THEORETICAL INVESTIGATION OF THE ABSORPTION OF OPTICAL RADIATION BY ORIENTED ICE PLATES IN THE IR RANGE

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The absorption coefficient α_{abs} *and the single scattering albedo* Λ *are theoretically investigated at different wavelengths in the IR range for the scattering volume consisting of the system of oriented ice plates. An approximation algebraic formula is derived for* α_{abs}. *It is shown that* α_{abs} *in the IR range duplicates all the peculiarities of fine structure in the dependence of ice absorption coefficient on the wavelength and becomes comparable to the scattering coefficient. It is found that* Λ *being identically equal to* 1 *in the visible range, can take the values from the interval* 0.5–0.8 *in the IR range.*

In Ref. 1 we investigated the extinction coefficient of the optical radiation that passed through a system of oriented ice plates. It was shown that the neutral behavior of the extinction coefficient in the visible range is broken in the IR range. The irregular part of the extinction coefficient whose value depends on the wavelength is connected with the complex refractive index of ice $\tilde{n} = n + i\kappa$. The values of $n - 1$ and κ for ice are comparable in the IR range,[2] so the refractive n and absorption κ indices equally affect the extinction coefficient.[1] Therefore the significant part of the intensity removed from the incident flux in the IR range by the system of ice plates is lost due to absorption. In this paper we perform the comparative estimate for the scattering volume consisting of the system of oriented ice plates.

Let us analyze the extinction and absorption characteristics when a wave propagates in the direction along the normal to the orientation plane of plates. Such a formulation of the problem makes it possible to reveal most easily the general features of the interaction of wave with the system of oriented ice plates. On the other hand, it is precisely the case in which we succeeded in obtaining the approximation algebraic formulas for the extinction coefficient α_{ext} and the absorption coefficient α_{abs}.

In the case in which a plane wave is incident normally on the base of a plate, the formulas for the extinction and absorption cross sections obtained by the physical optics method have the form[3]

$$\sigma_{ext} = 2\pi a^2(1 - \mathrm{Re}(T)), \quad (1)$$

$$\sigma_{abs} = \pi a^2(1 - |T|^2 - |R|^2), \quad (2)$$

where T and R are Fresnel's coefficients of transmission and reflection of a plane wave incident normally on a semitransparent layer, and a is the plate radius.

We have shown in Ref. 1 that when determining the extinction coefficient for the system of ice plates in the wavelength range 1–15 μm, it is sufficient to take into account only the refractive beams that have passed once through the plate thickness. Obviously, analogous conclusion can be made for the absorption coefficient. So in formulas (1) and (2) for the extinction and absorption cross sections we drop the terms that describe multiple internal reflections of electromagnetic field occurring between the plate bases. As a result, Eqs. (1) and (2) are reduced to a form

$$\sigma_{ext} = 2\pi a^2\{1 - \mathrm{Re}(t \exp[i\,k\,d(\tilde{n} - 1)])\}, \quad (3)$$

$$\sigma_{abs} = \pi a^2(1 - |r| - |t|^2 \exp(-2\,k\,d\kappa)), \quad (4)$$

where d is the plate thickness, $k = 2\pi/\lambda$ is the wave number, and t and r are the complex values defined by the expressions

$$t = \frac{4\tilde{n}}{(\tilde{n} + 1)^2}, \quad r = \left(\frac{\tilde{n} - 1}{\tilde{n} + 1}\right)^2. \quad (5)$$

The well–known empirical relationship between the thickness d and the diameter $2a$ of the plate crystal[4] of the form $d = B(2a)^{\alpha}$, where $B = 2.02$ and $\alpha = 0.449$, makes it possible to represent the integral characteristics of the scattering volume in the form of single integrals. In particular, for the extinction and absorption coefficients we have

$$\alpha_{ext} = N\int\limits_0^{\infty} f(a)\,\sigma_{ext}\,\mathrm{d}a, \quad (6)$$

$$\alpha_{abs} = N\int\limits_0^{\infty} f(a)\,\sigma_{abs}\,\mathrm{d}a, \quad (7)$$

where N is the plate number density in the scattering volume, $f(a)$ is the plate radius distribution function. If we assume that $f(a)$ obeys the gamma distribution, then each from integral expressions (6) and (7) can be reduced to the algebraic form. We obtained the algebraic formula for the extinction coefficient α_{ext} in Ref. 5. Let us represent it here in the form

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$$\alpha_{ext} = 2\,C\left(1 - \mathrm{Re}\left(\frac{t}{(1 + \mathrm{k}\,k\,x_2 - i(n-1)\,k\,x_2)^{x_1+1}}\right)\right), \qquad (8)$$

where $C = N\,\frac{\mu + 2}{\mu + 1}\,\pi \overline{a}^2$.

The average plate radius $\overline{a}$ is related to the parameters of the gamma distribution a_m and μ by the formula $\overline{a} = a_m(\mu + 1)/\mu$. The parameters x_1 and x_2 are uniquely related to μ and $\overline{a}$ and can be determined by minimization of the functional representing the difference between new and old plate size distribution functions.[5] A number of values x_1 and x_2 corresponding to some actual average radii $\overline{a}$ of ice plates and the degree μ of concentration of their radii around the average value were presented in Ref. 1.

The algebraic expression for the absorption coefficient similar to Eq. (8) can readily be derived by the procedure described in Ref. 5. A little manipulation yields

$$\alpha_{abs} = C\left(1 - |r| - |t|^2\,\frac{1}{(1 + 2\,\mathrm{k}\,k\,x_2)^{x_1+1}}\right) . \qquad (9)$$

Such an important characteristic of the scattering volume as the single scattering albedo Λ may be represented in terms of the extinction and absorption coefficients. It has the form

$$\Lambda = \frac{\alpha_{ext} - \alpha_{abs}}{\alpha_{ext}} . \qquad (10)$$

Let us numerically investigate two characteristics of the scattering volume, namely, the absorption coefficient and the single scattering albedo, taking into account that we have already considered the extinction coefficient in detail in Ref. 1.

The curves $n = n(\lambda)$ and $\kappa = \kappa(\lambda)$ for the refractive and absorption indices of ice constructed on the basis of the results presented in Ref. 2 are shown in Fig. 1. When passing from the visible to the IR range, the value of κ increases by a few orders of magnitude. It should be noted that the values of κ in the IR range remain relatively low and have no effect on the penetration of light through the air––ice interface. But at large optical depths which, as a rule, are characteristic of the atmospheric crystals, the value of κ in the vicinity of $\kappa = 10^{-1}$ becomes high enough to cause the strong absorption of optical radiation. It is easy to check from the analysis of the dependencies of the absorption index κ on the wavelength λ for different parameters of a disperse medium shown in Figs. 2–4. Spectral behavior of the curve $\alpha_{abs} = \alpha_{abs}(\lambda)$ depends primarily on the character of variations in the absorption index κ and weakly depends on the refractive index n. Actually, it follows from the analysis of the curves $\kappa = \kappa(\lambda)$ and $\alpha_{abs} = \alpha_{abs}(\lambda)$ that the positions of their local maximums on the spectral axis practically coincide. The fine structure of the dependence $\kappa = \kappa(\lambda)$ is not only reflected but also becomes more pronounced in the spectral behavior of each curve for the absorption coefficient. On the other hand, even resonance variation of the ice refractive index near $\lambda = 3$ μm insignificantly influences the absorption coefficient. In particular, resonance of the refractive index results only in the appearance of two closely spaced maxima instead of the single maximum in each curve $\alpha_{abs} = \alpha_{abs}(\lambda)$.

The larger thickness corresponds to larger radius of a plate and consequently to stronger absorption. However, the complete absorption of the optical radiation passing into the plate does not occur even for large plates ($\overline{a}$ = 250 μm). For this reason each curve $\alpha_{abs} = \alpha_{abs}(\lambda)$ retains the fine structure and duplicates all the peculiarities of the function $\kappa = \kappa(\lambda)$.

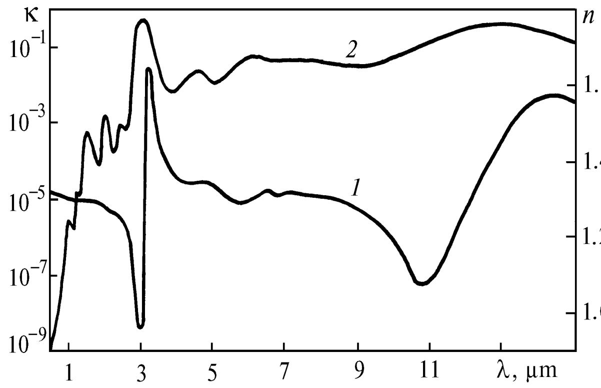

FIG. 1. Refractive and absorption indices as functions of the wavelength: 1) $n = n(\lambda)$; 2) $\kappa = \kappa(\lambda)$.

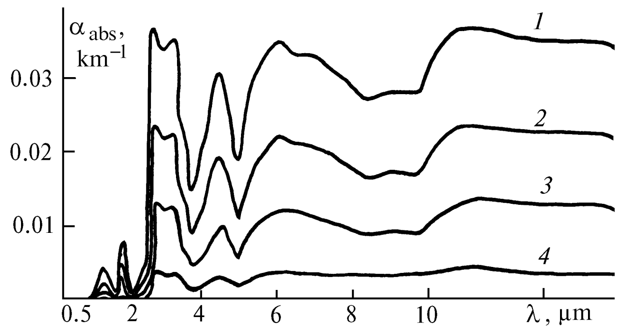

FIG. 2. Absorption coefficients for different average plate radii as functions of the wavelength for $N = 1\ l^{-1}$, $\mu = 5$, and $\overline{a}$ = 100 (1), 90 (2), 60 (3), and 40 μm (4).

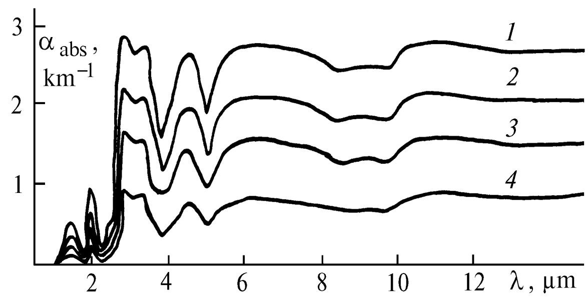

FIG. 3. Absorption coefficients for different average plate radii as functions of the wavelength for $\mu = 5$: 1) $N = 12.5\ l^{-1}$, $\overline{a}$ = 250 μm; 2) $N = 15\ l^{-1}$, $\overline{a}$ = 200 μm; 3) $N = 20\ l^{-1}$, $\overline{a}$ = 150 μm; and, 4) $N = 25\ l^{-1}$, $\overline{a}$ = 100 μm.

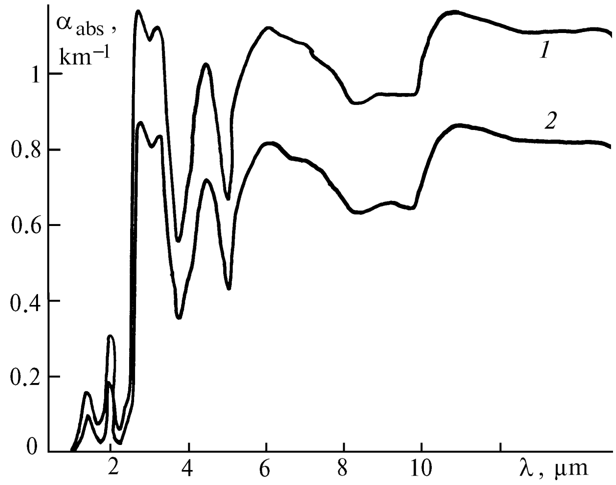

FIG. 4. Absorption coefficients for different values of the parameter μ *as functions of the wavelength.* $N = 25\ l^{-1}$, $\overline{a} = 100$ μm, *and* μ = 1 *(1) and* 8 *(2).*

In general, the curves $\alpha_{abs} = \alpha_{abs}(\lambda)$ for different $\overline{a}$ and μ duplicate each other but differ in scales. This means that it is necessary to relate the absorption of the optical radiation with the crystal volume rather than with linear dimensions. In other words, the crystal shape should have no effect on the absorption. For this reason the analogous dependences $\alpha_{abs} = \alpha_{abs}(\lambda)$ could be expected for any other shape of ice crystals occupying the same volumes.

The single scattering albedo Λ is shown in Figs. 5–7 as a function of the wavelength λ for different parameters of a disperse medium. Each curve has a stable minimum at λ ≈ 3 μm that corresponds to the greatest value of the ice absorption index κ in the wavelength range under investigation. The value of κ at the wavelength λ ≈ 3 μm is so high that it causes the complete transformation of optical radiation passed into any atmospheric crystal into the Joule heat. The value of κ is not so high at the rest of the wavelengths. For this reason the spectral behavior of the curves Λ = Λ(λ) as a whole substantially depends on the optical thickness of crystals. In particular, the greater values of the single scattering albedo Λ correspond to the smaller plates. It should be noted that in analogy with the absorption coefficients α_{abs}, the fine structure of the function κ = κ(λ) is reflected in the principal extrema of the curves Λ = Λ(λ).

It follows from the analysis of the dependences shown in Figs. 5–7 that the scattering volumes containing the atmospheric crystals must accumulate the intensity of the IR optical radiation in the form of heat energy. In this case we cannot say about a fixed general level of the absorbed intensity since its value significantly depends both on the wavelength and the average crystal volume.

In this paper we have performed the comparative analysis of the values of absorption coefficients α_{abs} calculated by three formulas, i.e., approximation formula (9) and integral representation (7) in which we have used two kinds of the absorption cross section given by Eqs. (2) and (4). Recall that the highest multiplicities of internal reflections are ignored in Eq. (4) and this form of the absorption cross section provides the basis for approximation formula (9) for the absorption coefficient.

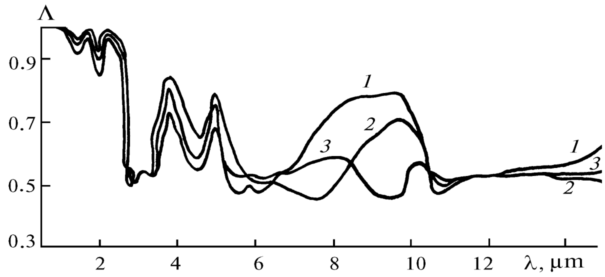

FIG. 5. Single scattering albedos for different average plate radii $\overline{a}$ *as functions of the wavelength.* μ = 5 *and* $\overline{a}$ = 50 *(1),* 100 *(2), and* 250 μm *(3).*

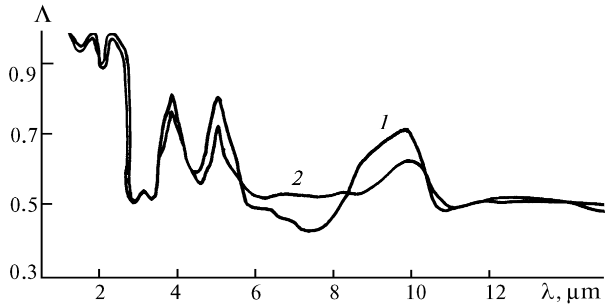

FIG. 6. Single scattering albedos for different values of the parameter μ *as functions of the wavelength.* $\overline{a}$ = 100 μm *and* μ = 8 *(1) and* 1 *(2).*

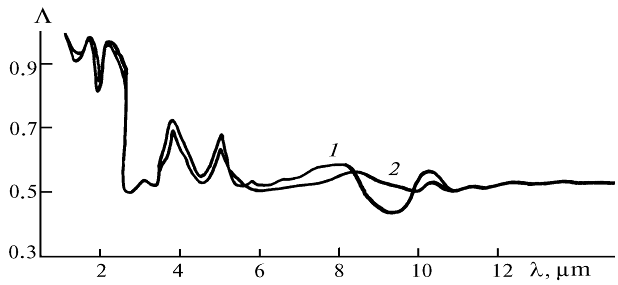

FIG. 7. Single scattering albedos for different values of the parameter μ *as functions of the wavelength.* $\overline{a}$ = 250 μm *and* μ = 8 *(1) and* 1 *(2).*

It is found that the values of the absorption coefficient α_{abs} calculated by different formulas at the same wavelength differ in no more than 1%. The greatest deviations of α_{abs} are obtained in the wavelength range with the small absorption index κ of ice (λ ≤ 2 μm), i.e., in the wavelength range in which it is necessary to take into account the internal reflections of higher multiplicity. In other words, neglect of the internal reflections results in larger error than the approximation of the integral expression for α_{abs} by the algebraic formula. The small error introduced by approximation formula (9) is due to the fact that, in contrast to analogous relation (8) for the extinction coefficient α_{ext}, it does not contain oscillating terms, all other factors being the same. The absorption coefficient yields no information about the crystal shapes but is connected with crystal volumes. Therefore approximation formula (9) derived for the particular case of scattering volume consisting of the oriented ice plates has wider field of application, i.e., it can be used for estimating the intensity of optical radiation absorbed by atmospheric crystals of arbitrary shape.

Summing up the aforementioned, we may conclude that strong absorption of optical radiation in the IR range by ice crystals does not allow us to neglect it even in the simplest interpretation schemes describing the radiation budget in the atmosphere.

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