INTENSITY STATISTICS OF LIGHT SCATTERRED BY AEROSOL SMOKE PLUMES

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Received December 30, 1992

In this paper we present our study of fluctuations of intensity of light scattered by particles of smoke plumes occurring in the atmosphere due to mixing of particles by large scale turbulent eddies. To do this we have calculated the mean intensity, relative variance, and temporal scale of fluctuations of intensity of scattered radiation in the focal plane of a receiving telescope. The results obtained enable a priori choice of the exposure time to be done for taking optical images of smoke plumes with a desired degree of averaging.

In Ref. 1 some possibilities of determining the power of industrial pollution emissions into the atmosphere and the particles concentration in smoke plumes from the measurements of intensity of backscattered optical radiation are considered. However, this approach does not take into account that the concentration of scattering particles, and, therefore, the intensity of received optical signal are random functions of coordinates and time due to the atmospheric turbulence. An analysis of the intensity fluctuations of optical radiation scattered by the particles of smoke plume can be useful both for estimating the exposure time for taking an averaged plume image and obtaining additional information about the object sounded.

In the paper we analyze mean distribution, variance, and temporal correlation scale of the intensity of optical radiation scattered by particles of a smoke plume and collected by a telescope in the image plane.

Problem statement. The optical beam propagating along the z axis of a Cartesian coordinate system $\{z', x', y'\}$ is assumed to be incident on the smoke plume of aerosol particles moving along the y axis due to wind. The radiation backscattered is recorded in the image plane of a telescope. According to Ref. 1 the variation of intensity of scattered radiation along the coordinate x in the telescope focal plane is described by the formula

$$U(x,\ t) = q_G \left\{1 - \exp[-\ 2\tau(x,\ t)]\right\}, \tag{1}$$

where q_G is the geometric factor determined by the parameters of receiving–transmitting system and by scattering properties of smoke plume particles;

$$\tau(x,\ t) = \sigma_{\rm eff} \int\limits_{-\infty}^{+\infty} {\rm d}z'\, \rho\left(z',\ \frac{L}{F_t}\, x,\ y,\ t\right) \tag{2}$$

is the optical thickness of the smoke plume along the z' axis; $\sigma_{\rm eff}$ is the sum of the scattering cross section, σ_s, and the absorption one, σ_a, for the case of small particles ($r_0 \lesssim \lambda/2$, where r_0 is the effective radius of particles, λ is the wavelength of radiation) while $\sigma_{\rm eff}$ is the absorption cross section only in the case of large particles ($r_0 \gg \lambda/2$) (see Ref. 1); ρ is the concentration of particles in the smoke plume at the point $\{z', x', y'\}$ and at time t; y is the distance from the smoke source, $y' = 0$, to the plane $y' = y$, from which the scattered optical radiation comes; and, L is the distance from the receiving telescope with the focal length F_t to the smoke plume axis.

The concentration ρ is a complicated function of space and time. Random spatiotemporal variations of ρ cause the intensity fluctuations U. In the subsequent discussion we consider the situation when the main contribution to the intensity fluctuations comes from large–scale turbulent eddies inducing the random displacements of the smoke plume, as a whole. We consider the scheme of sounding of the plant stack smoke plume, for example, so as it is observed from the side (the plume image along the vertical axis x). If we assume that the smoke coming out of the stack is not overheated and has no its own velocity, so that the smoke particles are entirely entrained by wind[6] we can use the Gaussian plume model for the concentration ρ (see Refs. 2–4)

$$\rho(z',\ x',\ y,\ t) = \frac{M}{\pi V a_z\, a_x} \exp\left\{-\frac{[z' - \tilde{z}(y,\ t)]^2}{a_z^2} - \frac{[x' - \tilde{x}(y,\ t)]^2}{a_x^2}\right\}, \tag{3}$$

where V is the mean velocity of wind whose direction is parallel to the Y axis and y is the distance from the stack mouth to the observation plane $\{z', x'\}$ along the plume axis;

$$M = V \int\limits_{-\infty}^{+\infty}\!\!\int {\rm d}z'\ {\rm d}x'\ \rho(z',\ x',\ y,\ t) \tag{4}$$

is the power of the emission[5];

$$\begin{Bmatrix} \tilde{z}(y,\ t) \\ \tilde{x}(y,\ t) \end{Bmatrix} = \frac{V}{M} \int\limits_{-\infty}^{+\infty}\!\!\int {\rm d}z'\ {\rm d}x' \begin{Bmatrix} z' \\ x' \end{Bmatrix} \rho(z',\ x',\ y,\ t) \tag{5}$$

are the coordinates of the centroid of the concentration distribution in the plane $\{z', x'\}$;

$$\begin{Bmatrix} a_z^2 \\ a_x^2 \end{Bmatrix} = \frac{2V}{M} \int\limits_{-\infty}^{+\infty}\!\!\int {\rm d}z'\ {\rm d}x' \begin{Bmatrix} (z' - \tilde{z})^2 \\ (x' - \tilde{x})^2 \end{Bmatrix} \rho(z',\ x',\ y,\ t) \tag{6}$$

are the squared effective dimensions of the smoke plume along the axes z' and x', respectively.

By substituting Eq. (3) into Eq. (2) and integrating we obtain

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$$\tau(x,\ t) = \frac{\sigma_{\mathrm{eff}}\, M}{\sqrt{\pi} V a_x} \exp\left\{ - \frac{\left[\dfrac{L}{F_t}\, x - \tilde{x}(y,\ t)\right]^2}{a_x^2} \right\} . \quad (7)$$

Models of plume parameters. In the general case the power M and the plume width a_x are random functions of distance y and time t and, hence, their fluctuations can influence the optical thickness of the plume $\tau(x,\ t)$. If smoke uniformly flows out from a stack the turbulent diffusion of particles along the wind can be neglected when estimating M. In this case the power of emission, M, does not depend on time and can be written as

$$M = M(0) \exp(-\alpha\, y/V) , \quad (8)$$

where $M(0)$ is the power of the emission in the source plane; α is the coefficient of concentration reduction due to particles interaction with the outer medium and their decay. The instant cross section (along the x axis) of the plume can be represented as a sum of the effective plume size in the source plane a_0 (a_0 is determined by the stack diameter) and $\Delta\tilde{x}(y,\ t)$ which is the plume lateral spread caused by the diffusion of smoke particles due to the small–scale turbulence. It is obvious that if one of the conditions

$$a_0^2 \gg \langle \Delta\tilde{x}^2 \rangle = \sigma_a^2 \quad (9)$$

or

$$\langle \tilde{x}^2 \rangle = \sigma_x^2 \gg \sigma_a^2 , \quad (10)$$

where $\langle \ldots \rangle$ denotes averaging over an ensemble; σ_a is the diffuse "instant" plume spread; σ_x^2 is the variance of the plume displacements, holds, the main contribution into the optical thickness fluctuations comes from random displacements $\tilde{x}(y,\ t)$. In this case a_x^2 in Eq. (7) may be considered as a constant value determined by the relation

$$a_x^2 = a_0^2 + \sigma_a^2 . \quad (11)$$

To estimate σ_x^2 and σ_a^2 we use the formulas obtained in Ref. 6:

$$\sigma_x^2 = 2\sigma_V^2\, t_L^2\, [y/V\, t_L - 1 + \exp(-\, y/V\, t_L)] , \quad (12)$$

$$\sigma_a^2 = \sigma_x^2 - (\sigma_V^2 - C_0\, \varepsilon_T^{2/3}\, a_0^{2/3})\, t_L^2\, [1 - \exp(-\, y/V\, t_L)]^2 , \quad (13)$$

where σ_V^2 and t_L are the variance and the Lagrange time of correlation of wind velocity vertical component, respectively; $C_0 \approx 0.9$; ε_t is the rate of turbulent energy dissipation. Let us estimate σ_x^2 and σ_a^2 for the case of neutral temperature stratification in the atmosphere when, according to the theory of the surface–layer turbulence,[7,8] simple relations can be used: $\sigma_V^2 = C_V^2\, u_*^2$, $V = (u_*/\kappa) \ln(h/z_0)$, $\varepsilon_T = u_*^2/(\kappa h)$, where u_* is the friction velocity; $\kappa = 0.4$ is the Karman constant; $C_V \approx 1$; z_0 is the underlying surface roughness parameter; and, h is the plume height. Using then the relationships for the turbulent exchange coefficient which are true for the neutral stratification[7,8]: $K_T = \kappa\, u_*\, h$ and $K_T = \sigma_V^2\, t_L$ and equaling their right–hand sides it is possible to obtain the Lagrange time of wind velocity vertical component $t_L \approx 0.4\, h/u_*$. Therefore, when $h = 30$ m, $z_0 = 0.3$ m, $y = 30$ m, and $a_0 \lesssim 2$ m condition (10) is fulfilled. If $V = 10$ m/s and $a_0 = 2$ m, then $\sigma_x^2 \simeq 6.1$ m, $a_x^2 \simeq 6.2\ \mathrm{m}^2$, and $(\sigma_x/a_x \approx 1)$ but for $a_0 = 0.5$ m we have $\sigma_x/a_x \approx 2$.

For the statistical moments of the functional $F(\tilde{x})$ the following relationships[9] can be used:

$$\langle F(\tilde{x}) \rangle = \int\limits_{-\infty}^{+\infty} \mathrm{d}\tilde{x}\ P(\tilde{x})\ F(\tilde{x}) , \quad (14)$$

$$\langle F(\tilde{x}_1)\ F(\tilde{x}_2) \rangle = \int\limits_{-\infty}^{+\infty}\!\!\int \mathrm{d}\tilde{x}_1\, \mathrm{d}\tilde{x}_2\ P(\tilde{x}_1,\ \tilde{x}_2)\ F(\tilde{x}_1)\, F(\tilde{x}_2) , \quad (15)$$

where $\tilde{x}_i = \tilde{x}(y,\ t_i)$ is the random plume displacement along the vertical axis x' for the distance y from the pollution source at the time t_i for which, in the coordinate system chosen, $\langle \tilde{x}_i \rangle = 0$, $P(\tilde{x})$ is the one–dimensional probability density for the distribution of the random value $\tilde{x}$, $P(\tilde{x}_1,\ \tilde{x}_2)$ is the two–dimensional probability density of the plume displacements $\tilde{x}_1$ and $\tilde{x}_2$ at the moments, t_1 and t_2.

The coordinate x is assumed to obey the Gaussian distribution law, that is[9]:

$$P(\tilde{x}) = (1/\sqrt{2\pi}\, \sigma_x) \exp[-\, \tilde{x}^2/(2\sigma_x^2)] , \quad (16)$$

$$P(\tilde{x}_1,\ \tilde{x}_2) = \left(2\pi\, \sigma_x^2 \sqrt{1 - K_x^2}\right)^{-1} \exp\left[- \frac{x_1^2 + x_2^2 - 2\, K_x\, \tilde{x}_1\, \tilde{x}_2}{2\sigma_x^2\, (1 - K_x^2)} \right], \quad (17)$$

where $K_x = K_x(t_1,\ t_2)$ is the temporal correlation coefficient of the plume displacements. In the case of stationary turbulence it depends on the difference $t_1 - t_2$ only.

The correlation coefficient K_x is determined by motion of two particles in the field of turbulence, which reach the observation plane at different times, t_1 and t_2. For the distances $y \ll t_L\, V$ it can be taken that the vertical velocity component $\tilde{V}_x(0,\ t)$ of a particle flying out of a stack at time t does not change as the particle is transferred by the mean flow. Hence, for the vertical displacement of the particle we can use the formula

$$\tilde{x}(y,\ t) = \tilde{V}_x(0,\ t)\ y/V . \quad (18)$$

In accordance with Eq. (18) the variance and the correlation coefficient of plume displacements, as a whole, are determined by the following expressions:

$$\sigma_x^2 = \langle \tilde{x}^2(y,\ t) \rangle = (\sigma_V^2 / V^2)\ y^2 , \quad (19)$$

$$K_x(t_1 - t_2) = \frac{\langle \tilde{x}(y, t_1)\, \tilde{x}(y, t_2)\rangle}{\sigma_x^2} = K_V(t_1 - t_2)\,, \tag{20}$$

where $\sigma_V^2 = \langle \tilde{V}_x^2(0, t)\rangle$ is the variance and $K_V(t_1 - t_2) = \langle \tilde{V}_x(0, t_1)\, \tilde{V}_x(0, t_2)\rangle / \sigma_V^2$ is the correlation coefficient of Euler wind velocity.

As follows from Eq. (20) under the condition of "frozen" turbulence ($y \ll t_L V$) the integral temporal scale of the displacements correlation,

$$t_x = \int_0^\infty \mathrm{d}t\, K_x(t) \tag{21}$$

is determined by the Euler integral temporal scale t_E completely. The above–mentioned example of evaluation of parameters σ_x^2 and a_x^2 shows that the condition ($y \ll t_L V$) is quite realizable.

Intensity statistics of scattered radiation in the focal plane of a receiving telescope. Mean intensity. By substituting Eq. (7) into Eq. (1) and using Eq. (14) we obtain for the mean intensity $\langle U\rangle$:

$$\langle U(x)\rangle = q_r \langle 1 - \exp(-2\tau(x, t))\rangle =$$

$$= q_G \left[1 - \int_{-\infty}^{+\infty} \mathrm{d}\tilde{x}\, P(\tilde{x}) \exp(-2\tau(x, \tilde{x})) \right]. \tag{22}$$

In the case of small optical thickness ($\tau \ll 1$) the exponent in Eq. (22) can be expanded into the Taylor series that in turn, can be truncated at the second term, i.e., $\exp(2\tau) \approx 1 - 2\tau$. As a result after integration over $\tilde{x}$ with the use of Eq. (16), we find

$$\langle U(x)\rangle = \frac{2\tau_0\, q_G}{\sqrt{1 + 2\sigma_x^2/a_x^2}} \exp\left\{ -\frac{x^2}{1 + 2\sigma_x^2/a_x^2} \right\}, \tag{23}$$

where

$$\tau_0 = \frac{\sigma_{\mathrm{eff}}\, M}{\sqrt{\pi}\, V\, a_x} \tag{24}$$

is the optical thickness in the absence of random displacements of the plume when $x = 0$, $X = (L/F_t)/(x/a_x)$ is the normalized coordinate in the focal plane of receiving telescope. It is clear from Eq. (23) that the mean intensity distribution along the x axis is of Gaussian form. The blurring of plume for the long–exposure measurements of intensity distribution $U(x)$ increases with the increasing ratio σ_x^2/a_x^2.

For large optical thickness ($\tau_0 \gg 1$) and under the condition that $\sigma_x^2 \ll a_x^2$ mean intensity at the plume image axis $\langle U(0)\rangle$ is close to the factor g_G and the distribution $\langle U(x)\rangle$ is not the Gaussian one, obviously.

Figure 1 presents the results of numerical calculations by formula (22) of the mean intensity distribution normalized by the factor q_G for different values of the parameters τ_0 and σ_x/a_x.

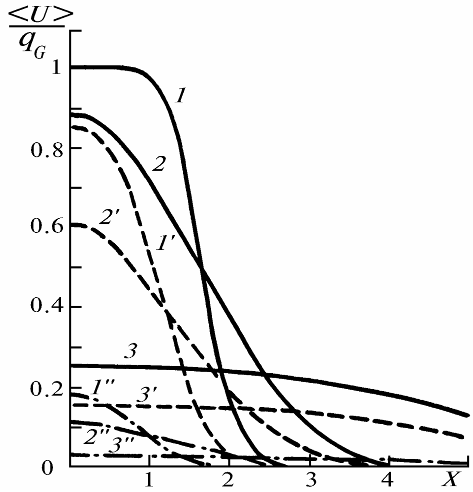

FIG. 1. Mean intensity distribution: 1, 1', and 1'') $\sigma_x/a_x = 0$; 2, 2', and 2'') $\sigma_x/a_x = 1$; 3, 3', and 3'') $\sigma_x/a_x = 5$; 1, 2, and 3) $\tau_0 = 5$; 1', 2', and 3') $\tau_0 = 1$; and 1'', 2'', and 3'') $\tau_0 = 0.1$.

Relative variance of intensity. From Eqs. (1) and (7) we have for the relative variance $\sigma_U^2 = \langle U^2(0)\rangle / \langle U(0)\rangle^2 - 1$

$$\sigma_U^2 = \frac{\langle \exp(-4\tau(0, t))\rangle - \langle \exp(-2\tau(0, t))\rangle^2}{[1 - \langle \exp(-2\tau(0, t))\rangle]^2}\,, \tag{25}$$

where in accordance with Eq. (14)

$$\langle \exp(-n2\tau(0, t))\rangle = \int_{-\infty}^{+\infty} \mathrm{d}\tilde{x}\, P(\tilde{x}) \exp(-n2\tau(0, \tilde{x}))\,, \tag{26}$$

$n = 1, 2$.

The asymptotic formulas can be obtained from Eqs. (25), (26), and (14) under the condition that $\sigma_x^2 \ll a_x^2$:

$$\sigma_U^2 = 2 f_1(\tau_0)\, (\sigma_x^4/a_x^4)\,, \tag{27}$$

where

$$f_1(\tau_0) = [2\tau_0 \exp(-2\tau_0) / (1 - \exp(-2\tau_0))]^2\,, \tag{28}$$

and for $\sigma_x^2 \gg a_x^2$:

$$\sigma_U^2 = f_2(\tau_0)\, (\sigma_x/a_x) - 1\,, \tag{29}$$

where the function f_2 for $\tau_0 \ll \exp(2\,\sigma_x^2/a_x^2)$ is determined by the expression

$$f_2(\tau_0) = 2^{3/2} \sum_{k=1}^{\infty} \frac{(-1)^k}{k!} \frac{(2\tau_0)^k}{\sqrt{k}} (2^{k-1} - 1) \Bigg/ \left(\sum_{k=1}^{\infty} \frac{(-1)^k}{k!} \frac{(2\tau_0)^k}{\sqrt{k}} \right)^2. \tag{30}$$

Functions $f_1(\tau_0)$ and $f_2(\tau_0)$ are presented in Fig. 2. It is clear that the relative variance σ_U^2 decreases with the increasing optical thickness τ_0. This effect is more distinct under the condition $\sigma_x^2 \ll a_x^2$, as it follows from the asymptotic formulas, for small displacements of a plume.

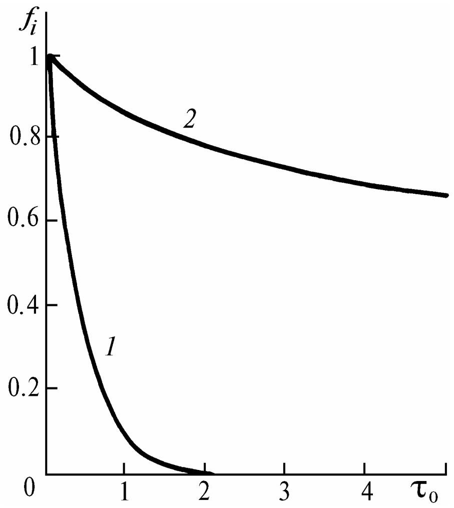

FIG. 2. Functions f_1 (curve 1) and f_2 (curve 2).

Figure 3 illustrates the variance σ_U^2 as a function of the ratio σ_x/a_x (a) and optical thickness τ_0 (b). As it follows from the presented data, starting from the values $\sigma_x/a_x = 2$ the magnitude σ_U^2 can exceed unity.

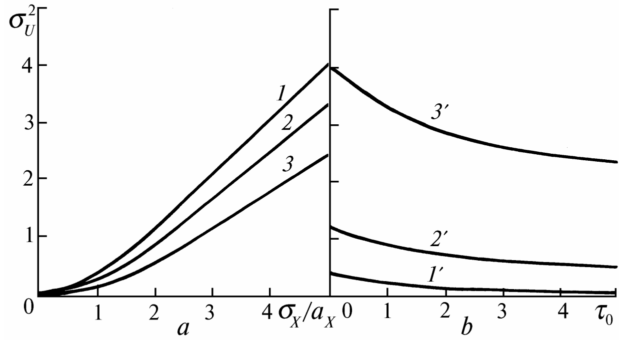

FIG. 3. The relative variance σ_U^2 as a function of the ratio σ_x/a_x (a) and optical thickness of a plume τ_0 (b): 1) $\tau_0 = 0.1$; 2) $\tau_0 = 1$; 3) $\tau_0 = 5$; 1′) $\sigma_x/a_x = 1$; 2′) $\sigma_x/a_x = 2$; 3′) $\sigma_x/a_x = 5$.

Temporal scale of the intensity correlation. For the temporal correlation coefficient $K_U(t_1, t_1) = \langle [U(t_1) - \langle U\rangle][U(t_2) - \langle U\rangle]/[\langle U^2\rangle - \langle U\rangle^2]\rangle$ in the case of $x = 0$ we have from Eqs. (1) and (7)

$$K_U(t_1, t_2) = \frac{\langle \exp(-2\tau(0, t_1)) \exp(-2\tau(0, t_2))\rangle - \langle \exp(-2\tau)\rangle^2}{\langle \exp(-4\tau)\rangle - \langle \exp(-2\tau)\rangle^2}, \tag{31}$$

where, in accordance with Eq. (17),

$$\langle \exp(-2\tau(0, t_1)) \exp(-2\tau(0, t_2))\rangle = \int\limits_{-\infty}^{+\infty}\int \mathrm{d}\tilde{x}_1\, \mathrm{d}\tilde{x}_2\, P(\tilde{x}_1, \tilde{x}_2) \times$$
$$\times \exp(-2\tau(0, \tilde{x}_1)) \exp(-2\tau(0, \tilde{x}_2)) . \tag{32}$$

Taking into account the stationarity of the process under study we obtain the temporal scale of intensity correlation, by analogy with Eq. (21), in the form

$$t_U = \int\limits_0^\infty \mathrm{d}t\, K_U(t) . \tag{33}$$

Using the expression

$$K_x(t) = \exp(-t/t_x) , \tag{34}$$

for the correlation coefficient of plume displacements we obtain from Eqs. (31)–(34), (16), (17), and (26) the following asymptotic formulas:

$$t_U = t_x/2 \tag{35}$$

under the condition $\sigma_x^2 \ll a_x^2$ and

$$t_U = t_x \frac{\ln 2}{\left[f_2(\tau_0)\dfrac{\sigma_x}{a_x} - 1\right]} \tag{36}$$

when $\sigma_x^2 \gg a_x^2$.

Figure 4 presents the results of numerical calculations by Eqs. (33) and (31) with the use of model (34) for the dependence of t_U on the ratio σ_x/a_x and the parameter τ_0. It is clear from Fig. 4 that the temporal scale of intensity correlation decreases with the plume displacement increase (with increasing ratio σ_x/a_x). For this, as it follows from Eqs. (36) and (29), the scale t_U is inversely proportional to the relative variance σ_U^2 for $\sigma_x^2/a_x^2 \gg 1$.

The measurement time T needed for obtaining the mean value $\langle U\rangle$ with a required relative error ε is determined by the known[6,7] relation

$$T = 2\, t_U\, \sigma_U^2/\varepsilon^2 . \tag{37}$$

As follows from Eqs. (27), (29), and (35)–(37) under the condition that $\sigma_x^2 \ll a_x^2$, the time of averaging T can be written as

$$T = (f_1(\tau_0)/\varepsilon^2)\, (\sigma_x^4/a_x^4)\, t_x , \tag{38}$$

and when $\sigma_x^2 \gg a_x^2$, the time T is completely determined, at a given error ε by the time of plume displacements correlation, t_x

$$T = t_x\, 2 \ln 2/\varepsilon^2 . \tag{39}$$

Let the initial characteristics of the pollution source be $a_0 = 0.5$ m, $h = 30$ m, and $\tau_0 = 3$ and neutral stratification of the atmosphere with the parameters $z_0 = 0.3$ m, $y_0 = 30$ m, $V = 10$ m/s, and $t_x = t_E = 3$ c[10] occur. Then in order to obtain an average smoke plume image near its axis with the error $\varepsilon = 0.1$ for the above conditions at 30 m from the source, it is necessary to carry out measurements during about three minutes. When the averaging time is shorter, the spectral brightness of a plume image can fluctuate and the variance of fluctuations can reach in the limit the values $\sigma_U^2 = 4$, as it follows from the results presented in Fig. 3. This should be taken into account when developing a technique of determining concentration of particles in plumes from remote optical measurements.[1,11]

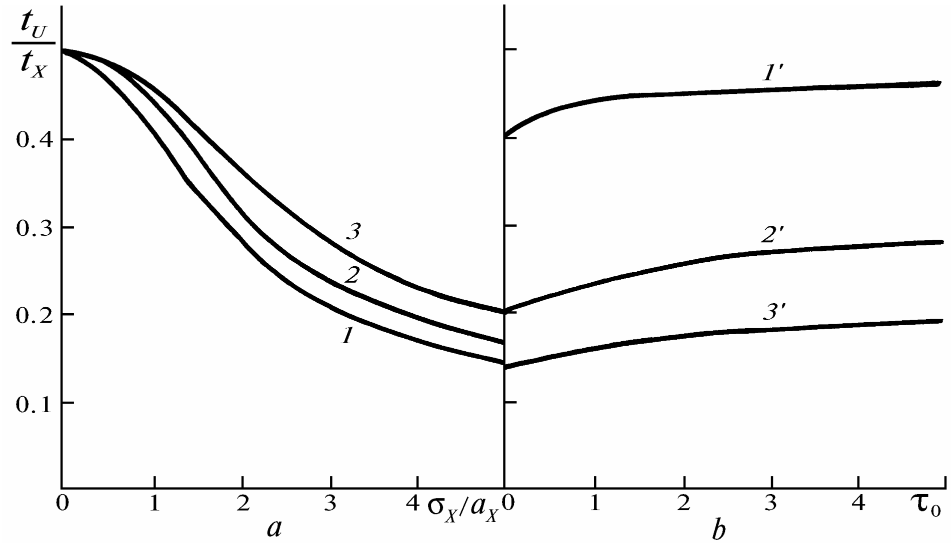

FIG. 4. The integral scale of intensity fluctuations correlation as a function of σ_x/a_x *(a) and* τ_0 *(b): 1)* $\tau_0 = 0.1$, *2)* $\tau_0 = 1$, *3)* $\tau_0 = 5$, *1')* $\sigma_x/a_x = 1$, *2')* $\sigma_x/a_x = 3$, *and 3')* $\sigma_x/a_x = 5$.

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