AN ACTIVE RECONSTRUCTION OF COHERENT IMAGES FROM PHASE–DISTORTED SIGNALS

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A noniterative method is proposed for reconstruction of coherent images distorted by the turbulent atmosphere. The method needs no reference source in the space of object. The method is based on the use of a spatiotemporal modulation of a sensing signal by a known function within the time interval of the frozen atmosphere. Such a modulation permits one to isolate time–constant phase perturbations of the signal field by the turbulent atmosphere.

Various options are considered of the proposed active method. One of such options has been simulated.

Holographic techniques for reconstruction of images, perturbed by the turbulent atmosphere, starting with the studies by Goodman and Waters,[1-3] assume, in particular, the use of iterative (multicycle) techniques of forming the reference signal in the plane of the sensed target. In this case a reference source is actively formed, while the image itself is reconstructed passively. However, there can occur situations,[3] in which the convergence of the iterative procedure cannot be reached. Naturally, the question arises whether it is possible or not to form a reference signal at the transmitting and receiving apertures that it could be actively controlled. If this is feasible one can avoid polycyclicity.

The present study deals with the technique, which needs for neither polycyclic formation of the reference signal, nor statistical algorithms for *a posteriori* processing. It is based on the idea of using sounding signals of a complicated spatiotemporal structure.

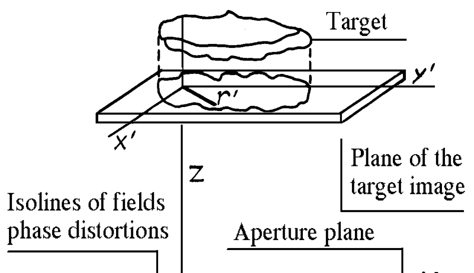

FIG. 1. Target, aperture, and field of phase distortions.

Let a sounding signal of a certain spatiotemporal structure (Fig. 1) be formed in the plane of a combined transmitting–receiving aperture:

$$\varepsilon_{ss}(\rho, t) = \Phi(\rho, t) . \qquad (1)$$

Some possibilities of forming such a signal will be discussed below. Here ρ is the spatial coordinate in the aperture plane. In this case, within the Fraunhofer and the thin–phase screen approximation[4] the field formed in the plane of the target image will have the form

$$E_s(r', t) = \frac{1}{\lambda R}\exp\left(j\frac{kr'^2}{2R}\right)E'(r') \times$$

$$\times \int \mathrm{d}r \exp\left(j\frac{k}{R}r'r\right)\exp(j\varphi_{a1}(\rho))\,\Phi(\rho, t) , \qquad (2)$$

where λ is the wavelength, $k = 2\pi/\lambda$ is the wave number, R is the range to a target, $\varphi_{a1}(\rho)$ describes the phase distortions introduced to the sounding signal by the turbulent atmosphere during its propagation towards the target, r' is the spatial coordinate in the target plane and $E(r')$ is the image of the target. In our further discussion we shall assume the signal to be reconstructed have the form

$$E(r') = \frac{1}{\lambda R}\exp\left(j\frac{kr'^2}{2R}\right)E'(r') . \qquad (3)$$

Denoting $r = kr'/R$, we reduce our mathematical model of the process to the Fourier transform. Taking the above designations into account one obtains the distribution of field scattered by the target over the receiving aperture in the form

$$\varepsilon_s(\rho, t) = \exp(j\varphi_{a2}(\rho))\int \mathrm{d}r\,\exp(jr\rho)E_s(r, t) , \qquad (4)$$

where $\varphi_{a2}(\rho)$ describes the phase distortions of the field occuring during the process of receiving the scattered field which, in general, differ from those taking place in transmitting the sounding signal. Based on relations (2) and (3), and the theorem of convolution,[5] relation (4) can be given in the form

$$\varepsilon_s(\rho, t) = \exp(j\varphi_{a2}(\rho))\int \mathrm{d}\rho'\varepsilon(\rho - \rho')\exp(j\varphi_{a1}(\rho))\,\Phi(\rho, t) , \qquad (5)$$

where $\varepsilon(\rho)$ is the spatial spectrum of the field $E(r)$.

Since the temporal dependence is introduced into the signal by an arbitrarily selected modulation (what makes the main idea of the active reconstruction), it may be eliminated by a transformation which lowers the rank of integral equation (5). In this connection the choice of a running δ–function, for $\Phi(\rho, \tau)$ is most obvious. However, the use of a weighted integration of the received signal such as, the procedure

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$$\varepsilon(\rho,\ \Delta\rho) = \int \mathrm{d}t\varepsilon_s(\rho,\ t)\ \Phi^*(\Delta\rho,\ t) =$$

$$= \exp\bigl(j\varphi_{a2}(\rho)\bigr)\ \varepsilon(\rho + \Delta\rho)\ \exp\bigl(j\varphi_{a1}(\Delta\rho)\bigr) \tag{6}$$

seems to be more general.
Assuming the function $\Phi(\rho\ t)$ to be orthogonal we have

$$\Phi(\rho,\ t) = \int \mathrm{d}t\ \Phi(\rho_1,\ t)\ \Phi^*(\rho_2,\ t) = \delta(\rho_1 - \rho_2)\ , \tag{7}$$

where $\Delta\rho$ is the minor spatial vector in the aperture plane and $\delta(\rho)$ is the Dirac δ–function.

Equation (6) contains two unknown functions of the coordinate ρ, of which the function $\varphi_{a2}(\rho)$ describes the multiplicative noise and $(\varepsilon(\rho))$ is the spatial spectrum of the sought–after image. We may, therefore, complete Eq. (6) to obtain a system of two equations by varying $\Delta\rho$ in the following manner

$$\begin{cases}\varepsilon(\rho,\ \Delta\rho) = \exp\bigl(j\varphi_{a2}(\rho)\bigr)\varepsilon(\rho + \Delta\rho)\ \exp\bigl(j\varphi_{a1}(\Delta\rho)\bigr)\ , \\ \varepsilon(\rho,\ 0) = \exp\bigl(j\varphi_{a2}(\rho)\bigr)\varepsilon(\rho + 0)\ \exp\bigl(j\varphi_{a1}(0)\bigr)\ ,\end{cases} \tag{8}$$

which takes the form

$$\begin{cases}\psi(\rho,\ \Delta\rho) = \varphi_{a2}(\rho) + \varphi(\rho + \Delta\rho) + \varphi_{a1}(\Delta\rho)\ , \\ \psi(\rho,\ 0) = \varphi_{a2}(\rho) + \varphi(\rho + 0) + \varphi_{a1}(0)\end{cases} \tag{9}$$

for the signal phase. By solving the system of equations (9) with respect to $\varphi(\rho)$ we obtain

$$\varphi(\rho+\Delta\rho) - \varphi(\rho+0) = \psi(\rho,\Delta\rho) - \psi(\rho,\ 0) - (\varphi_{a1}(\Delta\rho) - \varphi_{a1}(0))\ . \tag{10}$$

Note that the phase difference $\psi(\rho,\ \Delta\rho) – \psi(\rho,\ 0)$ is measurable, and it can be separated out using the signals $\varepsilon(\rho,\ \Delta\rho)$, and $\varepsilon(\rho,\ 0)$, since

$$\psi(\rho,\ \Delta\rho) - \psi(\rho,\ 0) = \arg(\varepsilon(\rho,\ \Delta\rho)\ \varepsilon^*(\rho,\ 0))\ .$$

One can see that in its meaning Eq. (10) is similar to the differential equation of the form

$$\frac{\mathrm{d}\varphi(\rho)}{\mathrm{d}\rho} = \left.\frac{\mathrm{d}\psi(\rho,\ \rho')}{\mathrm{d}\rho'}\right|_{\rho'=0} - \left.\frac{\mathrm{d}\varphi_{a1}(\rho)}{\mathrm{d}\rho}\right|_{\rho=0} \tag{11}$$

the solution of which is quite evident

$$\varphi(\rho) = \int\limits_0^{\rho} \mathrm{d}\rho''\ \left.\frac{\mathrm{d}\psi(\rho'',\ \rho')}{\mathrm{d}\rho'}\right|_{\rho'=0} - \left.\frac{\mathrm{d}\varphi_{a1}(\rho)}{\mathrm{d}\rho}\right|_{\rho=0} \cdot \rho + C\ . \tag{12}$$

Here C is the random phase; $\left.\dfrac{\mathrm{d}\varphi_{a1}(\rho)}{\mathrm{d}\rho}\right|_{\rho=0}$ is the unknown tilt of the wavefront.

Thus, the unknown differential constant $(\varphi_{a1}(\Delta\rho) – \varphi_{a1}(0))$ related to $\left.\dfrac{\mathrm{d}\varphi_{a1}(\rho)}{\mathrm{d}\rho}\right|_{\rho=0}$ results in a certain displacement of the image during its reconstruction what is insignificant for the problem under study.

The proposed technique of active reconstruction of an image can be easily performed using a correlation detector, which inherently transforms Eq. (6). Of course, not only the correlation processing but also a properly designed filtering would yield the results described by Eqs. (8) and (9). Indeed, let a spatiotemporal harmonic of the form

$$\Phi(\rho,\ t) = \exp\bigl(j(\Omega\rho - \omega_g)t\bigr)\ , \tag{13}$$

be chosen for $\Phi(\rho,\ t)$, where Ω is the spatiotemporal frequency and ω_g is a frequency "addition".

Substituting (13) into relations (2)–(4) in the plane of the receiving aperture, we obtain the signal in the form

$$\varepsilon_s(\rho,\ t) = \exp\bigl(j\varphi_{a2}(\rho)\bigr)\int \mathrm{d}\rho'\varepsilon(\rho - \rho') \times$$

$$\times \exp\bigl(j\varphi_{a1}(\rho')\bigr)\ \exp\bigl(j(\Omega\rho' - \omega_g)t\bigr)\ , \tag{14}$$

the temporal spectrum

$$\varepsilon_s(\rho,\ \omega) = \int \mathrm{d}t\ \exp(j\omega t)\varepsilon_s(\rho,\ t) \tag{15}$$

of which takes the form

$$\varepsilon_s(\rho,\ \omega) = \exp\bigl(j\varphi_{a2}(\rho)\bigr)\int \mathrm{d}\rho'\varepsilon(\rho - \rho') \times$$

$$\times \exp\bigl(j\varphi_{a1}(\rho')\bigr)\ \delta(w + \Omega\rho' - \omega_g) \tag{16}$$

if one takes relation (14) into account. By tuning the filter to the frequency $\omega_{\Delta\rho}$, we obtain

$$\varepsilon_f(\rho,\ t) = \exp\bigl(j\varphi_{a2}(\rho)\bigr)\ \exp\bigl(-\ j\omega_{\Delta\rho}t\bigr) \times$$

$$\times \exp\left(j\varphi_{a2}\left(\frac{\Omega(\omega_{\Delta\rho} - \omega_g)}{|\Omega|^2}\right)\right)\ \varepsilon\left(\rho + \left(\frac{\Omega(\omega_{\Delta\rho} - \omega_g)}{|\Omega|^2}\right)\right)\ . \tag{17}$$

Now, selecting

$$\begin{cases}\dfrac{\Omega(\omega_{\Delta\rho 1} - \omega_g)}{|\Omega|^2} = \Delta\rho, \\ \dfrac{\Omega(\omega_{\Delta\rho 2} - \omega_g)}{|\Omega|^2} = 0,\end{cases} \tag{18}$$

we arrive at a system of equations, similar to system (8). The temporal dependence can be eliminated, using, for example, a relevant homodyning.

A qualitative analysis of the derived–above expressions allows one to conclude that the reconstruction of an image is performed by regulating the position of the spectrum of an estimated image, independently of the multiplicative noise.

Because of the intrinsic properties of the Fourier transform[5] the change in the position of the signal spectrum is equivalent to the presence of a linear phase shift in the signal. The latter effect may be achieved by varying the spatial coordinate of a point source of the sounding signal. This is the basic operation in the techniques of active interferometry.[6] Based on this approach one can construct, avoiding the stage of phase conjugation, quite different algorithm, which in certain sense corresponds to an asymptotic case of relations (1)–(18).

Let the generator forming the sounding signal field emits two signals during the time interval of the frozen atmosphere

$$\begin{cases} \varepsilon_{ss_1}(\rho) = \mathrm{d}(\rho - \Delta\rho), \\ \varepsilon_{ss_2}(\rho) = \mathrm{d}(\rho - 0). \end{cases} \tag{19}$$

Taking into account the presence of phase distortions $\varphi_{a1}(\rho)$ at propagation of sounding signals, the field in the target plane may be written in the form:

$$\begin{cases} E_{s1}(r) = E(r)\int \mathrm{d}\rho \exp(jr\rho)\, \varepsilon_{ss_1}(\rho) \exp\big(j\varphi_{a1}(\rho)\big) = \\ \qquad = E(r)\exp(j\Delta\rho r)\exp\big(j\varphi_{a1}(\Delta\rho)\big), \\ E_{s2}(r) = E(r)\int \mathrm{d}\rho \exp(jr\rho)\, \varepsilon_{ss_2}(\rho) \exp\big(j\varphi_{a1}(\rho)\big) = \\ \qquad = E(r)\exp(j0r)\exp\big(j\varphi_{a1}(0)\big). \end{cases} \tag{20}$$

As a result, in the plane of the receiving aperture we obtain

$$\begin{cases} \varepsilon_{s1}(\rho) = \exp\big(j\varphi_{a2}(\Delta\rho)\big) \int \mathrm{d}\rho \exp(-jr\rho)\, E(r) \times \\ \qquad \times \exp(j\Delta\rho r)\exp\big(j\varphi_{a1}(\Delta\rho)\big) = \\ \qquad = \exp\big(j\varphi_{a2}(\rho)\big)\exp\big(j\varphi_{a1}(\Delta\rho)\big)\,\varepsilon(\rho - \Delta\rho), \\ \varepsilon_{s2}(\rho) = \exp\big(j\varphi_{a2}(\Delta\rho)\big) \int \mathrm{d}\rho \exp(-jr\rho)\, E(r) \times \\ \qquad \times \exp(j0r)\exp\big(j\varphi_{a1}(0)\big) = \\ \qquad = \exp\big(j\varphi_{a2}(\rho)\big)\exp\big(j\varphi_{a1}(0)\big)\,\varepsilon(\rho - 0). \end{cases} \tag{21}$$

Using a detector which records the sum of signals $\varepsilon_{c1}(\rho)$ and $\varepsilon_{c2}(\rho)$ we have

$$\left|\varepsilon_{c1}(\rho) + \varepsilon_{c2}(\rho)\right|^2 = \left|\varepsilon_{c1}(\rho)\right|^2 + \left|\varepsilon_{c2}(\rho)\right|^2 + \\ + \varepsilon^*_{c1}(\rho)\varepsilon_{c2}(\rho) + \varepsilon_{c1}(\rho)\varepsilon^*_{c2}(\rho). \tag{22}$$

Then, separating the interference term by one of the available techniques, one obtains for its phase the relation

$$\psi(\rho, \Delta\rho) = \varphi(\rho - \Delta\rho) - \varphi(\rho) + \varphi_{a1}(\Delta\rho) - \varphi_{a1}(0), \tag{23}$$

where

$$\varphi(\rho) = \arg\varepsilon(\rho). \tag{24}$$

By passing to a limit in relation (23) one obtains

$$\left.\frac{\mathrm{d}\psi(\rho, \rho')}{\mathrm{d}\rho'}\right|_{\rho'=0} = -\frac{\mathrm{d}\varphi(\rho)}{\mathrm{d}\rho} + \left.\frac{\varphi_{a1}(\rho)}{\mathrm{d}\rho}\right|_{\rho=0}. \tag{25}$$

Equation (25) is solved, like Eq. (23) relative to the function $\phi(\rho)$, that yields

$$\varphi(\rho) = \int\limits_0^{\rho} \mathrm{d}\rho'' \left.\frac{\mathrm{d}\psi(\rho'', \rho')}{\mathrm{d}\rho'}\right|_{\rho'=0} - \left.\frac{\mathrm{d}\varphi_{a1}(\rho)}{\mathrm{d}\rho}\right|_{\rho=0} \cdot \rho + C, \tag{26}$$

The unknown tilt of the wavefront $\left.\frac{\varphi_{a1}(\rho)}{\mathrm{d}\rho}\right|_{\rho=0}$) again does not affect the quality of the solution of the principal problem on reconstruction of an image.

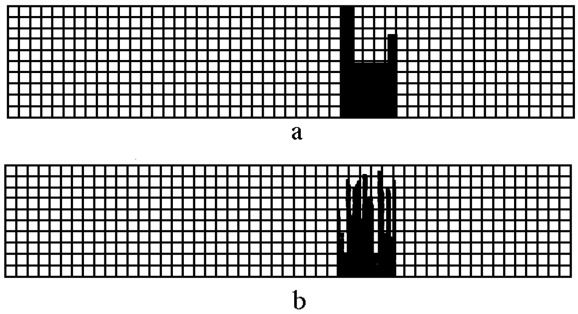

FIG. 2. Modulus (a) and phase (b) of the initial signal.

Figure $2a$ shows the distribution of modulus of the initial signal which is to be reconstructed. The general "field of view" contains 512 pixels. Figure $2b$ presents the simulated phase distribution of a signal reflected from a diffuse target. In their combination the modulus and the phase presented in Figs. $2a$ and $2b$ completely describe the initial field. It should be noted that in many problems on image reconstruction it is sufficient to extract information about the modulus alone.

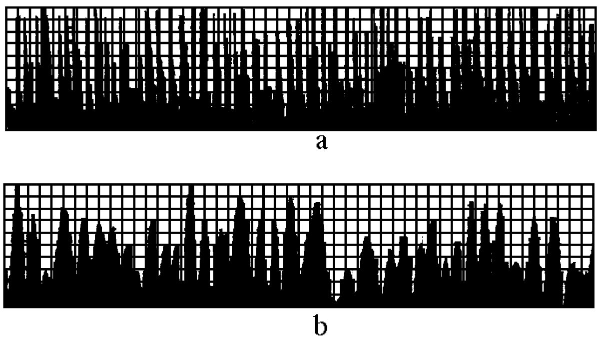

FIG. 3. Phase (a) and modulus (b) of the initial signal spectrum.

Figures $3a$ and $3b$ present the modulus and the phase of the initial field spectrum what corresponds to the propagation of a signal through an empty space. Figure $4a$ illustrates a result of differential processing of the phase of a received signal, i.e., presents the derivative of the estimated phase. The result of integrating the differential phase is shown in Fig. $4b$. Using the modulus of the spectrum $|\varepsilon(\rho)|$ (Fig. $3a$) and its reconstructed phase (Fig. $4b$) the image (see Fig. 5) is reconstructed using the Fourier–inversion, which satisfactorily agrees with the initial one (Fig. $2a$).

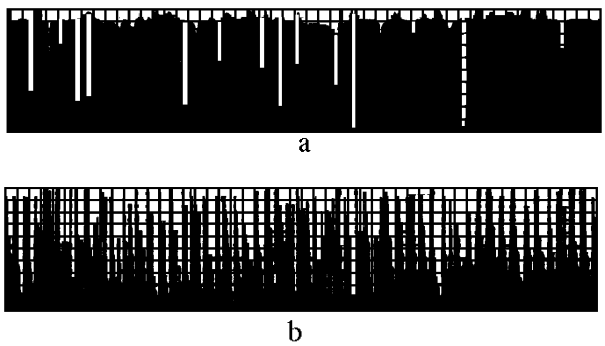

FIG. 4. Phases of differential (a) and reconstructed (b) signal spectra.

It is pertinent to note that the algorithm for active reconstruction of images from a single–cycle sensing recording with the quadratic signal detection, as described by relations (19)–(26) and illustrated by the mathematical simulation, does not make the general approach (see relations (1)–(18)) unreasonable. Being a particular case of more general approach, the technique of single–cycle sensing and recording has, naturally, a limited applicability. The limitation arises because of the assumption that the radiation source is a point source in the plane of transmitting that, of course, limits the operation range.

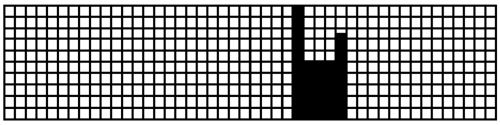

FIG. 5. Modulus of the reconstructed signal.

As a reult it is advisable to employ the approach described by relations (19)–(26) at a relatively short distance. At the same time a more general approach presented by relations (1)–(18) should be preferred at longer distances

Note also that the procedures of step–by–step integration of differential phase may be replaced by a parallel reconstruction using the least–squares technique, as was done in Ref. 7 in application to a shear interferometeric measurements.

Thus, a technique of active reconstruction of images distorted by the turbulent atmosphere is proposed and verified by a mathematical simulation. The technique needs neither for a reference source in the plane of a target, nor for iterative algorithms for signal processing.

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