LASER REFERENCE STARS AND THE PROBLEM ON MEASURING OF THE WAVEFRONT TILTS

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Received August 15, 1996

We discuss here a solution of the problem on stabilization of the position of a laser beam axis based on real time tracking of a natural star image in the focal plane of a telescope. In particular, we calculated the cross-correlation function between the vector, characterizing random shift of the power center of gravity of an optical beam propagated through a turbulent medium, and the vector, determining the center of gravity of a star image or any reference source formed by the same optical system. The cases with monostatic and bistatic laser reference stars are considered. The causes of inadequate correction when using a "pure" signal of backscattering are explained.

The development of theoretical and experimental studies with the use of the lasers for creating artificial reference stars, which have become popular in recent years,[1,2] have made the author to revise his results obtained 12 or 15 years ago. These results were published in Refs. 3–8. The results became available to the majority of scientists, including foreign ones, when the monographs[9,10] were published.

At that time, the author was solving the problem on stabilization of the position of a laser beam axis based on the real time tracking of a natural star image in a telescope focal plane. In particular, the cross-correlation function was calculated between the vector characterizing the random shift of power center of gravity of an optical beam, propagated through a turbulent medium, and the vector, determining the center of gravity of star image or any reference source formed with the same optical system. In this case it was assumed that this may be the image of a reference source–beacon or of an optical beam reflected from an object. In particular cases, these can be the images of a natural star, the image of a laser beam reflected diffusely or specularly.

If we formulate a special optical problem on reflection or scattering of laser radiation from atmospheric inhomogeneities, a conclusion may be drawn that the problem on correction of the wavefront tilt of a stellar radiation with the use of a laser reference star and the problem of the control of the laser beam axis position based on real reference star tracking are mathematically identical or very close.

Let us first state the main results obtained earlier in Ref. 3–8. It is known that random shifts of the power center of gravity of an optical beam are characterized by the vector:

$$\rho_c = \frac{1}{P_0}\int_0^x d\xi\,(x-\xi)\iint d^2R\, I(\xi, R)\,\nabla_R n_1(\xi, R), \quad (1)$$

where $n_1(\xi, \mathbf{R})$ denotes the fluctuations of the refractive index at the point $(\xi, \mathbf{R})$, $I(\xi, \mathbf{R})$ is the field intensity at the point $(\xi, \mathbf{R})$ from a source placed at the coordinate origin in the plane $\xi = 0$; x is the thickness of turbulence layer;

$$P_0 = \iint d^2R\, I(0,\mathbf{R})\ . \quad (2)$$

At the same time the stellar image random shifts, formed in the focal plane of the optical system (an equivalent thin lens with a focal length F and an area $\Sigma = \pi R_0^2$) in the phase approximation, are given by the following expression (when amplitude fluctuations are neglected):

$$\rho_F = -\frac{F}{k\Sigma}\iint_\Sigma d^2\rho\, \nabla_\rho S(0,\rho)\ , \quad (3)$$

where k is the radiation wave number; $S(0, \rho)$ denotes the optical wave phase fluctuations at a point ρ within the limits of the optical system aperture Σ. In this case it is assumed that an entrance pupil of the receiving aperture is located in the plane $\xi = 0$. For most of practical applications of the atmospheric turbulence the optical wave phase fluctuations are adequately described within the limits of smooth perturbation method, i.e.,

$$S(x, \rho) = k\int_0^x d\xi\,\times$$

$$\times \iint d^2\, n\,(\varkappa, \xi)\,\exp(i\varkappa\rho\gamma)\cos(\varkappa^2(x-\xi)\gamma/k), \quad (4)$$

0235-6880/96/11 910-06 $02.00

where

$$n_1(\mathbf{R}, \xi) = \iint d^2 n(\varkappa, \xi) \exp(-i\varkappa\mathbf{R}) ;$$

$$\gamma = \frac{1 + i\alpha\xi}{1 + i\alpha x} ; \quad \alpha = \frac{1}{ka_0^2} + \frac{i}{f} ;$$

γ and α are the values determined by the parameters (a_0 and f) of the optical beam formed. In two special cases the parameter γ is real and can easily be determined. For a plane wave $\gamma = 1$, for a spherical wave $\gamma = \xi/x$.

The calculation of correlation between random vectors ρ_c and ρ_F was most difficult, while being, at the same time, of great interest:

$$K = \langle \rho_c\, \rho_F \rangle ,$$

where $\langle ... \rangle$ denotes the averaging over the ensemble of random fluctuations of the function $n_1(\mathbf{R}, \xi)$. The papers[8–9] describe a series of scenarios of optical experiments, which differ only in the expression for the random phase S in Eq. (3). In the author's opinion, the problem on estimating the application efficiency of a laser reference star for correction of random tilts in the natural stellar image, formed by a telescope, can be solved using the functions already calculated in Ref. 3–9.

We characterize the random location of the star image formed in the telescope focal plane as the vector:

$$\rho_F^{\mathrm{pl}} = -\frac{F}{k\Sigma} \iint_\Sigma d^2 \rho\, \nabla_\rho S^{\mathrm{pl}}(0,\rho) , \tag{5}$$

where the superscript "pl" shows that this characteristic is calculated for the plane wave from a star. For an arbitrary optical scenario[3–5] of the laser reference star formation, the random vector describing the shift of the center of gravity of the reference star image can be described as follows:

$$\rho_m = \rho_c + \rho_F^{\mathrm{sph}} , \tag{6}$$

where ρ_F^{sph} is the location of a point source image. It is assumed that the reference star is created by focusing laser radiation and the reference star, being formed, is presented as a point source. By this is meant that a reference laser beam is rather wide ($\Omega = ka_0^2/x$, $\Omega \gg 1$) since only for a wide beam the focusing may be used, and an occurring reference source is not resolved by the telescope aperture.

It is assumed that the optical problem on radiation reflection or scattering on atmospheric inhomogeneities is not considered.

To change from linear measurements to angular ones we need to do the following normalization:

$$<(\varphi_F^{\mathrm{pl}})^2> = \frac{<(\rho_F^{\mathrm{pl}})^2>}{F^2} ,$$

$$<\varphi_c^2> = \frac{<\rho_c^2>}{x^2} , \quad <\varphi_c\, \varphi_F> = \frac{<\rho_c\, \rho_F>}{xF} .$$

It is natural that residual distortions of the random tilts of the wave front from a star (under condition that the artificial reference star and the natural star are in one isoplanar region) angular of the correction based on the "direct" tracking of the laser reference star is characterized by the following variance:

$$\langle e^2 \rangle = \langle (\rho_F^{\mathrm{pl}} - \rho_m)^2 \rangle = \langle (\rho_F^{\mathrm{pl}})^2 \rangle + \langle \rho_m^2 \rangle - 2\langle \rho_F^{\mathrm{pl}}\, \rho_m \rangle . \tag{7}$$

In this case variance of the measurement vector fluctuations and the correlation in Eq. (7) are:

$$<\varphi_m^2> = <\varphi_c^2> + <(\varphi_F^{\mathrm{sph}})^2> + 2 <\varphi_c\, \varphi_F^{\mathrm{sph}}> , \tag{8}$$

$$\langle \rho_F^{\mathrm{pl}}\, \rho_m \rangle = \langle \rho_F^{\mathrm{pl}}\, \rho_c \rangle + \langle \rho_F^{\mathrm{pl}}\, \rho_F^{\mathrm{sph}} \rangle . \tag{9}$$

By summing Eqs. (7), (8), and (9), one can obtain expression for all components of the variance $\langle e \rangle^2$. The monostatic and bistatic optical systems differ by the last terms in Eqs. (7)–(9).

All components of $\langle e^2 \rangle$ were obtained in Refs. 6, 7, except for the correlation $<\rho_F^{\mathrm{pl}}\, \rho_F^{\mathrm{sph}}>$ which is easily calculated. One can show, Refs. 3–5, that in the diffraction approximation

$$\langle \rho_c^2 \rangle = \pi^2 x^3 \int_0^1 d\xi\, (1 - \xi)^2 \times$$

$$\times \int_0^\infty d\varkappa \varkappa^3\, \Phi_n(\varkappa, x\xi) \exp\{-\varkappa^2 a^2 q^2(\xi)/2\} , \tag{10}$$

where $\Phi_n(\varkappa, \xi/x)$ is the spectral density of the refractive index fluctuations. When deriving Eq. (10) it is assumed that the reference laser beam is Gaussian, and

$$q(\xi) = [\xi^2\Omega^{-2} + (1 - x\xi/f)^2] ,$$

where $\Omega = ka^2/x$; a is the size and f is the curvature radius of phase front of the laser beam, forming the reference star. For a focused ($x = f$) laser beam

$$q(\xi) = [\xi^2\Omega^{-2} + (1 - \xi)^2]^{1/2} ,$$

if we use a wide laser beam ($\Omega \gg 1$) then we have

$$q(\xi) = (1 - \xi) .$$

As follows from calculations by formula (10)

$$<\varphi_c^2> = \frac{<\rho_c^2>}{x^2} = \pi^2\, 0.033\, 2^{7/6}\, c(1/6) a_0^{-1/3} x \times$$

$$\times \int\limits_0^1 d\xi (1-\xi)^{5/3} C_n^2(x\xi) \qquad (11)$$

provided that ($\Omega = ka^2/x \gg 1$, $x = f$).

The correlation $\langle\rho_F \rho_c\rangle$ for plane ($\gamma = 1$) and spherical ($\gamma = \xi/x$) waves is

$$<\rho_c\, \rho_F> = -\frac{\pi F}{P_0} \int\limits_0^x d\xi (x-\xi)\gamma \iint d^2R <I(\xi, R)> \times$$

$$\times \iint d^2\varkappa \varkappa^2 \Phi(\varkappa,\xi) \exp(-i\varkappa\mathbf{R}) \times$$

$$\times \exp(-\varkappa^2\gamma^2 R_0^2/2 - i\gamma\varkappa^2(x-\xi)/k) , \qquad (12)$$

i.e., it is expressed in terms of the laser beam mean intensity distribution

$$\iint d^2R <I(\xi, \mathbf{R})> = \pi a_0^2 \exp(-k^2 a_{\mathrm{eff}}^2/4) . \qquad (13)$$

As a result we obtain

$$<\rho_c\, \rho_F> = -2\pi^2\, 0.033 c(1/6) F \times$$

$$\times \int\limits_0^x d\xi (x-\xi)\gamma C_n^2(\xi)(\gamma^2 R_0^2/4 + a_{\mathrm{eff}}^2)^{-1/6}, \qquad (14)$$

$$a_{\mathrm{eff}}^2 = a_0^2 \left[(1-\xi/x)^2 + \Omega^{-2} + \Omega^{-2}\left(\frac{1}{2} D_S(2a_0)\right)^{6/5} \right]$$

for a wide Gaussian beam under not so strong turbulence $\left(\Omega^{-2}\left(\frac{1}{2}D_S(2a_0)\right)^{6/5} \ll 1\right)$ we obtain

$$a_{\mathrm{eff}} = a_0(1-\xi/x) ,$$

where $D_S(2a_0)$ is the structure function of the phase S.

It should be noted that in practice one can introduce three quite different schemes of the reference source formation, namely, a monostatic scheme, a bistatic scheme, and the so-called intermediate scheme when it is necessary to take into account the correlation between the waves propagating along two spaced paths.

MONOSTATIC SCHEME OF FORMING LASER REFERENCE STAR

Let us formulate the problem on correction of the angular displacements of star image φ_F^{pl}, formed with a telescope based on the measurements of angular displacements of the center of gravity of a laser reference star. For a monostatic scheme

$$\varphi_m = \varphi_c + \varphi_F^{\mathrm{sph}} .$$

It has already been demonstrated in a number of papers that the correction based on the "direct" correction, i.e., in the form:

$$\beta = \varphi_F^{\mathrm{pl}} - \varphi_m ,$$

is not optimal. The algorithm we recommended is as follows

$$\beta^{\min} = \varphi_F^{\mathrm{pl}} - A\varphi_m .$$

We shall formulate the problem on such a correction as the problem of seeking extremum for the variance of the form

$$\langle\beta^2\rangle = \langle(\varphi_F^{\mathrm{pl}} - A\varphi_m)^2\rangle =$$
$$= \langle(\varphi_F^{\mathrm{pl}})^2\rangle + A^2\langle(\varphi_m)^2\rangle - 2A\langle\varphi_F^{\mathrm{pl}}\varphi_m\rangle , \qquad (15)$$

where the coefficient A is determined from the condition of achieving optimal correction. This problem reduces to making preliminary calculations of the functional:

$$A = <\varphi_F^{\mathrm{pl}}\varphi_m> / <\varphi_m^2> .$$

In this case the following minimum occurs:

$$<\beta^2>_{\min} = <(\varphi_F^{\mathrm{pl}})^2> - <\varphi_F^{\mathrm{pl}}\varphi_m>^2 / <\varphi_m^2> . \qquad (16)$$

Correspondingly, when using our previous results, we have

$$<(\varphi_F^{\mathrm{pl}})^2> = \frac{<(\rho_F^{\mathrm{pl}})^2>}{F^2} =$$

$$= 2^{7/6}\, \pi^2\, 0.033 c(1/6) R_0^{-1/3} \int\limits_0^\infty d\xi\, C_n^2(\xi) ; \qquad (17)$$

$$<\varphi_m^2> = \frac{<\rho_m^2>}{x^2} = 2^{7/6}\, \pi^2\, 0.033 c(1/6)\, [R_0^{-1/3} + a_0^{-1/3} -$$

$$- 2^{7/6}\, (R_0^2 + a_0^2)\,]^{-1/6} \int\limits_0^x d\xi (1-\xi/x)^{5/3}\, C_n^2(\xi) ; \qquad (18)$$

$$<\varphi_F^{\mathrm{pl}}\varphi_m> = \frac{<\rho_F^{\mathrm{pl}}\, \rho_m>}{xF} = <\varphi_F^{\mathrm{pl}}\varphi_c> + <\varphi_F^{\mathrm{pl}}\varphi_F^{\mathrm{sph}}> ; \qquad (19)$$

$$<\varphi_F^{\mathrm{pl}}\varphi_c> = (-\, 2\, \pi^2\, 0.033 c(1/6)) \times$$

$$\times\, 2^{1/3} \int\limits_0^x d\xi\, C_n^2(\xi)(1-\xi/x)[R_0^2 + a_0^2(1-\xi/x)^2]^{-1/6}; \qquad (20)$$

$$<\varphi_F^{\mathrm{pl}}\varphi_F^{\mathrm{sph}}> = (2\pi^2\, 0.033 c(1/6)) 2^{1/3} R_0^{-1/3} \times$$

$$\times \int\limits_0^x d\xi\, C_n^2(\xi)(1-\xi/x)\, [1 + (1-\xi/x)^2]^{-1/6}. \qquad (21)$$

With the monostatic scheme of laser reference star formation we have, for the variance of residual distortions (16), the following expression

$$<\beta^2>_{\min} = 2^{7/6}\,\pi^2\,0.033c(1/6)R_0^{-1/3}\int\limits_0^\infty d\xi\, C_n^2(\xi)\,\times$$

$$\times\left\{1 - 2^{1/3}\frac{\left(\int\limits_0^x d\xi\, C_n^2(\xi)(1-\xi/x)\,([1+b^2(1-\xi/x)^2]^{-1/6} - [1+(1-\xi/x)^2]^{-1/6})\right)^2}{(1+b^{-1/3} - 2^{7/6}(1+b^2)^2)^{-1/6}\int\limits_0^x d\xi\, C_n^2(\xi)(1-\xi/x)^{5/3}\int\limits_0^\infty d\xi\, C_n^2(\xi)}\right\}. \tag{22}$$

Here the ratio $b = a_0/R_0$ is introduced. It is evident from Eq. (18) that the signal φ_m becomes practically noninformative at $R_0 = a_0$. In the case of the exact equality $R_0 = a_0$ we have $<\varphi_m^2> = 0$.

Therefore the correction at the level (16) is practically impossible. Strictly speaking, at $R_0 = a_0$ both the numerator and the denominator of the second component in Eq. (22) are equal to zero. A very interesting problem arises on seeking optimal ratio $b = a_0/R_0$ under conditions of minimum in residual angular displacements in the form (16). It is evident that the domain of permissible values of the parameter $b = a_0/R_0$ is the interval (0, 1) since the case when $b \gg 1$ is not good, as regards the energy considerations, and at $b = 1$ the signal φ_m bears no information.

The condition $b < 1$ corresponds to the condition $R_0 > a_0$. This has made it possible to combine both components of Eq. (19)

$$<\varphi_F^{\rm pl}\varphi_m> = (2\pi^2\,0.033c(1/6))2^{1/3}R_0^{-1/3}\int\limits_0^x d\xi\, C_n^2(\xi)\times$$

$$\times\,(1-\xi/x)[1 - (1+(1-\xi/x)^2)^{-1/6}]\,. \tag{23}$$

As a result we derive the following expression for Eq. (16):

$$<\beta^2>_{\min} = 2^{7/6}\,\pi^2\,0.033c(1/6)R_0^{-1/3}\int\limits_0^\infty d\xi\, C_n^2(\xi)\,\times$$

$$\times\left\{1 - 2^{1/3}\frac{\left(\int\limits_0^x d\xi\, C_n^2(\xi)(1-\xi/x)\,(1 - [1+(1-\xi/x)^2]^{-1/6})\right)^2}{(1+b^{-1/3} - 2^{7/6}(1+b^2)^{-1/6})\int\limits_0^x d\xi\, C_n^2(\xi)(1-\xi/x)^{5/3}\int\limits_0^\infty d\xi\, C_n^2(\xi)}\right\}. \tag{24}$$

It is easily seen that in expression (24) the expression before braces is the variance of the star angular shifts in the telescope in the absence of correction, i.e.,

$$<\beta^2>_{\min} = <(\varphi_F^{\rm pl})^2>\left\{1 - \frac{2^{1/3}f(x,\, C_n^2)}{[1+b^{-1/3} - 2^{7/6}(1+b^2)^{-1/6}]}\right\}, \tag{25}$$

where $b = a_0/R_0$, and

$$f(x,C_n^2) = \frac{\left(\int\limits_0^x d\xi\, C_n^2(\xi)\,[(1-\xi/x)\,(1+(1-\xi/x)^2)^{-1/6} - (1-\xi/x)]\right)^2}{\int\limits_0^\infty d\xi\, C_n^2(\xi)\int\limits_0^x d\xi\, C_n^2(\xi)(1-\xi/x)^{5/3}}\,.$$

At the first stage it is necessary to calculate, using models of the turbulent atmosphere, the function $f(x,\, C_n^2)$. Since the values of this function for different altitude models of the turbulence and different altitudes x of the reference star formation differ from unity, this value determines the optimal value $b_{\rm opt}$. We shall search for the optimal value $b_{\rm opt}$, on the one hand, for $b \subset (0, 1)$ and on the other hand, based on the condition of positive determination of the following expression:

$$\left[1-\frac{2^{1/3}f(x,C_n^2)}{[1+b^{-1/3}-2^{7/6}(1+b^2)^{-1/6}]}\right]>0\,.$$

TABLE I. Monostatic scheme. Calculation of the function f(x, C) for the altitude x of the reference star from 1 *to* 100 km *(first column) in the three modes*[11] *of the turbulent atmosphere. Second, third and fourth columns correspond to the mode of the best, medium, and worst (in the turbulent sense) atmosphere.*

x, km	f		
1	0.003281	0.0031231	0.0022437
2	0.0051637	0.0045658	0.00308
3	0.0063232	0.0054414	0.0035784
4	0.0070861	0.0060538	0.0039453
5	0.0076185	0.0065124	0.0042812
6	0.0080089	0.0068728	0.0045981
7	0.0083078	0.0071687	0.0049093
8	0.0085447	0.0074169	0.0052123
9	0.0087386	0.0076394	0.0055067
10	0.0089015	0.0078375	0.0057905
15	0.0094598	0.0086174	0.0070037
20	0.0098224	0.0091708	0.0078867
25	0.010091	0.0095794	0.0085215
30	0.0103	0.0098887	0.0089916
35	0.010475	0.010129	0.0093484
40	0.010622	0.010318	0.0096303
45	0.010746	0.010474	0.0098574
50	0.010852	0.010603	0.010044
55	0.010941	0.010711	0.0102
60	0.01102	0.010804	0.010332
65	0.011089	0.010884	0.010445
70	0.01115	0.010953	0.010544
75	0.011205	0.011014	0.010627
80	0.011253	0.011065	0.010703
85	0.011297	0.011113	0.01077
90	0.011336	0.011156	0.01083
95	0.011371	0.011195	0.010885
100	0.011404	0.01123	0.010934

It is natural to consider the correction efficient if the term in braces is less than unity. Our calculations, based on various models of the turbulent atmosphere, show (Table I) that in the range of x values from 1 to 100 km the function $f(x,\,C_n^2)$ varies from 0.002 to 0.01. If the function $f(x,\,C_n^2)$ is of the order of 0.01, then an effective correction is possible only when $b \le 1$, i.e., the laser beam size a_0 turns out to be comparable with the telescope aperture R_0. It should be noted that from the point of view of power this correction in the monostatic scheme is ineffective although practically it is hoped that the high level of correction can be achieved.

BISTATIC SCHEME OF THE REFERENCE STAR FORMATION

For the bistatic scheme it is assumed that the last terms in Eqs.(8), (9) are lacking, i.e.,

$$\langle\varphi_m^2\rangle = \frac{\langle\rho_m^2\rangle}{xF} = \langle(\varphi_F^{\rm sph})^2\rangle + \langle(\varphi_c)^2\rangle\,,$$

$$\langle\varphi_F^{\rm pl}\varphi_m\rangle = \frac{\langle\rho_F^{\rm pl}\rho_m\rangle}{xF} = \langle\varphi_F^{\rm pl}\varphi_F^{\rm sph}\rangle\,.$$

In the final analysis we have:

$$\langle\beta^2\rangle_{\min} = \langle(\varphi_F^{\rm pl})^2\rangle\left\{1-\frac{2^{1/3}f_1(x,C_n^2)}{[1+b^{-1/3}]}\right\}, \qquad (26)$$

where $b = a_0/R_0$, and

$$f_1(x,C_n^2) = \frac{\left(\int\limits_0^1 \mathrm{d}\xi\; C_n^2(x\xi)(1-\xi)(1+(1-\xi)^2)^{-1/6}\right)^2}{\int\limits_0^\infty \mathrm{d}\xi\; C_n^2(x\xi)\int\limits_0^1 \mathrm{d}\xi\; C_n^2(x\xi)(1-\xi)^{5/3}}\,.$$

The calculations of the function $f_1(x,C_n^2)$ are presented in Table II. For the bistatic scheme, as is seen from Eq.(26), any relationships between a_0 and R_0 are possible. The correction is more efficient at larger b values. It is reasonable that $b \gg 1$ cannot be practically realized, and for $b = 2$ we have

$$\langle\beta^2\rangle_{\min} = \langle(\varphi_F^{\rm pl})^2\rangle\{1-2^{-2/3}f_1(x,C_n^2)\}.$$

As in the case with the monostatic scheme the quality of correction based on bistatics, essentially depends on the altitude of a laser reference source and on the altitude profile of the structure constant of the atmospheric refractive index C_n^2.

At the first stage of planning an adaptive system operation based on the use of laser reference source, it is necessary to calculate the functions $f(x,\,C_n^2)$ and $f_1(x,\,C_n^2)$ based on models of the turbulent atmosphere. Since the values of these functions for various altitude models of turbulence and different altitudes x of the reference source location differ from unity, it is just these values that determine the optimal value of the parameter $b = b_{\rm opt}$. For different models of altitude profile of the structure parameter of the atmospheric turbulence, the functions $f(x,\,C_n^2)$ and $f_1(x,\,C_n^2)$ are calculated numerically. As the calculations made using three models of the turbulence show,[9] the function $f(x,\,C_n^2)$ has quite different behavior depending on the reference star height. The difference is due to different height behavior of the turbulence in the three models used. The calculations also show that this difference in $f(x,\,C_n^2)$ behavior takes place at low altitudes of the reference source. Therefore one should expect that this function is sensitive to variations in the height behavior of the structure constant of the refractive index just in the case of low altitudes of the reference source. The value in braces in Eq.(26) is minimized based on the data on the function $f(x,\,C_n^2)$.

TABLE II. Bistatic scheme. Calculation of the function f(x, C) for the altitude of the reference star x from 1 *to* 100 km *in the three modes*[11] *of the turbulent atmosphere.*

x, km	f		
1	0.5077	0.42758	0.27629
2	0.62776	0.52529	0.33881
3	0.66459	0.57396	0.39801
4	0.68036	0.60464	0.45814
5	0.68998	0.62874	0.51569
6	0.69767	0.65005	0.5674
7	0.70473	0.66941	0.61162
8	0.71155	0.68679	0.64735
9	0.71818	0.70195	0.67598
10	0.72457	0.71514	0.69835
15	0.75061	0.75572	0.75467
20	0.76773	0.77272	0.77327
25	0.7798	0.78114	0.78136
30	0.78741	0.7858	0.78557
35	0.79232	0.78862	0.78823
40	0.79554	0.79024	0.7894
45	0.79774	0.79144	0.79043
50	0.79931	0.79231	0.79117
55	0.80026	0.79293	0.79171
60	0.80111	0.79342	0.79213
65	0.80178	0.7938	0.79246
70	0.80229	0.7941	0.79273
75	0.80272	0.79435	0.79274
80	0.80306	0.79437	0.79291
85	0.8335	0.79453	0.79305
90	0.80359	0.79466	0.79316
95	0.8038	0.79478	0.79326
100	0.80399	0.79488	0.79336

Of course, the variations can occur in true value of the integral profile $C_n^2(\xi)$. Variations of the values

$$\Delta = \left\{1 - \frac{2^{1/3}f(x,C_n^2)}{[1 + b^{-1/3} - 2^{7/6}(1 + b^2)^{-1/6}]}\right\},$$

$$\Delta_1 = \left\{1 - \frac{2^{1/3}f_1(x,C_n^2)}{[1 + b^{-1/3}]}\right\}$$

are estimated based on the difference between the values of the functions $f(x, C_n^2)$ and $f_1(x, C_n^2)$ in the second, third and fourth columns of Tables I and II. In this case the absolute error in determining the value Δ is equal numerically to relative error in determining the functions, i.e., $\delta = \Delta(f + \delta f) - \Delta(f) = \delta f/f$. Starting with this equation, one can obtain the results that for the altitude $x < 15$ km the residual variance of the natural star image jitter due to the error of selection of the structure constant profile is of the order of 15 per cent of the value of the initial variance $\langle(\rho_F^{\mathrm{pl}})^2\rangle$; for the altitude range 20 km $> x >$ 40 km this error is no more than 4–6 per cent; for altitudes exceeding 60 km this error is no more than 2 per cent.

Thus, if we compare the variance of the stellar image jitter (for a plane wave) and the variance of the image jitter of a laser reference star formed by a narrow focused beam, it appears that the minimum of the functional (16) can be obtained by selecting the coefficient A.

The smoothing coefficient A is determined both by the ratio of the beam size a_0 and the telescope R_0, and by the factor

$$\frac{\left(\int\limits_0^x \mathrm{d}\xi\, C_n^2(\xi)[(1-\xi/x)\,(1+(1-\xi/x)^2)^{-1/6}-(1-\xi/x)]\right)^2}{\int\limits_0^x \mathrm{d}\xi\, C_n^2(\xi)(1-\xi/x)^{5/3}\int\limits_0^\infty \mathrm{d}\xi\, C_n^2(\xi)},$$

which can be determined using a model of $C_n^2(x\xi)$.

The time correlation function of cross-correlation of the data of measurements and the corrected error of stellar image jitter are also analyzed in Ref. 3.

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