ON QUALITATIVE CHANGES IN SIMPLE CLIMATIC MODELS PRODUCED BY PARAMETERIZATION

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Received November 5, 1991

For the simplest climatic model with two temperatures the steady states in the finite part of the plane are found. It is shown that at different parametrizations of the outgoing infrared radiation the position and the character of steady states can be changed down to their complete disappearance.

Simple energy–balance models are convenient for discussion of general climatic regularities and at the same time they underlie the detailed multidimensional models describing global climatic changes. In their first part they seem to be an interesting object for the qualitative analysis. Thus obtained information may be useful for studies based on the global models.

In the present note the steady states in the finite part of the plane are found for the simplest model of the radiative planetary regime with two temperatures. It is shown that at different parameterizations of outgoing radiation the position and the character of the steady states can be drastically changed down to their complete disappearance.

Let us consider the simplest climatic model[1] including the uniform horizontal surface at temperature T_s and the uniform air layer at temperature T_a over the surface. The heat balance is formed as a result of a heat exchange through radiation alone. Variations of the surface and air temperatures are determined by the equations

$$\begin{cases}\dot{T}_s = \sigma T_a^4(1 - D_t) - \sigma T_s^4 + F^{\downarrow}D_s,\\ \dot{T}_a = \sigma T_s^4(1 - D_t) - 2\sigma T_a^4(1 - D_t) + F^{\downarrow}(1 - D_s).\end{cases} \quad (1)$$

The dot denotes the differentiation over time, D_t is the atmospheric transmission function for the long–wave radiation, D_s is the atmospheric transmission function for the short–wave solar radiation, and $F^{\downarrow}$ is the balance of the solar energy at the upper boundary of the atmosphere. Figure 1 explains the origin of different terms in Eq. (1). Let us introduce the following designations

$$T_s = x\ ,\ T_a = y\ ,\ \sigma(1 - D_t) = a\ ,\ F^{\downarrow}D_s = b\ ,\ F^{\downarrow}(1 - D_s) = c. \quad (2)$$

Using these designations one can write a system of equations (1) in the form

$$\begin{cases}\dot{x} = ay^4 - \sigma x^4 + b = P\ ,\\ \dot{y} = ax^4 - 2ay^4 + c = Q\ .\end{cases} \quad (3)$$

A system of equations (3) is the autonomous system of two nonlinear differential equations. To study it we use the methods of qualitative theory of differential equations.[2,3] The steady state coordinates in the finite part of the plane are found from the equations

$$P = 0\ ,\ \text{and}\ Q = 0 \quad (4)$$

and are equal to

$$x_0 = \pm((2b + c)/(2\sigma - a))^{1/4}\ ,$$
$$y_0 = \pm((\sigma c + ab)/a(2\sigma - a))^{1/4}\ . \quad (5)$$

Standard analysis of the trajectory behaviors in the vicinity of these steady states shows that there are two saddles and the stable and unstable nodes, see Fig. 2. This picture remains unchanged at the variations of numerical values of $F^{\downarrow}$, D_t, and D_s.

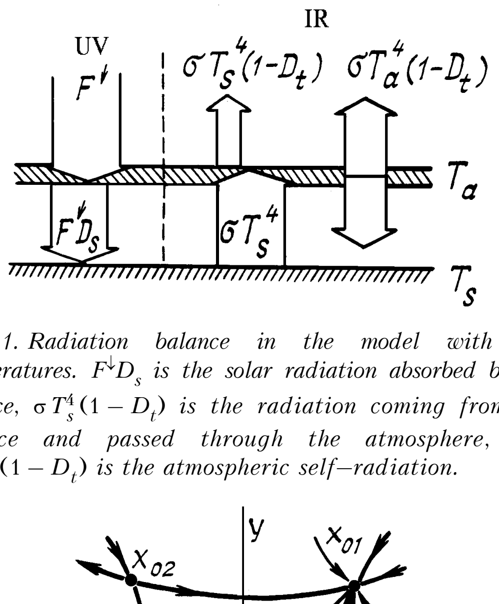

FIG. 1. Radiation balance in the model with two temperatures. $F^{\downarrow}D_s$ is the solar radiation absorbed by the surface, $\sigma T_s^4(1 - D_t)$ is the radiation coming from the surface and passed through the atmosphere, and $\sigma T_a^4(1 - D_t)$ is the atmospheric self–radiation.

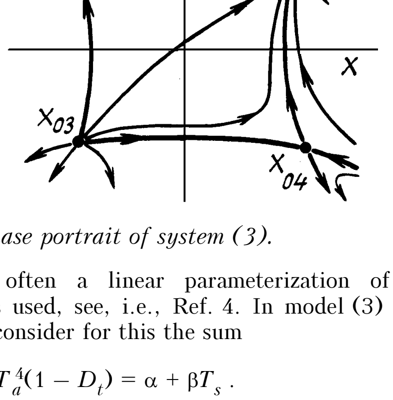

FIG. 2. Phase portrait of system (3).

Very often a linear parameterization of outgoing radiation is used, see, i.e., Ref. 4. In model (3) it is quite natural to consider for this the sum

$$\sigma T_s^4 D_t + \sigma T_a^4(1 - D_t) = \alpha + \beta T_s\ . \quad (6)$$

In this case a system of equations (3) are reduced to

$$\begin{cases} \dot{x} = -a_1x^4 + \beta x + b_1 = P_1 \\ \dot{y} = a_1x^4 - 2\beta x + c_1 \end{cases} \quad (7)$$

with

$a_1 = \sigma(1 + D_t)$, $b_1 = a + b$, $c_1 = -2\alpha + c$.

System (7) is, strictly speaking, not a system of equations, because y is the function of x. A fragment of its phase portrait is depicted in Fig. 3. It can be seen that the isolated steady states are absent, and trajectories tend to go into infinity under any initial conditions.

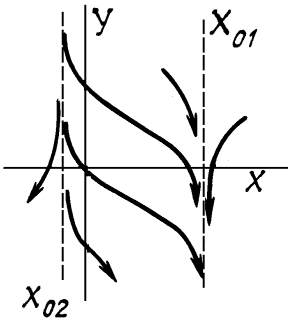

FIG. 3. Phase portrait of system (7). x_{01} and x_{02} are the roots of equation $P_1 = 0$.

It is possible, in principle, to parametrize separate constituents of the outgoing radiation. Thus, if a linear parameterization of the atmospheric long–wave radiation is adopted

$$\sigma T_a^4(1 - D_t) = \varepsilon T_a , \quad (8)$$

we obtain

$$\begin{cases} \dot{x} = \varepsilon y - \sigma x^4 + b , \\ \dot{y} = ax^4 - 2\varepsilon y + c \end{cases} \quad (9)$$

instead of Eqs. (3). A system of equations (9) has two steady states

$$x_0 = \pm ((c + 2b)/(2\sigma - a))^{1/4} ,$$
$$y_0 = \pm (ab + \sigma c)/\varepsilon(2\sigma - a) \quad (10)$$

and one of them located in the positive quadrant is a stable node and the other is a saddle. Similar picture is obtained, when the radiation going from the surface $\sigma T_s^4 D_t$, is parameterized by a linear function.

Thus, the parameterizations changing the nonlinearity type can yield drastic, qualitative changes in the system behavior, e.g., the change of the number and character of steady states. The possibility of such qualitative changes must be taken into account when constructing the climatic models with different levels of parameterization.

ACKNOWLEDGMENT

The author acknowledges Prof. E.P. Gordov for fruitful discussions.

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