SOME PROBLEMS OF COMPENSATION FOR THE DISTORTIONS OF OPTICAL BEAMS. SELF-ACTION AND RANDOM DISTORTIONS OF PROFILED BEAMS

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Propagation of profiled light beams through a nonlinear or turbulent medium is investigated in the approximation of optically thin layer. Regions have been identified where transfer to the profiled beams causes a decrease of their radius in the receiving plane. The standard deviation of the beam center of gravity has been derived as a function of the correlation length for the density fluctuations in the turbulent layer of a medium.

INTRODUCTION

Profiling of optical beams is well known to be a method of control of the beam parameters, for example, of compensation for the nonlinear beam distortions. The profiled beams were not studied in detail for a long time, apparently, because of sophisticated practical realization of their amplitude profile in experiments. However, in recent years several techniques have been proposed to create a desired pulse shape and beam profile in real systems,[1–5] including the control of the optical beam wave front.[4] Therefore, interest in an analysis of mechanisms of non-Gaussian beam propagation has increased.[6–11] Various problems of self–action of profiled optical beams were also considered analytically and by numerical modeling in our previous papers (see Refs. 7 and 12–22), where it was shown in particular that the profiled beams undergo much less distortions as compared with the Gaussian beams. Note that numerical experiments were carried out with the use of the difference schemes constructed and substantiated by us and outlined, for example, in Ref. 23.

Two problems that have not yet been discussed are considered in the present paper: 1) optimization of the initial radius of the profiled beam and 2) its distortions during propagation through a layer of a turbulent medium.[19] It should be noted that the efficiency of optimization of the Gaussian beam radius as applied to the problems of light energy transfer was discussed in Refs. 24 and 25.

1. ON THE EFFICIENCY OF OPTIMIZATION OF THE PROFILED BEAM RADIUS

To analyze the distortions of optical beams that have passed through a thin nonlinear layer, we make use of the formulas derived in Ref. 26 for calculation of the position of the beam center of gravity X_c and beam radii a_x and a_y

$$X_c(z) = \frac{z}{2Q^2} \iint\limits_{-\infty}^{\infty} S_x f^2 \, dx \, dy , \qquad (1)$$

$$a_x^2 = a_x^2(0) + \frac{z^2}{4Q^2} \iint\limits_{-\infty}^{\infty} f_x^2 dx dy + \frac{z}{2Q^2} \iint\limits_{-\infty}^{\infty} f^2 \{2xS_x + \frac{z}{2} S_x^2\} \, dx dy . \qquad (2)$$

The formula for a_y is analogous to Eq. (1) with substitution of $\partial/\partial x$ by $\partial/\partial y$. Here z is the beam propagation coordinate normalized by $l_d = ka^2/2$; k is the wave number; a is the initial beam radius; $f(x, y)$ is the beam profile upon entering a nonlinear medium; $S(x, y)$ is the beam phase after it has passed through a nonlinear layer; $S = S_c + S_{nl}$, S_{nl} is the additional run–on of the phase upon exiting the medium; S_c is the phase of the beam in the cross section $z = 0$; Q is the f distribution norm; x, y are the transverse coordinates normalized by a. In the case of axial symmetry it is expedient to transfer to the radius $r^2 = x^2 + y^2$.

Upon entering the nonlinear layer, the light has either hyper-Gaussian or hyperhollow profile, respectively:

$$f_G = \exp\{-b(x^m + y^m)\} , \qquad (3)$$

$$f_h = (x^m + y^m) f_G . \qquad (4)$$

Note that $b = 1$ corresponds to the amplitude profile of a reference beam subsequently used to compare the optical beam parameters.

Substituting the expressions for f_G and f_h into Eq. (1), it is not difficult to derive the following dependence for the position of the center of gravity of the optical beam with the plane phase front in the cross section $z = 0$ that has passed through the layer of a moving medium with thermal nonlinearity

$$X_{c\,G}(z) = \frac{\theta_{nl}\, z}{2^3} \frac{b^{2/m}}{I(m)} , \quad X_{c\,h}(z) = \frac{z\,\theta_{nl}}{2^8} \frac{b^{2/m}}{I(m)} \chi(m) , \qquad (5)$$

where θ_{nl} is the excess beam divergence,

$$I(m) = \Gamma^2(1 + 1/m) , \quad \chi(m) = \frac{m(2 + 2/m)(3 + 2/m)}{1 + 2/m} , \qquad (6)$$

$\Gamma(x)$ is the gamma-function. The functions $I(m)$ and $\chi(m)$ are tabulated in Table I (the function $\delta(m)$ is described below). One can see directly from Table I that the displacements of hyperhollow beams with m being equal to 2 and 4 are smaller by 3 and 1.5 times than the displacements of the corresponding Gaussian beams. It is essential that the position of the center of gravity as a function of b obeys the same law for both f_G and f_h. If $b > 1$, then with increase of m the parameter $b^{2/m}$ decreases. If $b < 1$, the reverse dependence is observed.

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Therefore, a choice of the optimal profile depends directly on the initial beam width (for a given b, the optimal m can be indicated). Note that for the constant initial beam power the parameter b determines the peak beam intensity.

TABLE I. Functions $I(m)$, $\chi(m)$, and $\delta_G(m)$ versus the parameter m.

m	2	4	6	8	10
$I(m)$	0.7854	0.9064	0.93	0.9417	0.95135
$\chi(m)/2^5$	0.375	0.728	1.09	1.46	1.772
$\delta_G(m)\cdot 10^2$	7.46	2.543	1.444	1.1074	0.9011

As distinct from the displacement of the beam center of gravity for the square of the beam width

$$a^2(z) = a_x^2 + a_y^2 - X_c^2 , \quad (7)$$

an optimal value of b exists for which the width is minimum. So, for propagation of the optical beam with initially flat–top profile is accompanied by the change of the square of the width by the law

$$a^2(z) = \frac{2^{1-2/m}}{3}\frac{\Gamma(1+3/m)}{b^{2/m}\,\Gamma(1+1/m)} + \frac{z^2\Gamma(2-1/m)\,mb^{2/m}}{\Gamma(1+1/m)\,2^{3-2/m}} + $$

$$+\frac{z^2\theta_{nl}^2\,b^{4/m}}{I^2(m)\,2^6}\left(\frac{2^{4/m}}{3^{2/m}} + \frac{m\Gamma(2-1/m)\,\Gamma(1+1/m)\,2^{4/m}}{3^{3-1/m}} - 1\right). \quad (8)$$

Note that a_x^2 and a_y^2 make the same contribution to the first two terms in Eq. (8).

It is clear that for every set of the parameters m, z, and θ_{nl} the optimal value of $b^{2/m}$ exists for which $a^2(z)$ reduces to a minimum. This value is determined by solving the cubic equation for $\xi = b^{2/m}$:

$$-\frac{c_1}{\xi^2} + c_2 + 2\,c_3\,\xi = 0. \quad (9)$$

For brevity, in Eq. (9) the coefficients of $b^{2/m}$ from Eq. (8) are denoted by c_i, i = 1, 2, 3, respectively. It is interesting to note that even in the case of optical beam propagation through a linear medium ($\theta_{nl} = c_3 = 0$), the distribution with the parameter

$$(b_{opt})_G = \left(\frac{2^{4-4/m}\,\Gamma(1+3/m)}{3\,m\,\Gamma(2-1/m)}\right) z^{-m/2} = \varphi_G(m)\,z^{-m/2} \quad (10)$$

other than unity is optimal.

It follows from Eq. (10) that it is necessary to transfer to wider beams for fixed m as z increases. The coefficient $\varphi_G(m)$ as a function of the parameter m is shown in Fig. 1 by solid curve. An analysis of Fig. 1 and expression (10) shows that when $z < 1$ (the path length is smaller than the diffraction length of the reference beam) the same value of b can be optimal for different m. For $z > 1$ such a situation is never realized.

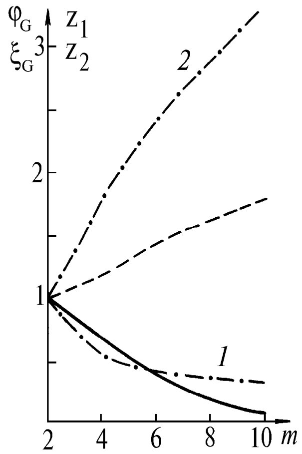

FIG. 1. Functions $\varphi_G(m)$ (solid curve), $\xi_G(m)$ (dashed curve), and z_1 and z_2 (dot–dash curves 1 and 2, respectively) versus the parameter m for hyper–Gaussian beams.

The choice of $(b_{opt})_G$ (i.e., initial beam width) leads to the following value of the optical beam radius at the receiver:

$$a^2(z,\, b_{opt}) = z\left(\frac{m}{3}\frac{\Gamma(1+3/m)\,\Gamma(2-1/m)}{\Gamma^2(1+1/m)}\right)^{1/2} = z\xi(m). \quad (11)$$

The $\xi(m)$ dependence is depicted in Fig. 1 by dashed curve.

As is clear from Fig. 1, the minimum beam width in the cross section z of a linear medium increases with m. Nevertheless, with optimal choice of the parameter b the beam width at the receiver decreases as compared with its value for b = 1 by a factor of

$$\eta = \frac{1}{2}(b_{opt}^2 + 1/\,b_{opt}^2) = \frac{1}{2}\left(\frac{\varphi_G^2(m)}{z^m} + \frac{z^m}{\varphi_G^2(m)}\right). \quad (12)$$

Therefore, for flat–top beams the requirements for a choice of the optical beam initial radius become more stringent. It is interesting to compare the widths of the Gaussian beam (b = 1) and profiled one for optimal b. It is easy shown that their ratio is equal to

$$\eta_G = 0.5\,(1 + z^2)/\,z\,\xi(m)\,. \quad (13)$$

So, in the propagation path segment

$$\xi(m) - \sqrt{\xi^2(m) - 1} = z_1 \le z \le z_2 = \xi(m) + \sqrt{\xi^2(m) - 1} \quad (14)$$

there is no point in going to hyper–Gaussian beams ($\eta \le 1$) with plane initial wave front.

For the Gaussian beam, η equals unity in a single point. As m increases, the width of this zone also increases and runs into the values 0.3–3.3 for the hyper–Gaussian beam with m = 10. Therefore, in a linear medium an optical beam with initially flat–top amplitude distribution and plane phase front should be used in either the near zone (the path length $\leq (0.15\text{–}0.28)\, ka^2$) or the far zone (the path length $\geq (0.9\text{–}1.63)\, ka^2$), respectively, as m increases. All mentioned above is illustrated by Fig. 1, where the dependence of z_1 and z_2 on the parameter m is shown by dot–dash curves. For path lengths between curves 1 and 2 transfer to a flat–top beam does not decrease the optical beam radius as compared with the Gaussian beam radius.

In more general case ($c_3 \neq 0$) an analysis of Eq. (9) in detail is difficult because we must solve the cubic equation. However, the most important dependence of ξ_{opt} on the excess divergence, propagation path length, and parameter m can be obtained. To this end, Eq. (9) is reduced to the form

$$\delta_G(m)\, \theta_{\text{nl}}^2\, z\, \eta = 1/\eta^2 - 1, \tag{15}$$

by introducing new variable $\eta = \xi/\xi_{\text{opt l}}$, where $\xi_{\text{opt l}}$ is the optimal value of ξ in the case of propagation through a linear medium,

$$\delta_G(m) = \sqrt{\Gamma(1+3/m)/3}\, \left[\Gamma^3(1+1/m)\, \Gamma^{3/2}(2-1/m)\, m^{3/2}\, 2^{4/m}\right]^{-1} \times$$

$$\times \left[\frac{2^{4/m}}{3^{2/m}} - 1 + m\, \Gamma(2 - 1/m)\, \Gamma(1 + 1/m)\, 2^{4/m}/3^{3-1/m}\right]. \tag{16}$$

The function $\delta_G(m)$ is tabulated in Table I.

Using the graphic approach to solve Eq. (15), it is not difficult to see that when a beam passes through a thin nonlinear layer, the optimal value of η is smaller than unity, and η is closer to zero, the larger is the parameter $z\theta_{\text{nl}}^2\delta_G(m)$. It can be easy seen that for sufficiently large values of this parameter (when $\eta \ll 1$), its optimal value η_{opt} is equal to $1/(\theta_{\text{nl}}^{2/3} z^{1/3} \delta_G^{1/3}(m))$.

Propagation of the hollow beams was analyzed in the same manner.

2. AMPLITUDE COMPENSATION FOR THE RANDOM PHASE DISTORTIONS OF OPTICAL BEAMS

One of the most complex problems of nonlinear adaptive optics is the problem of compensation for the distortions of light beams propagating through a turbulent medium. An analysis of the effect of the initial beam profile on the beam power characteristics in the receiver cross section was not practically carried out in the literature devoted to this problem. Such an investigation in the case of optical beam propagation through a thin layer of a turbulent medium is made below (see also Ref. 19). Note first that compensation for the beam distortions by a flexible mirror controlling the lowest–order aberration modes is feasible when the correlation length and the beam radius are approximately equal. As their ratio decreases, the number of the controllable modes of a mirror increases quickly. Therefore, it is expedient to optimize the other beam parameters, for example, its profile. For this purpose calculations of the position of the beam center of gravity were done by formula (1) for various initial amplitude distributions from the class

$$f(x, y) = ((1 - c) + c(x^m + y^m))\, \exp[-2(x^m + y^m)] \tag{17}$$

(the number 2 in the exponent is for comforable calculations) when the beam passes through the layer of a turbulent medium and receives the random run–on of the phase $S_\varphi(x, y)$ upon exiting the medium. Let S_φ be the Gaussian random variable with the correlation function

$$S_\varphi(x, y)\, S_\varphi(x', y') = \sigma^2 \exp[-((x - x')^2 + (y - y')^2)/\rho_c^2], \tag{18}$$

where ρ_c is the correlation length normalized by a; σ is the fluctuation variance with zero mean $\overline{S_\varphi} = 0$ (the bar atop denotes averaging). In Eq. (17), c is the coefficient varying from 0 up to 1. The calculated results are shown in Figs. 2 and 3 as the dependence of $J_c = X_c^2(z)$ on the parameter ρ_c in the cross section $z = 1$ with $\sigma = 1$.

The J_c dependence for the hyper–Gaussian beams ($c = 0$) is shown in Fig. 2*a*. The number adjacent to curves indicate the values of the parameter m. Four characteristic regions can be identified in Fig. 2*a* which are practically the same for the examined beam profiles. So, as ρ_c decreases from 6 to 4, the standard deviation of the position of the beam center increases. If ρ_c is within the interval 2 – 4, J_c does not change. Note that profiling leads to decrease in J_c by a factor of 1.5 (for $m = 10$) as compared with the displacement of the Gaussian beam center.

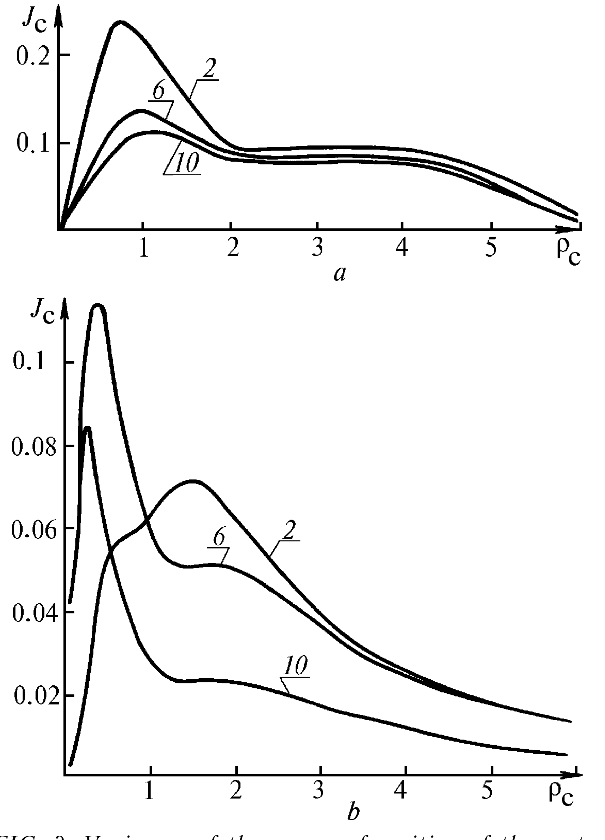

FIG. 2. Variances of the square of position of the center of the hyper-Gaussian (a) and hyperhollow (b) beams as functions of the correlation length.

In the third region J_c increases as ρ_c decreases. Moreover, the position of the left boundary of the region and minimum value of J_c are determined by the beam profile. It is important that as m increases, the amplitude of the displacement maximum is halved and is shifted towards larger ρ_c for m = 10. The case with $\rho_c < 1$ is of great interest for practice. It is essential that for $m = 10$ the standard deviation of the beam center in maximum is smaller by a factor of 2.5 than the corresponding value for the Gaussian beam.

In the last region J_c decreases when ρ_c tends to zero. This fact is connected with the beam spreading at the receiver and its smearing due to small fluctuations.

In the case of the hyperhollow beams ($c = 1$) wandering of the beam center of gravity (see Fig. 2*b*) differs from the above–mentioned wandering of flat–top beams. First of all it should be noted that the region with flat–topped dependence of J_c on ρ_c is absent for the hollow beams. Moreover, for $\rho_c > 3$ the standard deviation of the center of the hyperhollow beam with $m = 10$ is approximately halved in comparison with deviation of the hollow beam with $m = 2$. As distinct from the flat–top beams, in the given case the values of J_c coincide for $\rho_c \geq 4$ when $m \leq 6$. One more distinct feature is the dependence of J_c maximum on ρ_c. As m increases, this maximum is shifted towards smaller ρ_c, and its amplitude increases. Note that when $m = 10$, the maximum variance of the beam center displacement is greater by a factor of 2.3 than the corresponding value attained for the hollow beam with $m = 2$. For $\rho_c \geq 0.5$ the hyperhollow beam with $m = 10$ must be used without question. Emphasize that the maximum J_c of the hollow beam is shifted in the region with the correlation length larger than the beam radius that also could give practical advantages when the hollow beams are used.

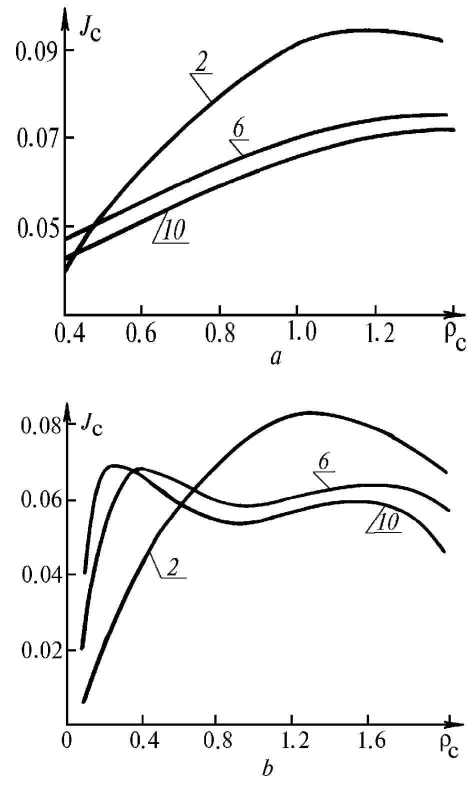

FIG. 3. The same as in Fig. 2. c = 0.835 *(a) and* 0.91 *(b).*

Analogous calculations were done for $c = 0.1$, 0.2, 0.5, 0.8, and 0.9. The typical dependence of the standard deviation of the beam center of gravity is shown in Figs. 3*a* and 3*b*, for $c = 0.835$ and 0.91, respectively. As follows from the result of calculations, there is no point in the use of beams with $c \leq 4$, because for practically important case $\rho_c < 1$ J_c is not smaller than that in the cases mentioned above. Beams with $c \sim 0.8$ are promising for the given application. However, the specific value of the parameter c at which the smallest value of J_c is observed, depends on m (see Fig. 3).

Thus, profiling of a light beam makes it possible to decrease essentially the variance of the fluctuations of the optical beam center of gravity.

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