)\right)\right] \hat{\Phi}_\varepsilon(\kappa_1; k_1, k_2)\right\}, \quad (4)$$

where

$$\boldsymbol{\eta}_1(x, \zeta) = \tilde{A}(x, \zeta)\, \boldsymbol{\eta}_1 - \frac{F_0}{k_1} A(x, \zeta)\, \kappa ;$$

$$\boldsymbol{\eta}_2(x, \zeta) = \tilde{A}(x, \zeta)\, \boldsymbol{\eta}_2 - \frac{F_0}{k_2} A(x, \zeta)\, \mathbf{q} ;$$

$$\kappa(x, \zeta) = B(x, \zeta)\, \kappa - \frac{k_1}{F_0} \tilde{B}(x, \zeta)\, \boldsymbol{\eta}_1 ;$$

$$\mathbf{q}(x, \zeta) = B(x, \zeta)\, \mathbf{q} - \frac{k_2}{F_0} \tilde{B}(x, \zeta)\, \boldsymbol{\eta}_2 ;$$

$$A(x, \zeta) = U_1\left(\frac{\zeta}{F_0}\right) U_2\left(\frac{x}{F_0}\right) - U_2\left(\frac{\zeta}{F_0}\right) U_1\left(\frac{x}{F_0}\right) ;$$

$$B(x, \zeta) = U_1'\left(\frac{\zeta}{F_0}\right) U_2\left(\frac{x}{F_0}\right) - U_2'\left(\frac{\zeta}{F_0}\right) U_1\left(\frac{x}{F_0}\right) ;$$

$$\tilde{A}(x, \zeta) = U_1\left(\frac{\zeta}{F_0}\right) U_2'\left(\frac{x}{F_0}\right) - U_2\left(\frac{\zeta}{F_0}\right) U_1'\left(\frac{x}{F_0}\right) ;$$

$$\tilde{B}(x, \zeta) = U_1'\left(\frac{\zeta}{F_0}\right) U_2'\left(\frac{x}{F_0}\right) - U_2'\left(\frac{\zeta}{F_0}\right) U_1'\left(\frac{x}{F_0}\right) .$$

The functions $U_1\left(\frac{x}{F_0}\right)$ and $U_2\left(\frac{x}{F_0}\right)$ are here the partial solutions of the equation

$$U''(x) - \frac{F_0^2}{F^2(x)} U(x) = 0$$

with the boundary conditions

$$U_1(0) = U_2'(0) = 1, \quad U_2(0) = U_1'(0) = 0,^{4,5} \quad \text{and} \quad F_0 = F(x = 0)$$

is the "initial" value of the focal distance in the lens–like medium.

Let us consider two identical Gaussian beams with different carrier frequencies ($\lambda_1 \neq \lambda_2$) which propagate along the optical axis of the lens–like medium, i.e.,

$$E_0(\mathbf{q}; k_l) = E_0 \exp\left\{-\frac{\mathbf{q}^2}{2a_0^2} - \frac{ik_l}{2R_0}\mathbf{q}^2\right\}, (l = 1, 2) ,$$

where E_0 is ihe field amplitude at ihe center of the output - aperture, a_0 is ihe initial radius of the oplica[beam, $\mathbf{R}_0$ is the wavefront curvature radius of the optical beam at the output aperture. In this case it may be assumed that

$$\Gamma_4^{(0)}(x, \mathbf{R}_1, \boldsymbol{\eta}_1, \mathbf{R}_2, \boldsymbol{\eta}_2; k_1, k_2) =$$
$$= \Gamma_2^{(0)}(x, \mathbf{R}_1, \boldsymbol{\eta}_1; k_1)\, \Gamma_2^{(0)}(x, \mathbf{R}_2, \boldsymbol{\eta}_2; k_2) , \quad (5)$$

where

$$\Gamma_2^{(0)}(x, \mathbf{R}, \boldsymbol{\eta}; k) = \frac{E_0^2 a_0^2}{a^2(x; k)} \times \exp\left\{-\frac{R^2 + \eta^2/4}{a^2(x; k)} + iS(x; k)\, \mathbf{R}\boldsymbol{\eta}\right\}$$

is the second–order coherence function for the monochromatic wave propagating through the lens–like medium without random inhomogeneities[4–5]:

$$a(x;k) = a_0\left\{\left[U_1\left(\frac{x}{F_0}\right) - \frac{F_0}{R_0}U_2\left(\frac{x}{F_0}\right)\right]^2 + \frac{F_0^2}{k^2a_0^4}U_2^2\left(\frac{x}{F_0}\right)\right\}^{1/2}$$

is the running radius of the optical beam and $S(x;k) = \frac{1}{a(x;k)}\frac{da(x;k)}{dx}$ is ihe running wavefront curvature of the optical beam. Substituting Eq. (5) into Eq. (4) for $\mathbf{R}_1 = \mathbf{R}_2 = \eta_1 = \eta_2 = 0$ we obtain the intensity fluctuation frequency correlation function for the two Gaussian beams at the optical axis of the lens–like medium. If $I(x, \mathbf{q}; \kappa) = E(x, \mathbf{q}; \kappa)\, E^*(x, \mathbf{q}; \kappa)$ is the optical beam intensity then the intensity fluctuation correlation function for Ihe frequency–diversity waves has the form

$$B_I(k_1, k_2) = W(x, 0, 0, 0, 0; k_1, k_2) =$$

$$= \langle I(x, 0; k_1)\, I(x, 0; k_2)\rangle - \langle I(x, 0; k_1)\rangle \langle I(x, 0; k_2)\rangle =$$

$$= 2\pi^2E_0^4k_1k_2a_0^4\int_0^x d\zeta\frac{a^{-2}(\zeta;k_1)\,a^{-2}(\zeta;k_2)}{M(x,\zeta;k_1)\,M(x,\zeta;k_2)}\int_0^\infty d\kappa\,\kappa\,\times$$

$$\times\exp\left\{-\frac{F_0^2A^2(x,\zeta)}{2}\left[\frac{1}{k_1^2a^2(\zeta;k_1)M(x,\zeta;k_1)} + \frac{1}{k_2^2a^2(\zeta;k_2)M(x,\zeta;k_2)}\right]\kappa^2\right\}\left\{\Phi_\varepsilon(\kappa;k_1,k_2)\,\times\right.$$

$$\times\cos\left[\frac{F_0A(x,\zeta)}{2}\left(\frac{N(x,\zeta;k_1)}{k_1M(x,\zeta;k_1)} - \frac{N(x,\zeta;k_2)}{k_2M(x,\zeta;k_2)}\right)\kappa^2\right] -$$

$$\left.-\hat{\Phi}_\varepsilon(\kappa;k_1,k_2)\cos\left[\frac{F_0A(x,\zeta)}{2}\left(\frac{N(x,\zeta;k_1)}{k_1M(x,\zeta;k_1)} + \frac{N(x,\zeta;k_2)}{k_2M(x,\zeta;k_2)}\right)\kappa^2\right]\right\},$$

where

$$M(x,\zeta;k_l) = N^2(x,\zeta;k_l) + \frac{F_0^2A^2(x,\zeta)}{k_l^2a^4(\zeta;k_l)};$$

$$N(x,\zeta;k_l) = F_0A(x,\zeta)\,S(\zeta;k_l) - B(x,\zeta),\ (l = 1, 2).$$

For $\kappa_1 = \kappa_2$ Eq. (6) describes the variance of the intensity fluctuations of the optical beam at the optical axis of the lens–like medium

$$\sigma_I^2(x;k) = W(x,0,0,0,0;k,k) =$$

$$= \langle I^2(x,0;k)\rangle - \langle I(x,0;k)\rangle^2.$$

We shall calculate the variance and the frequency correlation function of the intensity for two cases:

1) continuous medium with the Kolmogorov spectrum of the inhomogeneities of the dielectric permittivity[12]

$$\Phi_\varepsilon(\kappa;k_l,k_r) = \hat{\Phi}_\varepsilon(\kappa;k_l,k_r) = 0.033\,C_\varepsilon^2\,\kappa^{-11/3}\exp\left[-\frac{\kappa^2}{\kappa_m^2}\right], \qquad (7)$$

where is the structural parameter of the dielectric permittivity fluctuations in the medium, $\kappa_m = 5.92/l_0$, and l_0 is the inner scale of the turbulence and

2) discrete monodispersed scattering medium[13,14]

$$\Phi_\varepsilon(\kappa;k_l,k_r) = -\hat{\Phi}_\varepsilon(\kappa;k_l,k_r) = \frac{m_0a^4}{2\pi\,k_lk_r}\exp\left\{-\frac{1}{4}a^2\kappa^2\right\}, \qquad (8)$$

where m_0 is the density and a is the radius of the scatterer.

To start with, we consider the features peculiar to the behavior of the variance of Gaussian optical beam intensity fluctuations propagating along the optical axis of the lens-tike medium with continuous inhomogeneities of the dieteclrk permittivity (Eq. (7)) in comparing with the homogeneous (on the average) medium. Substituting Eq. (7) into Eq. (6) and evaluating the integrals, we obtain the relation for the absolute variance of the Gaussian beam intensity fluctuations

$$\sigma_I^2(x,k) = \frac{36}{5}\Gamma\left(\frac{7}{6}\right)0.033\pi^2E_0^4C_\varepsilon^2k^2\kappa_m^{-5/3}a_0^4\,\times$$

$$\times\int_0^x d\zeta\frac{a^{-4}(\zeta;k)\,C^{5/6}(\zeta;k)}{M^2(x,\zeta;k)}\left\{\left[1 + \frac{D^2(x,\zeta)}{C^2(x,\zeta)}\right]^{5/12}\right\}\times$$

$$\times\cos\left[\frac{5}{6}\arctan\frac{D(x,\zeta)}{C(x,\zeta)}\right] - 1, \qquad (9)$$

where

$$C(x,\zeta) = 1 + \frac{F_0^2A^2(x,\zeta)\,\kappa_m^2}{k^2a^2(\zeta;k)\,M(x,\zeta;k)};$$

$$D(x,\zeta) = 1 + \frac{F_0A(x,\zeta)\,N(x,\zeta;k)}{x\,M(x,\zeta;k)}D;$$

$D = x\kappa_m^2/\kappa$. In the case of a wide ($\kappa a_0^2/x \gg 1$) collimaled ($R_0 = \infty$) beam in which $a(x;k) \simeq a_0U_1\left(\frac{x}{F_0}\right)$ and $S(x;k) \simeq \frac{1}{F_0}\frac{U_1'\left(\frac{x}{F_0}\right)}{U_1\left(\frac{x}{F_0}\right)}$ we find for the ratio of the normalized variances of the intensity fluctuations of optical wave propagating through the lens–like medium $\left[\beta^2\left(\frac{x}{F_0}\right) = \sigma_I^2(x;k)/\langle I(x,0;k)\rangle^2\right]$ and through the homogeneous eddy medium $\left[\beta_\infty^2 = \lim_{F_0\to\infty}\beta^2\left(\frac{x}{F_0}\right)\right]$ the following relation:

$$\gamma_p(\xi) = \frac{\beta_p^2(\xi)}{\beta_{p\infty}^2} = \int\limits_0^1 d\eta \left\{ (1 + \tilde{D}^2(\xi, \eta))^{5/12} \times \right.$$

$$\left. \times \cos\left[\frac{5}{6}\arctan(\tilde{D}(\xi, \eta))\right] - 1) \right\} \times$$

$$\times \left\{ \int\limits_0^1 d\eta \left[(1 + \eta^2 D^2)^{5/12} \cos\left[\frac{5}{6}\arctan(\eta D)\right] - 1 \right] \right\}^{-1}, \qquad (10)$$

where

$$\tilde{D}(\xi, \eta) = \frac{U_1(\xi\eta)}{\xi U_1(\xi)} \left[U_1(\xi\eta)\, U_2(\xi) - U_1(\xi) U_2(\xi\eta)\right] D;$$

$\xi = x/F_0$ is the ratio of the path length to the "initial" value of the local focal distance in the lens–like medium.

It follows from Eq. (9) that in the case of "quasispherical" wave $(\kappa a_0^2/x \ll 1)$ in which $a(x; k) \approx \frac{F_0}{k a_0} U_2\left(\frac{x}{F_0}\right)$ and $\frac{1}{F_0} \frac{U_2'\left(\frac{x}{F_0}\right)}{U_2\left(\frac{x}{F_0}\right)} k)$ the similar ratio of the normalized variances of the intensity fluctuations has the following fom:

$$\gamma_s(\xi) = \frac{\beta_s^2(\xi)}{\beta_{s\infty}^2} = \int\limits_0^1 d\eta \left\{ (1 + \hat{D}^2(\xi, \eta))^{5/12} \times \right.$$

$$\left. \times \cos\left[\frac{5}{6}\arctan(\hat{D}(\xi, \eta))\right] - 1 \right\} \times$$

$$\times \left\{ \int\limits_0^1 d\eta \left[(1 + \eta^2(1 - \eta^2) D^2)^{5/12} \times \right. \right.$$

$$\left. \left. \times \cos\left[\frac{5}{6}\arctan\left(\eta(1 - \eta) D\right)\right] - 1 \right] \right\}^{-1}. \qquad (11)$$

where

$$\hat{D}(\xi, \eta) = \frac{U_2(\xi\eta)}{\xi U_2(\xi)} \left[U_1(\xi\eta)\, U_2(\xi) - U_1(\xi) U_2(\xi\eta)\right] D.$$

The analysis of relations (10) and (11) demonstrates that in the region of "geometric optics" ($\tilde{D}(\xi, \eta) \ll 1$ and $(\hat{D}(\xi, \eta) \gg 1)$ we have

$$\gamma_p(\xi) \approx \frac{11}{6} \xi^{-5/6} U_1^{-5/6}(\xi) \times$$

$$\times \int\limits_0^1 d\eta\, U_1^{5/6}(\xi\eta) \left[U_1(\xi\eta)\, U_2(\xi) - U_1(\xi) U_2(\xi\eta)\right]^{5/6}$$

and

$$\gamma_s(\xi) \approx \frac{\Gamma\left(\frac{11}{3}\right)}{\Gamma^2\left(\frac{11}{3}\right)} \xi^{-5/6} U_2^{-5/6}(\xi) \times$$

$$\times \int\limits_0^1 d\eta\, U_2^{5/6}(\xi\eta) \left[U_1(\xi\eta)\, U_2(\xi) - U_1(\xi) U_2(\xi\eta)\right]^{5/6}.$$

Figure 1 shows the results of calculation of the ratios of the normalized variances of intensity fluctuations of the wide collimaled beam ($\gamma_p(\xi)$) given by Eq. (10) and the "quasispherical" wave ($\gamma_s(\xi)$) given by Eq. (11) for different values of D and two models of varying the local focal distance of the lens–like medium with distance: $F(x) = F_0$ (solid lines) and $F(x) = F_0\left(1 + \frac{x^2}{F_0^2}\right)$ (dashed lines).

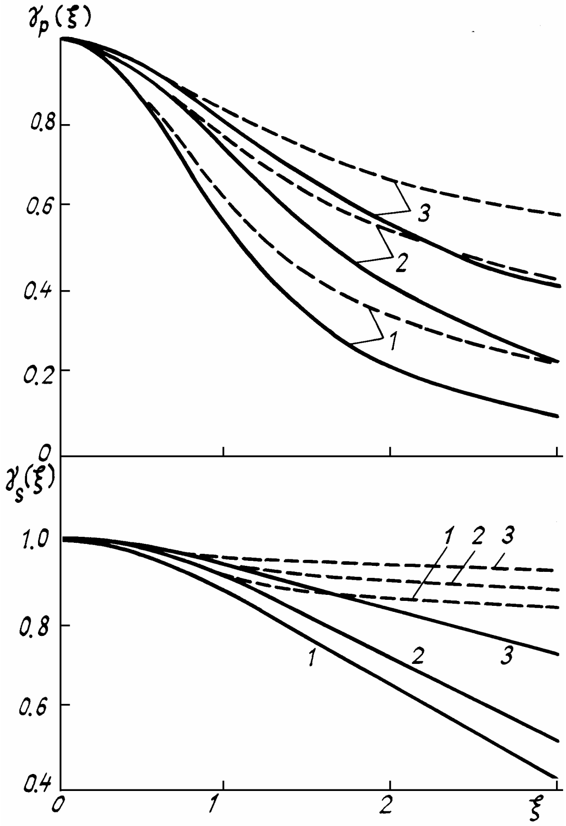

FIG. 1. The ratios of the normalized variances of intensity fluctuations for optical wave propagating through the lens–like medium and through the regularly homogeneous medium continuous random inhomogeneities for a wide collimated beam and the "quasispherical" wave vs ξ:
1) $D \leq 1$, *2)* $D = 10$, *and 3)* $D \gg 1$.

The effect of weakening the intensity fluctuations of the optical wave propagating through the lens–like medium (produced by beam defocusing) is reduced with increase of D. It should be stressed that the results presented here reveal the groundlessness of the conclusion made in Ref. 10 about the possibility of evalualing the effect of defocusing in the lens–like medium (the refraction channel) on the intensity fluctuations with the help of calculated the intensity fluctuations behind a thin lens. Such an approximation is possible only for a lens–like medium in which the local focal distance increases rapidly along the path. e.g., for $F(x) = F_0\left[1 + \left(\frac{x^2}{F_0^2}\right)\right]$. In this case the defocusing effect on the optical radiation is produced only by a snort initial section of the propagation path, and its effect may indeed be approximated by that of a thin lens. If the focal distance of the lens–like medium increases slowly with x, or remains constant, or even decreases, such an approximation of the lens–like medium by a thin lens becomes invalid. The results of calculation of $\gamma_{ps}(\xi)$, shown in Fig. 2 for $D \gg 1$ and various models of the profile of the local focal distance $F(x)$ also support this conclusion. The value of $\gamma_{ps}(\xi)$ was calculated for

1) $F(x) = F_0\left[1 + \left(\frac{x}{F_0}\right)^2\right]$;

2) $F(x) = F_0\left[1 + \frac{2}{3}\left(\frac{x}{F_0}\right)^2\right] \Big/ \sqrt{1 + \frac{1}{3}\left(\frac{x}{F_0}\right)^2}$;

3) $F(x) = F_0$;

4) $F(x) = F_0 \Big/ \left[1 + \left(\frac{x}{F_0}\right)^2\right]$;

5) $F(x) = F_0\left[1 - \left(\frac{x}{F_0}\right)^2\right] \Big/ \sqrt{1 + 2\left(\frac{x}{F_0}\right)^2}$ $(x < F_0)$.

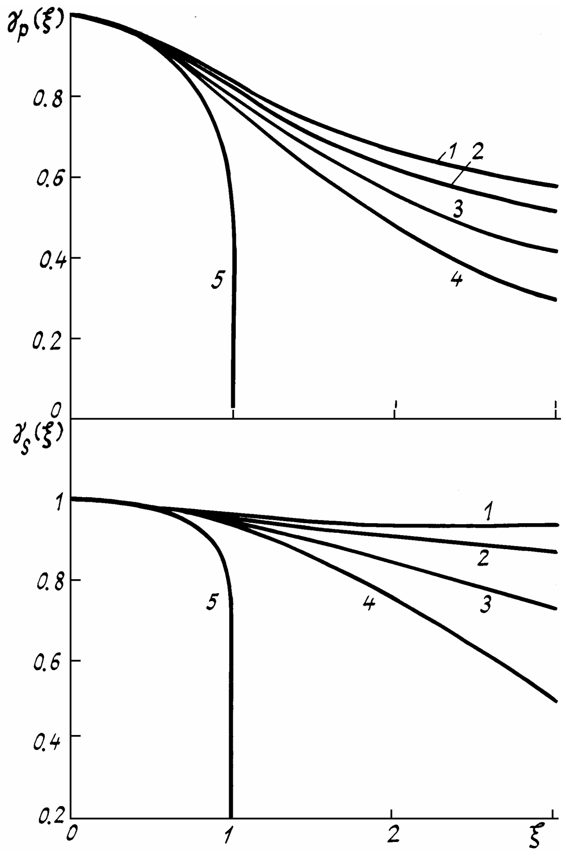

FIG. 2. The ratios of the normalized variances of intensity fluctuations $\gamma_p(\xi)$ and $\gamma_s(\xi)$ for $D \gg 1$ and different models of $F(x)$:

1) $U_1(\xi) = \sqrt{1 - \xi^2}$, *2)* $U_1(\xi) = (1 + \frac{2}{3}\xi^2)^{3/4}$, *3)* $U_1(\xi) = \mathrm{ch}(\xi)$,

4) $U_1(\xi) = \exp(\frac{1}{2}\xi^2)$, and *5)* $U_1(\xi) = 1/\sqrt{1 - \xi^2}$ $(\xi < 1)$.

The functions $U_1(\xi)$ corresponding to them can be found in legends It can be seen from Figs. 1 and 2 that the intensity fluctuations decrease more rapidly in a wide collimated beam than in a "quasispherical" wave.

Analysis of the case of the discrete scattering medium (Eq. (8)) leads lo the following conclusions. The absolute variance of the intensity fluctuations of a wide collimated beam for $d = L/(\kappa a)^2 \gg 1$ decreases by a factor of $1/U_1^4(\xi)$ compared to the case of the regularly hemogeneous medium ($F_0 \to \infty$), and the intensity fluctuations of a "quasispherical" wave — by a factor of $(\xi/U_2(\xi))^4$. Figure 3 shows the results of calculations of the attenuation factor of intensity fluctuations ($\beta(\xi) = \sigma_I^2(x,0,\kappa)/\lim\limits_{F_0 \to \infty} \sigma_I^2(x,0,\kappa)$ of the wide collimated beam ($\beta_p(\xi)$) and of the "quasispherical" wave ($\beta_s(\xi)$) propagating through the discrete scattering medium with different regularities of varying the local focal distances along the path for $d \gg 1$. The profiles of $F(x)$ used there coincide with those shown in Fig. 2. A more rapid decrease of the intensity fluctuations of the wide collimated beam than of the "quasispherical" wave can be seen again in the discrete scattering medium, which is similar lo the case of the continuous medium.

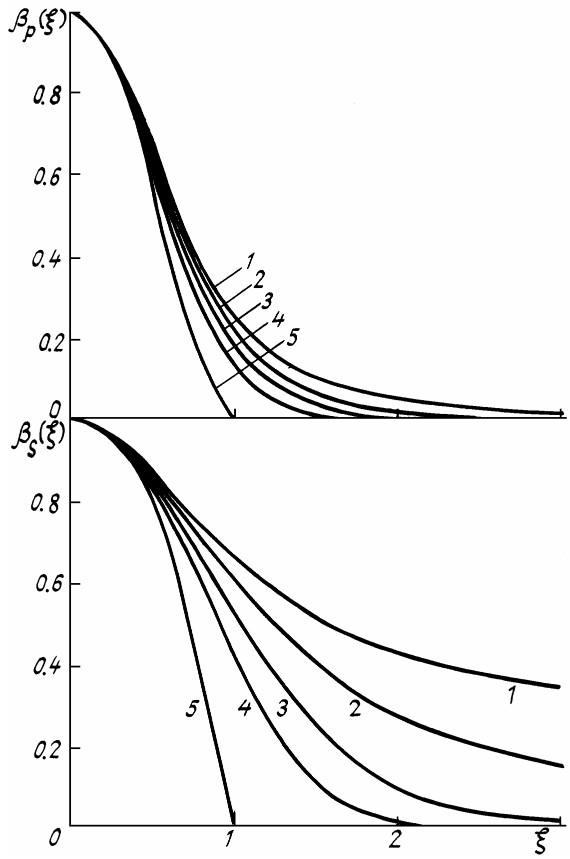

FIG. 3. The dependence of the ratios of the absolute variances of the intensity fluctuations $\beta_p(\xi)$ and $\beta_s(\xi)$ in the discrete scattering medium on the functional form of the profileof $F(x)$:

1) $U_1(\xi) = \sqrt{1 - \xi^2}$, *2)* $U_1(\xi) = (1 + \frac{2}{3}\xi^2)^{3/4}$, *3)* $U_1(\xi) = \mathrm{ch}(\xi)$,

4) $U_1(\xi) = \exp(\frac{1}{2}\xi^2)$, and *5)* $U_1(\xi) = 1/\sqrt{1 - \xi^2}$ $(\xi < 1)$.

In contrast to the variance of the intensity fluctuations of the optical beam which propagates through the lens–like medium with random inhomogeneities described by Eq. (7), the normalized intensity fluctuation frequency correlation function $b_I(\kappa_1,\kappa_2) = B_I(\kappa_1,\kappa_2)/[\sigma_I(x,0;\kappa_1)\sigma_I(x,0;\kappa_2)$ appears to be practically independent of the refraction properties of the medium. Using Eqs. (6) and (7), it may be shown that for $D(x, \xi) \ll 1$

$$b_I(\kappa_1,\kappa_2) \simeq 1, \qquad (12)$$

and for $D(x, \xi) \gg 1$ and $D(x, \xi)\,\Omega \gg 1$

$$b_I(\kappa_1,\kappa_2) \simeq \frac{1-\Omega^{5/6}}{(1-\Omega^2)^{5/6}}, \qquad (13)$$

where $\Omega = (\kappa_1 - \kappa_2)/(\kappa_1 + \kappa_2) = (\lambda_2 - \lambda_1)/(\lambda_1 + \lambda_2)$ is the relative wave number difference.

It is not difficult to find that the obtained results (12) and (13) identically coincide with the results for the regularly homogeneous medium ($F_0 \to \infty$) presented in Ref. 15.

As for the intensity fluctuation frequency correlation of optical wave, which propagates through a lens–like medium with discrete scatterers, for the continuous waves ("quasiplanar" and "quasispherical" waves), for which $a(x,\kappa_1) \simeq a(x;\kappa_2) \simeq a(x) \gg 1$ and $S(x,\kappa_1) \simeq S(x;\kappa_2) \simeq S(x)$, for $d \gg 1$ and $\Omega \ll d^{-1}$ we have

$$b_{I\,ps}(k_1,k_2) \simeq 1 - \frac{\Omega^2}{\Omega^2_{avs}(\xi)}, \qquad (14)$$

where

$$\Omega_{av}(\xi) = \frac{\xi U_1(\xi)}{4\bar{d}\sqrt{\int\limits_0^1 d\eta\, U_1^2(\xi\eta)\,[U_1(\xi\eta)\,U_2(\xi) - U_1(\xi)U_2(\xi\eta)]^2}}$$

is the characteristic scale of frequency correlation for a wide collimated beam (the "quasiplanar" wave),

$$\Omega_{as}(\xi) = \frac{\xi U_2(\xi)}{4\bar{d}\sqrt{\int\limits_0^1 d\eta\, U_2^2(\xi\eta)\,[U_1(\xi\eta)\,U_2(\xi) - U_1(\xi)U_2(\xi\eta)]^2}}$$

is the same parameter for the "quasispherical" wave, where $\bar{d} = \dfrac{x}{\bar{k}a^2}$ and $\bar{k} = \dfrac{2k_1k_2}{k_1+k_2}$ is the wave number corresponding to the average wavelength.

Figure 4 shows the factors of increasing the scale of intensity fluctuation frequency correlations in a lens–like medium with discrete scatterers $\alpha_{ps}(\xi) = \Omega_{avs}(\xi)/\lim\limits_{F_0\to\infty}(\xi)$. Various profiles of $F(\xi)$ were considered (the used models of the focal distance in the lens–like medium were the same as the models used to calculate $\gamma_{ps}(\xi)$, see Fig. 2).

Thus, the existence of regular inhomogeneities of dielectric pemurtivity in the discrete scattering media improved the degree of the intensity fluctuation frequency correlations compared to the regularly homogeneous medium. Indeed, because of the defocusing properties of the lens–like medium, the spatial scale ρ_c of the shadow pattern on the receiver will be larger than the actual size of the scatterer ($\rho_c > a$). Hence, as demonstrated in Ref. 16. the scale of the frequency correlation will also be larger:

$$\Omega_c > \kappa\rho_c^2/x > \kappa a^2/x\ (\alpha_{ps}(\xi) > 1).$$

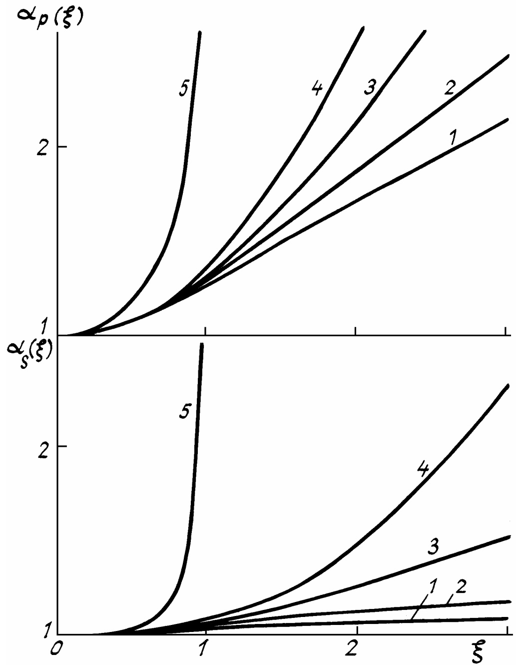

FIG. 4. The characteristic scales of the intensity fluctuations frequency correlation of the wide collimated beam ($\alpha_p(\xi)$) and of the "quasispherical" wave ($\alpha_s(\xi)$) propagating through the discrete scattering medium with various profiles of $F(x)$:

1) $U_1(\xi) = \sqrt{1-\xi^2}$, *2)* $U_1(\xi) = (1 + \frac{2}{3}\xi^2)^{3/4}$,

3) $U_1(\xi) = \mathrm{ch}(\xi)$, *4)* $U_1(\xi) = \exp(\frac{1}{2}\xi^2)$, *and*

5) $U_1(\xi) = 1/\sqrt{1-\xi^2}$ $(\xi < 1)$.

The analysis of the intensity fluctuation characteristics of the Gaussian beam propagating through the lens-like medium demonstrates that the intensity fluctuations of optical beam in a lens-like medium are weaker than Ihe fluctuations in a regularly homogeneous medium. Moreover, these intensity fluctuations decrease the faster, the larger is the initial divergence of Ihe optical beam. The intensity fluctuation frequency correlation of the optical beam propagating through the lens – tike medium with continuous random inhomogeneities coincides with that for the regularly homogeneous medium, white the existence of the regular refraction in homogeneity in the discrete scattering medium results in an increase of Ihe scale of the intensity fluctuation frequency correlations.

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