QUALITY OF IMAGING OF OBJECTS ILLUMINATED WITH A COHERENT LIGHT IN A RANDOMLY INHOMOGENEOUS MEDIUM

V.A. Banakh and B.N. Chen

Institute of Atmospheric Optics,
Siberian Branch of the Russian Academy of Sciences, Tomsk
Received July 7, 1994

The effect of correlation of counter waves on the quality of imaging through a randomly–inhomogeneous medium is analyzed in the paper. It is shown that correlation of the wave incident on the object and reflected one causes an improvement of a point–source image quality as well as an improvement in resolution of the image of a two–point object. The effect becomes stronger in the case when the transmitting aperture and the aperture of receiving telescope are close.

It is known that the spatial and temporal variations of the refractive index of a random medium cause the distortions of an object image. The image is blurred, its contour becomes indistinct, and the information on small–scale details of an object is lost.

Great attention is paid to the problem of imaging through a turbulent medium. The works in this field conditionally fall into three main classes. First of them includes the works devoted to study of the long–exposure incoherent imaging of emitting or illuminated objects.[1] Analysis of imaging in these works is based on a convolution of the object intensity distribution with the long–exposure point spread function (PSF) of the system "telescope+random medium". The problem in this case is reduced to estimating the long–exposure PSF of the system "telescope+random medium".

Second class integrates studies of short–exposure images of sources of incoherent light or incoherently illuminated objects. Applied researches in this direction are mainly based on Labeyrie method.[2,3]

The next class of works can be related to the field of coherent imaging,[4] where we are concerned with the images of objects illuminated with a coherent light. One of the directions of such investigations is the coherent imaging when the light incident on an object and light, received by a telescope, pass along the same path. In this case, the direct and return waves correlate due to passage through the same inhomogeneities of a random medium.[5–10] The effect of correlation of counter waves on a "quality" of coherent imaging through turbulent medium is under study in this paper.

Consider now the object shown in Fig. 1 with amplitude reflection coefficient described by the function $O(\rho', \mathbf{r})$. Here ρ', and $\mathbf{r}$ are the 2D–vectors lying in the plane perpendicular to the direction of light propagation. This object is illuminated with a coherent light from a source located at a distance L from it. Light field distribution in the plane of emitting aperture of the source is described by the function $U_0(t)$. The object is viewed with a telescope from distance L with the amplitude transmission function over its aperture $T(\rho)$.

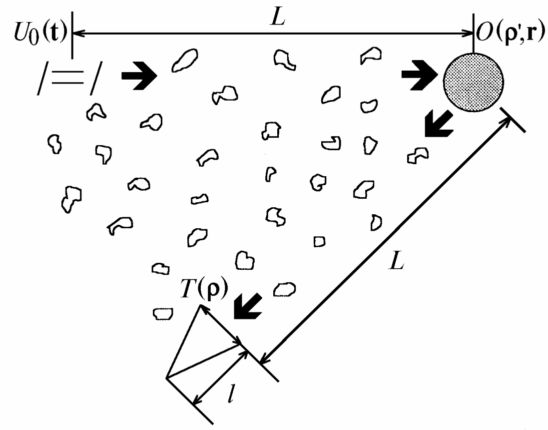

FIG. 1. Imaging scheme

Let us define the image quality as a functional[10]

$$\theta(l) = \int d^2\omega\, \omega^2\, |N(l, \omega)|^2 \Big/ \int d^2\omega\, |N(l, \omega)|^2. \qquad (1)$$

Here $N(l, \omega)$ is the scaled spatial spectrum of mean intensity of light in the image plane l behind the telescope lens

$$N(l, \omega) = S(l, \omega) / S(l, 0),$$

$S(l, \omega) = \int d^2\rho'' \langle I(l, \rho'')\rangle \exp(i\omega\rho'')$ is the spatial spectrum of the mean intensity, ω is the spatial frequency. The mean intensity $\langle I(l, \rho'')\rangle$ in the plane l behind the telescope lens is described by the following expression[11]

$$\langle I(l, \rho'')\rangle = \left(\frac{\kappa}{2\pi l}\right)^2 \int d^2t_{1,2}\, U_0(\mathbf{t}_1)\, U_0^*(\mathbf{t}_2) \int d^2\rho'_{1,2} \times$$

$$\times \int d^2r_{1,2} \langle O(\rho'_1, \mathbf{r}_1)\, O^*(\rho'_2, \mathbf{r}_2)\rangle \int d^2\rho_{1,2}\, T(\rho_1)\, T(\rho_2) \times$$

$$\times G_d(x,x_0;\rho_1',\mathbf{t}_1)\,G_d^*(x,x_0;\rho_2',\mathbf{t}_2)\,G_b(x_0,x;\rho_1,\mathbf{r}_1)\times$$

$$\times G_b^*(x_0,x;\rho_2,\mathbf{r}_2)\rangle\exp\left[\frac{i\kappa}{2l}\left(1-\frac{l}{F_t}\right)(\rho_1^2-\rho_2^2)-\frac{i\kappa}{l}(\rho_1-\rho_2)\rho''\right], \quad (2)$$

where $G_d(x, x_0; \rho, \mathbf{t})$ and $G_b(x_0, x; \rho, \mathbf{r})$ are Green's functions for the line–of–sight propagation from the source to the object and backward from the object to the telescope, respectively; F_t is the focal length of the telescope; $\kappa = 2\pi/\lambda$ is the wave number; x_0 determines the position of the source plane on the x' axis, and x determines the object plane.

In what follows we will consider the turbulent atmosphere with Kolmogorov spectrum of the refractive index fluctuations causing strong intensity fluctuations on the propagation path as a random medium.[11] The optical source is assumed to emit a Gaussian beam with the effective radius a and the wave–front curvature radius F. For the function $T(\rho)$ we also use the Gaussian model with the effective radius a_t.

It is known[11] that for the strong intensity fluctuations we can write

$$\langle I(l,\rho'')\rangle = \langle I(l,\rho'')\rangle_1 + \langle I(l,\rho'')\rangle_2,$$
$$S(l,\omega) = S_1(l,\omega) + S_2(l,\omega). \quad (3)$$

The first term in Eq. (3) describes the mean intensity of the beam in the case of no correlation between the incident and reflected waves. The second term is due to correlation of the counter waves, and it determines specific features of images of coherently illuminated objects as compared to the images of objects illuminated incoherently.

The influence of processes described by the second term in Eq. (3) on the quality of imaging is more convenient to be studied for the case of a point object. The spatial spectrum, $S_0(l, \omega)$, of a point object is a constant. Inhomogeneous medium and diffraction on the apertures of the telescope and the illuminating source play the role of high–frequency filters. Therefore the spatial spectrum of the object image is obtained from the object spatial spectrum as a result of the filtration of its high–frequency components.

For the point object, we have

$$O(\rho, \mathbf{r}) = 4\pi/\kappa^2\,\delta(\mathbf{r})\,\delta(\rho - \mathbf{r}), \quad (4)$$

where $\delta(\mathbf{r})$ is the Dirac delta–function. Having substituted Eq. (4) into Eq. (2) for the term $S_1(l, \omega)$ in Eq. (3) we obtain

$$S_1(l,\omega)=\text{const}(g^2+2p)^{-1}\exp\left[-\omega^2/\omega_0^2\,(1+\Omega_t^2Q^2+2p\Omega_t/\Omega)\right], \quad (5)$$

where $\omega_0 = 2a_t\kappa/l$; $g^2 = 1+\Omega^2(1-L/F)^2$; $Q = 1+L(1/l-1/F_t)$; $p = 2\Omega/(3g)$; $\Omega = \kappa a^2/L$, and $\Omega_t = \kappa a_t^2/L$ are the Fresnel numbers of the transmitting aperture a_r and of the lens of the telescope a_t, respectively, $q = 0.82\beta_0^{-12/5}$; $\beta_0^2 = 0.31C_\varepsilon^2\,\kappa^{7/6}L^{11/6}$ is the parameter characterizing the turbulent conditions of propagation along the atmospheric path, C_ε^2 is the structure constant of the refractive index fluctuations. In the considered case of strong intensity fluctuations, the parameter β_0^2 exceeds unit significantly.

As follows from Eq. (5), the best image of the point object is formed in the plane l^* behind the telescope lens, for which the relation[12]

$$Q = 1 + L/l^* - L/F_t = 0. \quad (6)$$

is valid. In optics the relationship (6) is known as the formula for a thin lens, and it determines the plane of sharp image of the point object. The amplitude of the component $S_1(l, \omega)$ is proportional to $\beta_0^{-12/5}$, and the characteristic spatial scale of its decrease in the transverse plane is of the order of

$$\omega_1 \sim \omega_0\,(1 + Q^2 + 2p\,\Omega_t/\Omega)^{-1/2}.$$

The component $S_2(l, \omega)$ is expressed in the form

$$S_2(l,\omega) =$$
$$=\text{const}\,A\,B\left[\Omega^2\left(1-\frac{L}{F}\right)^2+A\,\Omega\,\Omega_t\left(1-\frac{L}{F}\right)^2p^2+B^2\right]^{-1}\times$$
$$\times\exp\left\{-\frac{\omega^2}{\omega_0^2}\left[C+A\,B\,\Omega_t^2\,Q^2-B\,p^2\,\Omega_t\,\Omega^{-1}\left[\Omega^2\left(1-\frac{L}{F}\right)^2+\right.\right.\right.$$
$$\left.\left.\left.+A\,\Omega\,\Omega_t\left(1-\frac{L}{F}\right)^2p^2+B^2\right]^{-1}\left[1+A\,\Omega\,\Omega_t\left(1-\frac{L}{F}\right)Q\right]^2\right]\right\}, \quad (7)$$

where the following designations are introduced:

$$A = [1 + p(1 + \Omega_t/\Omega)]^{-1};\quad B = 1 + p;\quad C = 1 + p\,\Omega_t/\Omega.$$

It follows from Eq. (7) that $S_2(l, \omega)$ is proportional to the quantity $\beta_0^{-24/5}$, and the characteristic scale of its decrease ω_2 is proportional to ω_0. Thus, although the amplitude of the component S_2 is essentially lower than that of the component S_1 at $\beta_0^2 \gg 1$, the characteristic scale of its decrease far exceeds the characteristic scale of the component S_1 decrease ($\omega_2/\omega_1 \sim (1 + \Omega_t^2Q^2 + 2p\,\Omega_t/\Omega)^{1/2} \gg 1$).

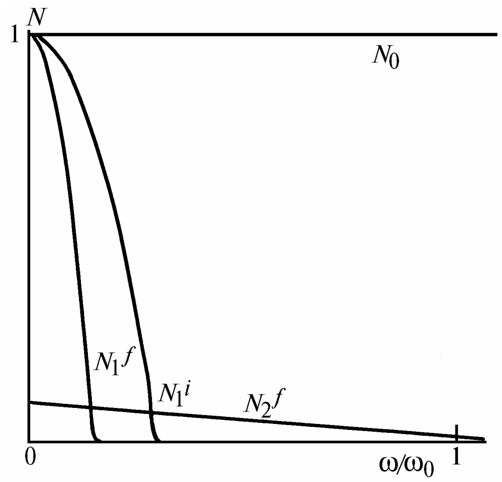

FIG. 2. Schematic illustration of the terms S_1 and S_2 behavior.

The behavior of S_1 and S_2 is illustrated schematically in Fig. 2. Here N_0 is the normalized spatial spectrum of the point object, $N_1^i = S_1(l^*, \omega)/S_1(l, 0)$ and $N_1^f = S_1(F_t, \omega)/S_1(l, 0)$ are the spatial spectra of this object image without considering the correlation of counter waves in the plane of sharp image ($l = l^*$) and in the focal plane of the telescope ($l = F_t$), respectively. $N_2^f = S_2(F_t, \omega) / S_1(l, 0)$ is the relative contribution of the second component to the spatial spectrum of the object image in the focal plane of the telescope.

As the image size in the plane l^* is minimum, the width of the spectrum S_1 in this plane $S_1(l^*, \omega)$ exceeds that in the focal plane $S_1(F_t, \omega)$. At the same time, it follows from Fig. 2 that the component $S_1(F_t, \omega)$ contains the information on high–frequency portion of the spatial spectrum of the point object, filtered by inhomogeneous media in the absence of the counter waves correlation. Consequently, the account of S_2 must lead to improvement of the object image quality in the focus of telescope.

This fact was tested by calculations. Having used the Eqs. (1), (5), (7) we calculate the functional $\theta_1(l^*)$, which characterizes the point–object image quality in the sharp–image (conjugated) plane without consideration of the counter wave correlation, and the function $\theta(F_t)$, which characterizes the image quality of the same object in the focal plane of the telescope accounting for counter wave correlation. Let the parameter M be defined as the ratio of the quantity $\theta(F_t)$ to the $\theta_1(l^*)$. It is clear that $M > 1$ corresponds to improvement of the image quality, and $M < 1$ corresponds to worsening of the image quality.

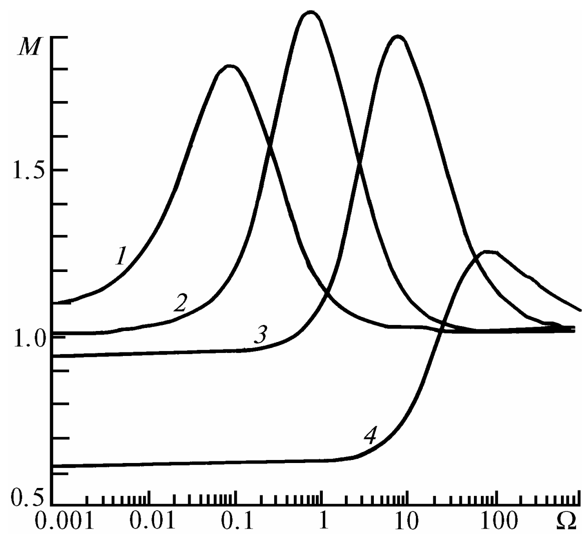

FIG. 3. Dependence of the quantity $M=\theta(F_t)/\theta_1(l^*)$ for a point object on the Fresnel ratio of the aperture of a source of coherent light: $\Omega_t = 0.1$ (1); 1 (2); 10 (3); 100 (4).

Figure 3 presents the dependence of M on the Fresnel number of the source illuminating the point object with coherent light at different Fresnel numbers of the telescope lens. The parameter β_0^2 was equal to 50.

From Fig. 3, it is clear that the improvement of image quality according to the criterion (1) takes place when the apertures of the coherent source and the telescope are equal ($\Omega = \Omega_t$). In this case, the contribution of the coherent term S_2 is comparable with that of the term S_1. Thus, the correlation of counter waves under conditions of strong intensity fluctuations can lead to improvement of image quality in the focal plane as compared to that in the conjugated plane l^*. The difference in apertures ($\Omega \gg \Omega_t$ or $\Omega \ll \Omega_t$) does not result in an improvement of the image quality.

This has a good physical explanation first presented in Ref. 13 for the mean image intensity. Really, it is clear that the long–range correlation[14] makes the counter rays coherent (correlative) only in the region limited by the size of output aperture a of a optical source. Therefore, a lens of a size less than $2a$ collects not all the coherent rays. Use of a telescope with the lens of larger than $2a$ dimensions results in the relative decrease of the coherent component of scattered radiation $< I(l, \rho'')>_2$ as compared with the increased contribution from the incoherent component $< I(l, \rho'')>_1$.

Now let us examine the effect of correlation of counter waves on the resolution of the coherently illuminated objects viewed in a turbulent atmosphere. Assume the object to be a set of two point scatterers and present the function $O(\rho, \mathbf{r})$ in the following form:

$$O(\rho, \mathbf{r}) = (2\pi/\kappa^2) [\delta(\mathbf{r} - \mathbf{r}_0) + \delta(\mathbf{r} + \mathbf{r}_0)] \delta(\mathbf{r} - \mathbf{r}_0), \quad (8)$$

where $2\mathbf{r}_0$ is the distance between scatterers.

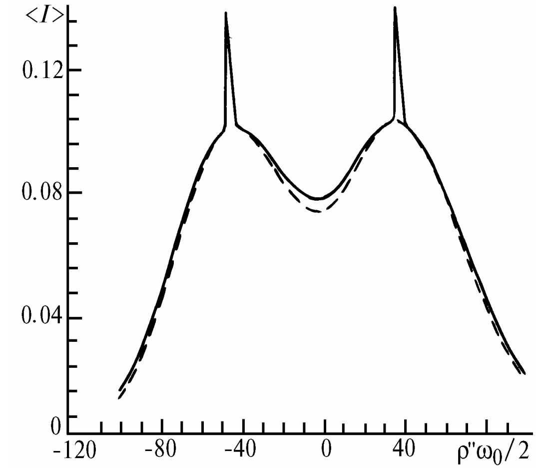

FIG. 4. Intensity distribution in the image of a two–point object at $r_0/\rho_n = 150, \Omega = 10$, $\Omega_t = 10$, and $\beta_0^2 = 50$.

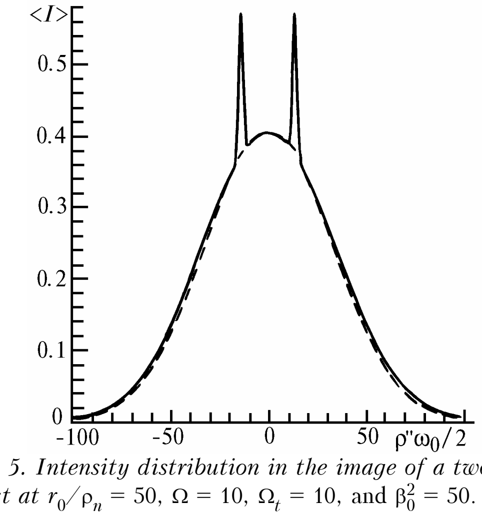

FIG. 5. Intensity distribution in the image of a two–point object at $r_0/\rho_n = 50$, $\Omega = 10$, $\Omega_t = 10$, and $\beta_0^2 = 50$.

Using Eqs.(2) and (8) we can obtain the expression for the mean intensity distribution in the image of a two–point object. Shown in Figs. 4 and 5 are the results of calculations using the obtained expression of such an image intensity distribution in the conjugated plane and the focal plane for different separations between scatterers. The distance in the plane transverse to the optical axis of the telescope scaled by the quantity $l/\kappa a_t$ is used as abscissa.

The dashed curves in Figs. 4 and 5 for the conjugated plane and solid curves without peaks for the focal plane in the same figures correspond to the images of a two–point object being obtained without considering correlation of the counter waves. It is seen from the figures that in this case the resolution in the image decreases with decreasing separation r_0 until completely unresolved picture.

Now consider the image of a two–point object when the wave incident on the object and the reflected one correlate. In this case, as it follows from the results of calculations presented in Figs. 4 and 5, the resolution of the object is improved (see Fig. 4) and it persists even for such separation between points at which the two scatterers become unresolved when viewed in the conjugate plane or in the case of abscence of the counter waves correlation.

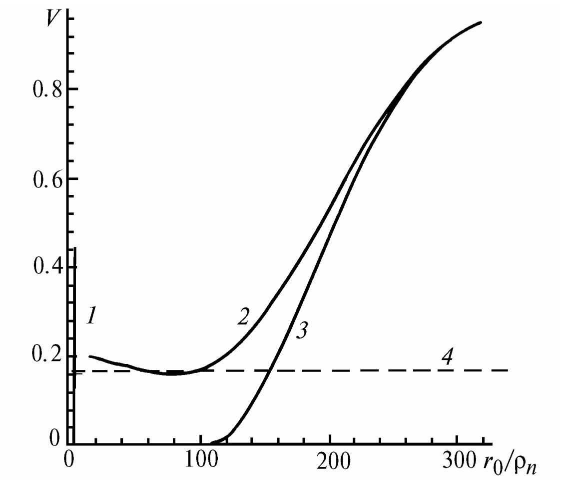

FIG. 6. Visibility function of a two–point object ($\Omega = 10$, $\Omega_t = 10$) *in the plane of shape image at* $\beta_0^2 = 0$, *in the focal plane at* $\beta_0 = 50$ *with and without regard for correlation of counter waves (2 and 3, respectively), and resolution effect by Rayleigh criterion.*

For quantitative estimation of the improvement in the telescope resolution effect due to correlation of counter waves we introduce the function of "visibility" by formula $V = (I_{max} - I_{min}) / (I_{max} + I_{min})$, where I_{max} denotes the maximum intensity in the image, and I_{min} denotes the minimum intensity at $\rho' = 0$.

Figure 6 demonstrates the visibility function for a two–point object corresponding to different imaging schemes. The separation range scaled by the parameter ρ_n is shown in abscissa. Parameter ρ_n is the coherence length of the plane wave passed the path of length L in a turbulent medium:

$$\rho_n^2 = L / (1.22\ \kappa\ \beta_0^{12/5}) .$$

It follows from Fig. 6 that the resolution effect increases essentially due to the correlation of counter waves if the apertures of a source of coherent light and the telescope are close in size.

Thus, the correlation of wave illuminating the object and the wave reflected from it can lead to essential increase of a coherent image quality and resolution effect of a telescope.

REFERENCES

1. J. Goodman, *Statistical Optics* [Russian translation], (Mir, Moscow, 1980), 528 pp.
2. A. Labeyrie, Astronomy and Astrophysics **6**, 85–90 (1970).
3. A.A. Tokovinin, *Star Interferometers* (Nauka, Moscow, 1988).
4. P.A. Bakyt et al., *Theory of Coherent Imaging* (Radio i Svyaz', Moscow, 1987), 264 pp.
5. T. Mavroidis, J.C. Dainty, M.J. Northcott, J. Opt. Soc. Am. A. **7**, No. 3, 348–355 (1990).
6. B.S. Agrovskii et al., J. Opt. Soc. Am.A. **8**, No. 7, 1141–1147 (1991).
7. C.J. Solomon et al., J. Mod. Opt. **38**, No. 11, 1993–2008 (1991).
8. T.Mavroidis, C.J. Solomon, and J.C. Dainty, J. Opt. Soc. Am. A. **8**, No. 7, 1003–1013 (1991).
9. R. Mazar, A. Bronshtein, and L. Kodner, Proc. SPIE **1968**, 249–257 (1993).
10. A.N. Bogaturov, Proc.SPIE **1968**, 270–281 (1993).
11. V.E. Zuev, V.A. Banakh, and V.V. Pokasov, *Optics of Turbulent Atmosphere* (Gidrometeoizdat, Leningrad, 1988), 272 pp.
12. M. Born and E. Wolf, *Principles of Optics* (Pergamon, New York, 1959).
13. V.A. Banakh, Izv. Vyssh. Uchebn. Zaved., ser. Radiofiz. **29**, No. 12, 1507–1509 (1986).
14. Yu.A. Kravtsov and A.I. Saichev, "*Effects of twofold propagation of waves in randomly inhomogeneous media*", Preprint No. 20(382), Institute of Radiotechnics and Electronics, Moscow (1984).