STUDY OF THE MODE COMPETITION IN CONTINUOUS SPECTRUM OF GENERATION OF A PULSED SUPERBROADBAND DYE LASER WITH A POINT SELF-RECONSTRUCTING CAVITY

A.G. Zhiglinskiy, **I.A. Zhuvikina, and E.V. Novikov**

Scientific Research Institute of Physics at the St. Petersburg State University
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Based on balance equations, peculiar features of multichannel continuous generation in a dye laser with broken mode competition operating in a stationary regime have been studied. The classification of spectrum types has been proposed. In several specific cases the analytical solutions have been obtained.

In this paper we consider a laser with a point self-reconstructing cavity. Its optical arrangement is shown in Fig. 1. Beams, emitted from the reference plane (RP) and having traveled through the cavity once or twice, come back to the emission point. We call the reference plane as the plane of point self-reconstruction. If a diffraction grating is introduced into the cavity, point coordinates in the reference plane become dependent on the wavelength which is governed by the wavelength of a beam having autocollimationally reflected from the grating and passed through a point of the active medium.

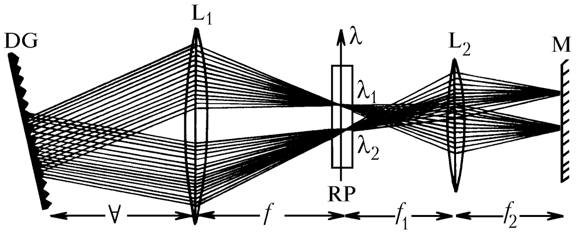


FIG. 1.

The case of lasing in a cavity with broken mode competition in the stationary regime with pumping of two sections was considered in Ref. 5.

Our paper is aimed at solving direct and inverse problems of formation of a certain spectrum when a wide section of the active medium is pumped.

The direct problem is to find a spectrum of lasing based on given parameters of a cavity, a medium, and pump. The inverse problem is to determine the parameters needed for the formation of spectrum of a given type based on the type of spectrum and its intensity distribution. In so doing we assume that the length of the pumped section in the direction normal to the cavity optical axis is much greater than the resolution of the intracavity spectral device in the active medium plane. Therefore, we use continuous variables related to a point of active medium to describe the lasing process. Propagation of radiation in the cavity is described within the framework of the geometric optics.[1,4]

Let us call the pencil of rays emitted from a point in the point self-reconstruction plane and coming back to the same point as the lasing channel. To define the lasing channel, we use the coordinates of points of rays intersection with the active medium λ' and λ'' expressed in terms of wavelengths. At $\lambda' = \lambda''$ rays are autocollimationally reflected from the grating and we refer this channel to as an autocollimational one, otherwise, as a circular one. Let us introduce new variables λ (the channel wavelength) and μ based on the parameters λ' and λ'' as

$$\lambda = (\lambda' + \lambda'')/2, \quad \mu = (\lambda' - \lambda'')/2.$$

Further consideration is worthwhile to be carried out in terms of variables x and y which are related to the variables λ and μ by the $\pi/4$ rotation transformation:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{\sqrt{2}}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \lambda \\ \mu \end{pmatrix}; \quad \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \sqrt{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

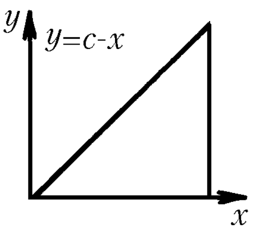
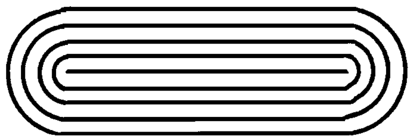
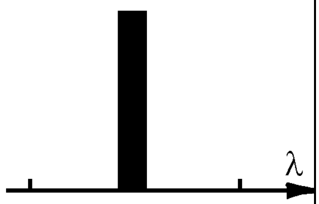
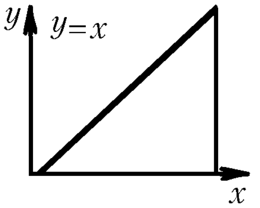
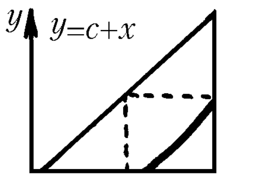
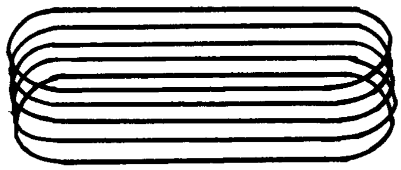
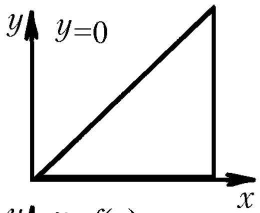
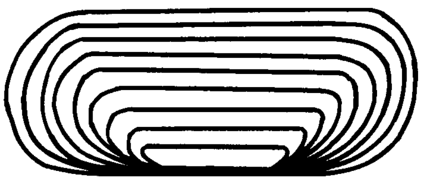
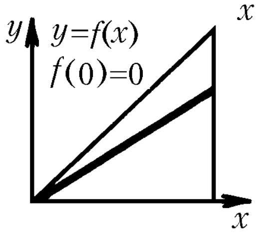
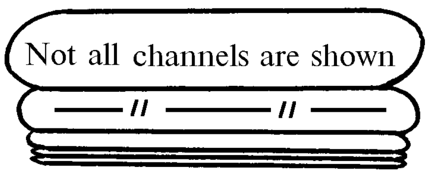
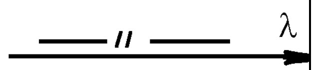
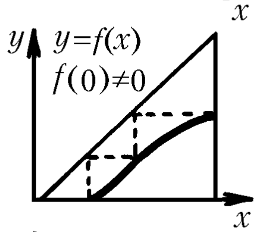
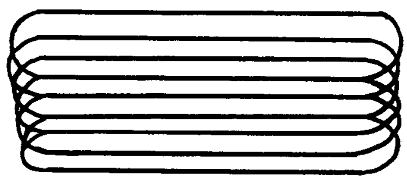
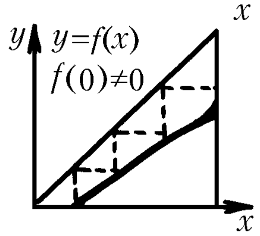
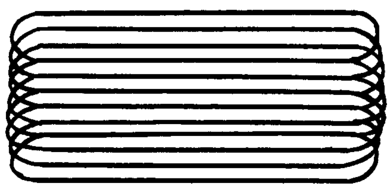
Then it follows from the definition of λ, μ, x, and y that

$$x = \lambda'/\sqrt{2}; \quad y = \lambda''/\sqrt{2}.$$

We will schematically denote the circular channels by a closed oval. For the autocollimational channel the oval degenerates into a straight line.

Let us now consider the plane (x, y). Every point in this plane corresponds to a lasing channel. The set of all possible channels forms a triangular domain (Figs. 2 and 3). The set of points in the triangle, which lies on the straight line $y = x$, corresponds to autocollimational channels. The triangle vertex lying at the right angle corresponds to the channel with the maximum separation between the intersection points in the medium which is equal either to the active medium length in the direction normal to the resonator optical axis or to the cavity pupil diameter in the point self-reconstruction plane.

TABLE I.

No.	Type of spectrum	Channel configuration	Width of spectrum
1			
2			
3			
4			
5			
6			
7			

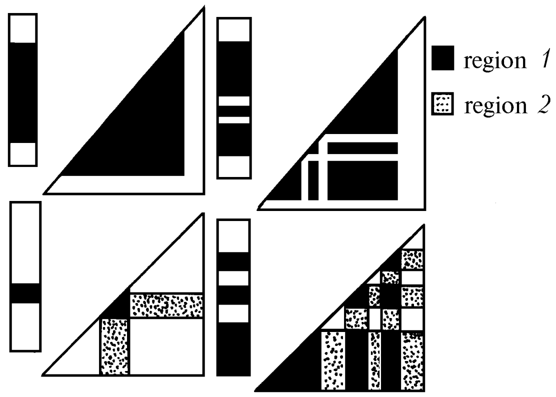


FIG. 2.

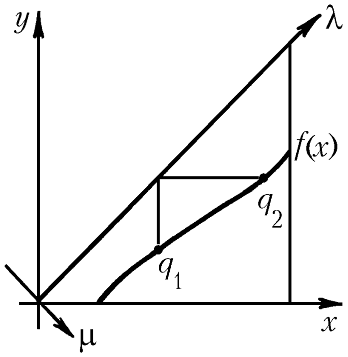


FIG. 3.

When a wide section of the active medium is pumped, channels may be formed, whose both coordinates are within the pumped section. The domain we call the pumping triangle corresponds to this set of channels. The formation of channels which have only one coordinate within the pumped section is also possible. Figure 2 shows the correspondence between pumping in the medium and channel representations. Double pumped channels lie in the regions *1*, whereas once pumped channels are in the regions *2*.

In the stationary regime of lasing not all channels survive but only the set which is represented by a line. We call it the curve of initiated channels. To be sure that the set of points corresponding to initiated channels is the line rather than a 2-D domain, a passage to the limit should be done from a discrete pumping case. In so doing the existence of 2-D domains is excluded because of the requirement that in the stationary regime no more than N channels generate (N is the number of medium divisions).

Within the pumping triangle we describe the curve of initiated channels by the function $y = f(x)$.

Table I presents the types of possible channels configurations according to the ways of their intersection in the medium. If rays do not intersect, the configuration corresponds to the uncoupled channels. They may be enclosed channels *1*, autocollimational channels *2*, or an inverted configuration of channels *3*. If all channels intersect at the only point, this case is referred to as focusing *4*. If all channels are coupled like an infinite chain, this case is called as completely coupled *5*. If the chain is not infinite, but consists of two or three sections, these variants are called as two, three, etc. times coupled cases *6*, *7*. Various combinations of these configurations are also possible.

The complete system of balance equations describing the lasing continuum has the following form:

$$\mathrm{d}n(x,t)/\mathrm{d}t = b_{01}(n - n(x,t))\,W(x) - \\ - \left[\int b(x,y)\,\varepsilon\, q(x,y,t)\,dy + \frac{1}{\tau}\right] n(x,t), \quad (1)$$

$$\mathrm{d}q(x,y,t)/\mathrm{d}t = F(x,y,t)\,q(x,y,t), \quad (2)$$

where

$$F(x,y,t) = \frac{\varepsilon}{2} b(x,y)[n(x,t)+n(y,t)] - 1/[\tau(x,y)];$$

b_{01} is the absorption probability per one pumping quantum; $n - n(x, t)$ is the density of dye molecules in the ground state; $W(x)$ is the density of pumping quanta coming per unit time to the point of the medium with coordinate x; $b(x, y)$ is the probability of stimulated downward transition per one photon in the channel (x, y); $\tau(x, y)$ is the mean photon lifetime in the channel (x, y); ε is the coefficient of medium filling with radiation; $n(x, t)$ is the number of dye molecules at the upper lasing level, and simultaneously the population inversion because the lower lasing level decays very fast; $q(x, y, t)$ is the photon density in the medium for the channel (x, y).

The system of equations (1) and (2) governs the evolution of the $q(x, y, t)$ function. It can be solved only numerically.

In the stationary mode

$$\mathrm{d}/\mathrm{d}t \equiv 0; \quad n(x,t) = n(x); \quad q(x,y,t) \equiv q(x,y).$$

It follows from Eq. (2) that for lasing channels

$$F^{\mathrm{g}}(x,y)=0 \text{ or } n(x)+n(y) = \frac{2}{\varepsilon\, b(x,y)\,\tau(x,y)} - \frac{2}{\varepsilon}\Gamma(x,y). \quad (3)$$

The threshold condition has the form

$$q^{\mathrm{g}}(x,y) > 0. \quad (4)$$

Equality (3) can be considered as the condition for $f(x)$ curve run, because just for the lasing channels on this curve

$$F^{\mathrm{g}} = F(x, f(x)) \equiv 0 \text{ and } q(x, f(x)) > 0.$$

For the channels, not lasing under the stationary regime, the following condition is fulfilled

$$q^{\mathrm{n}}(x,y) = 0, \quad y \neq f(x)$$

as well as the condition of negative increment

$$F^{\mathrm{n}}(x,y) < 0, \quad y \neq f(x). \quad (5)$$

Taking into account the fact that population inversion is removed only by photons from the lasing channels, Eq. (1) takes the form

$$b_{01}(n - n(x))\ W(x) - \\ - \left[\sum_l \varepsilon\, b(x, f(x))\ q_l(x, f(x)) - \frac{1}{\tau}\right] n(x) = 0. \qquad (6)$$

Summation in Eq. (6) is done over the channels which intersect the active medium at the point $x\sqrt{2}$ (there can be one or two points depending on the $f(x)$ curve run, see Fig. 3). If $f(x) \equiv 0$ on the $[0, a]$ section (the case of focusing), then the sum transforms into integral.

Below we present examples of analytical types of spectra where we assume that the initiated channels pass through the pumped section of the active medium two times. Having additionally defined the pumping function with zero, the extension becomes possible to the cases when lasing channels pass through the pumped section only once or several wide bands are pumped (Fig. 2).

Completely coupled channels.

The curve of initiated channels has the form

$$f(0) = 0, \quad 0 < f(x) < 0, \quad f'(x) \neq 0, \quad x \in [0, \mathrm{a}].$$

This case is nondegenerate, since the balance equations for the number of photons and population inversion can be solved separately. Analysis of the solution shows that for the threshold condition to be fulfilled the pump must be infinitely large at the point (0, 0).

Autocollimational channels, $y = x$.

The solution of the system of balance equations has the form:

$$q(x, x) = \frac{b_{01}\, n\, W(x)}{b(x, x)\, \Gamma(x, x)} - \frac{1}{\varepsilon\, \tau\, b(x, x)}\,. \qquad (7)$$

Conditions for occurrence of the type of the spectrum:

$$\Gamma(x, x) + \Gamma(x', x') < 2\, \Gamma(x, x')\ , \quad W(x) > \frac{\Gamma(x, x)}{b_{01}\, \tau\, \varepsilon\, n}$$

or

$$\Gamma(x, x) < \Gamma(x, x'), \quad \Gamma(x', x') < \Gamma(x, x'). \qquad (8)$$

This case is nondegenerate too. The condition for negative increment does not include the pumping, and its redistribution over the active medium does not change the spectrum type.

Enclosed and inverted configurations, $y = f(x)$.

The solution of the system of balance equations has the form:

$$q(x, f(x)) = \frac{n\, b_{01}\ W(x) + W(f(x))}{b(x, f(x))\ \Gamma(x, f(x))} - \frac{2}{\varepsilon\, \tau\, b(x, f(x))}\,. \qquad (9)$$

Conditions for occurrence of this type of the spectrum:

$$B(x,f(x))W(x)+B(x',f(x'))W(x')<B(x,f(x'))[W(x)+W(x')]; \qquad (10a)$$

$$W(x) + W(f(x)) > \frac{2\, \Gamma(x, f(x))}{n\, b_{01}\, \tau\, \varepsilon}\,; \qquad (10)$$

$$B(x, f(x)) < B(x, f(x')), \quad B(x', f(x')) < B(x, f(x')). \qquad (10a)$$

Here we introduce the function

$$B(x, y)\ \mathrm{a}\ \Gamma(x, y)/[W(x) + W(y)].$$

Inequalities (10a) are sufficient for providing negative increment. They are equivalent to the existence of the plane stationary minimum at the curve $f(x)$ in the pumping triangle, i.e., those channels survive which have maximum lifetime, gain, and pumping. If there is no plane minimum, the exact inequality should be used. This case is degenerate; pumping enters into the condition for negative increment and its redistribution may cause the spectrum type transformation.

Channel focusing, $f(x|\equiv□0)$.

This case is degenerate, i.e., we believe that channels do not intersect, but are placed close to each other within the physically infinitely small section of the active medium. The system can be solved using a limiting transition from the previous case:

$$q(x, 0)\ \mathrm{a}\ n\, b_{01} \frac{W(x) + W(0)}{2\, \Gamma(x, 0)\ b(x, 0)} - \frac{1}{\tau\, \varepsilon\, b(x, 0)}\,. \qquad (11)$$

The conditions for occurrence of this type of spectrum are analogous to the previous ones, but because $f(x)$ is at the boundary of the pumping triangle, sufficient conditions necessitate the boundary minimum of the function

$$B(x, 0)W(x) + B(x', 0)W(x') < B(x, x')[W(x)+W(x')];$$

$$W(x) + W(0) > \frac{2\, \Gamma(x, 0)}{n\, b_{01}\, \tau\, \varepsilon}; \quad B(x, 0) < B(x, x'),$$

$$B(x', 0) < B(x, x'). \qquad (12)$$

If the domain of inequality fulfillment does not span the entire triangle, the combined spectrum can be formed, when in different sections of pumping the conditions for different spectrum types are fulfilled. In these cases the curve of initiated channels is either broken or discontinuous.

Coupled channels.

The solution for occurrence of this type of spectrum can be analyzed only by solving numerically the functional inequalities. As an example, we present here the balance equations for the channels coupled two times (see sixth row in the table). For higher degrees of coupling, the escalation of the number of equations in the system takes place:

$$b_{01} \frac{n}{n(x)}\ W(x) - \left[\frac{\varepsilon}{2}\, b\, q(x, f(x)) + \frac{1}{\tau}\right] \mathrm{a}\ 0;$$

$$-\frac{1}{\tau} + b_{01}\,\frac{n}{n(f(x))}\,W(f(x)) -$$

$$-\left[\frac{\varepsilon}{2}\,b\,q(x,\,f(x)) + \frac{\varepsilon}{2}\,b\,q(f(x),\,f(f(x)))\right] \mathrm{a}\;0;$$

$$b_{01}\,\frac{n}{n(f(f(x)))}\,W(f(f(x))) -$$

$$-\left[\frac{\varepsilon}{2}\,b\,q(f(x),\,f(f(x))) + \frac{1}{\tau}\right] \mathrm{a}\;0; \tag{13}$$

$$n(x) + n(f(x))\;\mathrm{a}\;\frac{2\,\Gamma(x,\,f(x))}{\varepsilon}\,,$$

$$n(f(x)) + n(f(f(x)))\;\mathrm{a}\;\frac{2\,\Gamma(f(x),\,f(f(x)))}{\varepsilon}\,.$$

The equations for the number of photons and population inversion are solved simultaneously for the variables $n(x)$, $n(f(x))$, $q(x, f(x))$, and $q(f(x), f(f(x)))$. This is a degenerate case. The system of equations can be reduced to a cubic equation. With increasing degree of coupling, the degree of algebraic equation increases in proportion to the number of channel intersections.

In all cases of lasing the increment surface, which is established in the stationary regime, necessarily has a plane zero maximum on the curve of initiated channels in accordance with the fact that for channels lying on $f(x)$ the equality of gain and losses is achieved:

$$\frac{\partial}{\partial x}\,F(x,\,y)\,\Big|_{f(x)} \equiv \frac{\partial}{\partial y}\,F(x,\,y)\,\Big|_{f(x)} \equiv 0.$$

Thus, we propose the approach to analyzing multichannel lasing. The system of balance equations is formulated. In a number of important cases, the analytical solutions of this system are obtained, which describe the spectrum of output radiation as well as the conditions of its occurrence. In this case the solution of direct and inverse problems is possible in the domain of parameters determined by inequalities (8), (10), and (12).

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