RADIATIVE TRANSFER IN A STRATIFIED SCATTERING MEDIUM

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A behavior of the signal–to–noise ratio δ is investigated vs. the optical dimensions of a medium and probability of photon survival as well as the effect of the underlying surface and external sources when radiation passes through a multilayer medium.

The solution of many applied problems related to vision in scattering media calls for estimating the maximum visibility range depending on optical characteristics of a medium and experimental geometry. Such estimates must start from the concept of optimum detection[1,2] and its specific realization in the form of engineering methods of calculation that offer the final formulas convenient for an analysis. Theoretical description of this problem is based on the solution of the radiative transfer equation derived for media unbounded in the transverse direction (with respect to radiation propagation). Quantitative data for calculating the specific situations in bounded media are obtained by the numerical methods; it makes the complete analysis of the problem difficult. The solution of the problem becomes much more complicated when describing the radiation propagation in layered inhomogeneous or stratified scattering media. The necessity of obtaining this solution stems from the fact that natural media have clearly defined inhomogeneous, in particular stratified, structure. We studied previously the simplest variant of such a structure, namely, a two–layer medium.[3,4] In this paper, we deal with the model of a three–layer scattering medium of finite volume and study the dependence of the signal–to–noise ratio on the optical and geometric parameters of the medium.

Let us consider the scattering medium consisting of three layers, each being a rectangular parallelepiped in form with arbitrary optical lengths of its edges τ_x, τ_y, and τ_z. The direction of radiation propagation coincides with the x axis. The scattering medium is also characterized by the quantum survival probability Λ and the scattering phase function with the asymmetry ratio $a = (\eta + 2\mu)/(\beta + 2\mu)$, where η, β, and μ are the integral parameters of the scattering phase function in a six–flux representation.[6]

In practice, the most widespread method of determining the signal–to–noise ratio δ is the comparison between the energy characteristics of the object under study and the background. We define the signal–to–noise ratio in analogy with Refs. 2 and 5.

The formula for calculating the signal–to–noise ratio can be derived using the recursion relations[4]

$$\delta = \frac{\exp(-\tau_x)\{[1 - r_1(\tau,\Lambda,a)\,r_2(\tau,\Lambda,a)]\,[1 - r_2(\tau,\Lambda,a)\,r_3(\tau,\Lambda,a)] - }{t_1(\tau,\Lambda,a)\,t_2(\tau,\Lambda,a)\,t_3(\tau,\Lambda,a) - \exp(\tau_x)\{[1 - r_1(\tau,\Lambda,a)\times}$$

$$\frac{- t_2^2(\tau,\Lambda,a)\,r_1(\tau,\Lambda,a)\,r_3(\tau,\Lambda,a)\}}{\times\, r_2(\tau,\Lambda,a)]\,[1 - r_2(\tau,\Lambda,a)\,r_3(\tau,\Lambda,a) - t_2^2(\tau,\Lambda,a)\,r_2(\tau,\Lambda,a)\,r_3(\tau,\Lambda,a)\,]\,\}},$$

where τ_x is the total optical depth of the medium; $t_1(\tau, \Lambda, a)$, $t_2(\tau, \Lambda, a)$, $t_3(\tau, \Lambda, a)$, $r_1(\tau, \Lambda, a)$, $r_2(\tau, \Lambda, a)$, and $r_3(\tau, \Lambda, a)$ are the coefficients of transmission and reflection of the 1st, 2nd, and 3rd layers of the scattering medium. These coefficients can be determined by any one of the methods accounting for spatial boundedness of the medium, e. g., by the methods described in Refs. 7–9.

The above formula allows one to study the effect of optical dimensions, photon survival probability, scattering phase function, and sequence of layers on the value δ. The range of variation of the parameters of the medium considered in this paper is as follows: optical depth of each layer is 1–10, scattering phase function changes from spherical one (a = 1) to that of C1 cloud (a = 12), and Λ = 0.5–1. We also studied the effect of reflecting surfaces bounding the medium, background llumination, and receiver aperture.

The behavior of δ normalized to the signal–to–noise ratio for the medium with optical depth $\tau_x = 3$ ($\tau_{xi} = 1$) is considered here for a better comparision of the results. Let us study the behavior of the signal–to–noise ratio depending on the optical dimensions of the medium, the quantum survival probability, the degree of anisotropy of the scattering phase function, and the stratification of the scattering medium. Shown in Figs. 1 and 2 are the plots of the signal–to–noise ratio vs. optical dimensions of the medium. They depict δ as a function of the optical depth of one of the layers of the layered inhomogeneous medium.

It follows from the data obtained that with the optical depth increase, as for a homogeneous medium, δ decreases. This conclusion is valid regardless of the serial number of the layer whose optical dimensions increase. It should be noted that if one and the same layer is placed first as the first layer and then as the last one, the values of δ remain unchanged (all other factors being the same).

The effect of the layered structure is illustrated by Figs. 1 and 2; whence it follows that at constant total optical depth of the medium and variable characteristics of one of the layers, the difference in δ (with the parameters studied) may reach an order of magnitude.

From these figures, it is apparent that the scattering phase function has much less of an effect on the value of the signal–to–noise ratio than the other factors, since the alternation of the layers with different scattering phase functions changes δ by a factor of no more than 1.5–2 (Fig. 1).

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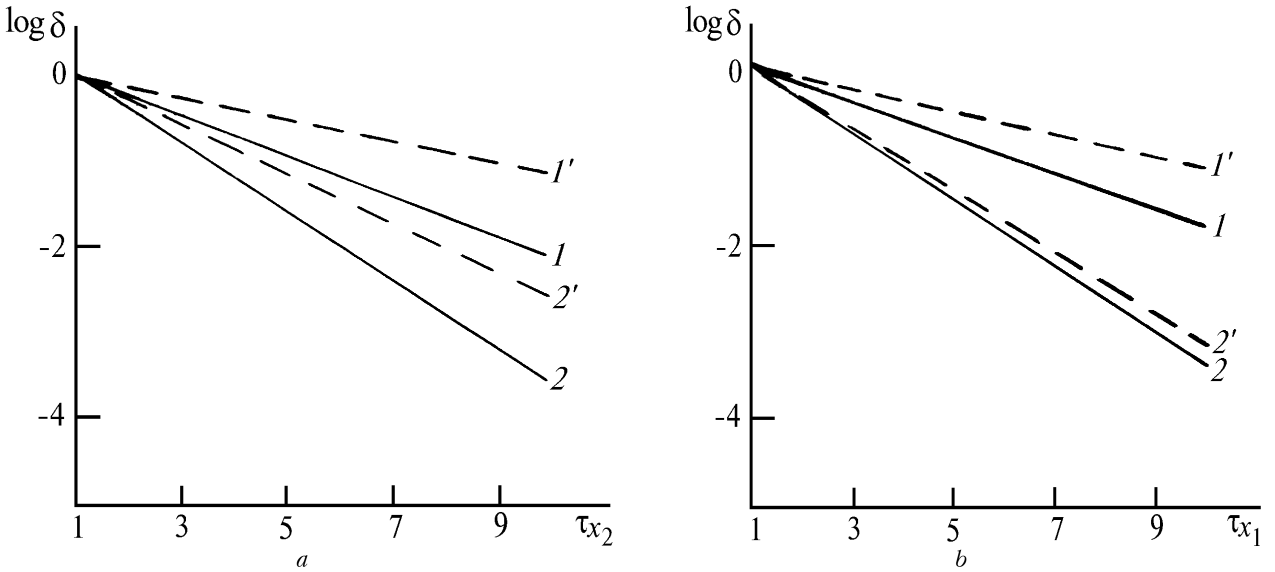

FIG. 1. Signal–to–noise ratio vs. the optical depth of the medium at $\tau_{y,z}=5$: *a)* $a_2=1$, $a_1=a_3=12$ (*1 and 1′*); $a_2=12$, $a_1=a_3=1$ (*2 and 2′*); $\Lambda=1$ (*1 and 2*) and 0.8 (*1′ and 2′*); $\tau_{x_1}=\tau_{x_3}=1$; *b)* $a_2=12$, $a_1=a_3=1$ (*1 and 1′*); $a_2=1$, $a_1=a_3=12$ (*2 and 2′*); $\Lambda=1$ (*1 and 2*) *and* 0.8 (*1′ and 2′*); $\tau_{x_2}=\tau_{x_3}=1$.

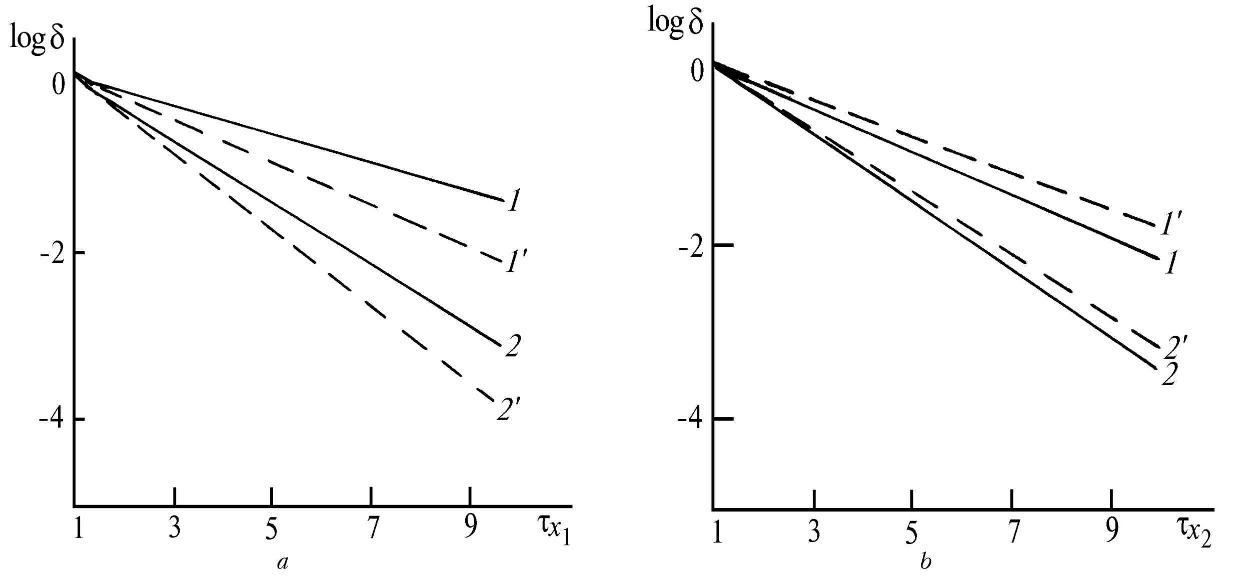

FIG. 2. Signal–to–noise ratio vs. the optical depth of the medium at $\tau_{y,z}=5$: *a)* $a_{1-3}=1$, $\tau_{x_2}=\tau_{x_3}=1$ (*1 and 1′*); $a_{1-3}=12$, $\tau_{x_2}=\tau_{x_3}=1$ (*2 and 2′*); $\Lambda_{1,3}=0.8$, $\Lambda_2=1$ (*1 and 2*), $\Lambda_{1,3}=1$, $\Lambda_2=0.8$ (*1′ and 2′*); *b)* $a_{1-3}=1$, $\tau_{x_1}=\tau_{x_3}=1$ (*1 and 1′*); $a_{1-3}=12$, $\tau_{x_1}=\tau_{x_3}=1$ (*2 and 2′*); $\Lambda_{1,3}=0.8$, $\Lambda_2=1$ (*1 and 2*), $\Lambda_{1,3}=1$, $\Lambda_2=0.8$ (*1′ and 2′*).

The absorption in the medium has the most significant effect on the value δ (Figs. 2 and 3). The increase of the absorption results in the δ increase (see, e.g., Fig. 3) and is independent of the layered structure of the medium. Physical interpretation of this phenomenon is clear. With the absorption increase in the medium, the intensity of multiply scattered light field decreases. As a result, δ increases. In this case the process is independent of the absorbing layer position in the medium.

Let us analyze the behavior of the signal–to–noise ratio as a function of the optical dimensions of the layered inhomogeneous medium, the reflection coefficient of surfaces, the strength of external radiation sources, and the receiver aperture. The strength of external sources in calculations changed by three orders of magnitude from $0.1\,I_0$ to $100\,I_0$. Here I_0 is the intensity of a reference signal. The medium was characterized by the following parameters:

the 1st layer: $\tau_{x_1}=1$, $\tau_{y_1}=\tau_{z_1}=5$, $a_1=1$, $\Lambda_1=1$;

the 2nd layer: $\tau_{x_2}=1$, $\tau_{y_2}=\tau_{z_2}=5$, $a_2=12$, $\Lambda_2=1$;

the 3rd layer: $\tau_{x_3}=1–10$, $\tau_{y_3}=\tau_{z_3}=5$, $a_3=1$, $\Lambda_3=1$.

The calculational results reveal that the value δ decreases with the increase in optical depth of any layer.

The increase in external source strength also results in the δ decrease.

Moreover, at sufficiently large depth of one of the layers and hence of the entire medium, the effect of the external sources becomes insignificant. This is true for a normal illumination from external sources with respect to the direction of reference signal .

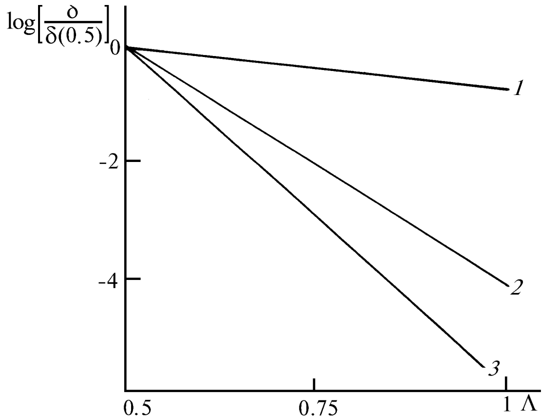

FIG. 3. Signal–to–noise ratio vs. the photon survival probability: $\tau_{y,z} = 5$, $\tau_{x_2} = \tau_{x_3} = 1$, $a_{1,3} = 1$, *and* $a_2 = 12$. *Here* $\tau_{x_1} = 1$ (*1*), 5 (*2*), *and* 10 (*3*).

An analysis of the δ dependence on the reflection coefficient of lateral surfaces bounding the scattering medium shows that the increase in this coefficient is analogous in its action to the increase in the transverse optical dimensions of the medium and consequently results in the δ decrease.

The receiver aperture decrease results in a sharp increase of the normalized signal–to–noise ratio, what can be explained by the decrease in multiply scattered portion of light entering a receiver. This is true if we ignore limitations imposed by the system sensitivity. It should be added that all conclusions have been drawn for instrumental recording of radiation. For visual observation, one must take into account physiological peculiarities of eyesight.

The analysis of the investigation results enables us to conclude the following.

The signal–to–noise ratio decreases due to increase in the optical dimensions of a medium, reflection coefficients of surfaces bounding the medium, strength of external sources, and receiver aperture.

The signal–to–noise ratio is improved with the absorption increase in the medium (as in the case of improving the quality of photography when photographic material is made black), less sharply pronounced stratification, and lesser degree of anisotropy of the scattering phase function.

Thus the joint action of all factors is a multiparameteric problem, whose solution calls for the development of a special algorithm.

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