SIMULATIONS OF TURBULENT-INDUCED LOG-AMPLITUDE FLUCTUATIONS: THE KARHUNEN-LOEVE APPROACH

V.V. Voitsekhovich, V.G. Orlov, and S. Cuevas

Instituto de Astronomia UNAM, Mexico
Received July 29, 1996

A problem of simulations of the turbulent-induced log-amplitude fluctuations is considered. A method based on the Karhunen-Loeve functions allowing one to generate the samplings of log-amplitude fluctuations associated with weak-turbulence conditions is described. A validity of the approach presented is illustrated by comparison of the theoretical statistical characteristics to those obtained from simulations.

1. INTRODUCTION

Methods of quasirandom simulations of the turbulent-induced distortions are often used in atmospheric and adaptive optics.[1–8] These simulations allow one to solve a number of problems where it is difficult to apply an analytical treatment. The existing atmospheric applications are usually restricted to the generation of wavefront distortions. However, there are also some interesting problems dealing with an effect of log-amplitude fluctuations. Below we describe a generator of log-amplitude fluctuations operating on the basis of the Karhunen-Loeve (K-L) functions which allows one to simulate the log-amplitude fluctuations with given correlation function. We assume through the paper that these fluctuations are isotropic and produced by weak turbulence on the vertical propagation path.

2. KARHUNEN-LOEVE FUNCTIONS FOR LOG-AMPLITUDE FLUCTUATIONS

The K-L approach is convenient for simulations, because a random process of interest can be represented as a linear superposition of orthogonal functions with statistically independent coefficients, so each coefficient can be generated randomly. In order to construct a set of K-L functions $L(\boldsymbol{\rho})$, we need to solve the homogeneous integral equation in which a kernel is given by the correlation function $B(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$ of a random process under consideration:

$$\int\limits_G \mathrm{d}\boldsymbol{\rho}_1\, B(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)\, L(\boldsymbol{\rho}_1) = \lambda L(\boldsymbol{\rho}_2), \tag{1}$$

where G denotes the area in which the K-L functions have to be constructed.

Our main concern here is to simulate the log-amplitude fluctuations on the two-dimensional circular area, so the associated K-L functions have to be constructed at the same area. A similar problem has been solved by Wang and Markey[9] who calculated the K-L functions for the turbulent-induced wavefront distortions. It has been shown that the two-dimensional integral equation is reduced to the one-dimensional equation if the random process of interest is assumed to be isotropic, i.e.,

$$B(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = B(|\boldsymbol{\rho}_1 \$ \boldsymbol{\rho}_2|). \tag{2}$$

Since the log-amplitude fluctuations also can be considered as isotropic, we apply the same approach (we omit some details referring the reader to the Wang and Markey's paper). If the condition (2) is appeared, the functions L can be represented as a product of radial and angular factors:

$$L(\boldsymbol{\rho})\ K_p^q(\rho)\ \exp(iq\varphi), \tag{3}$$

where ρ and φ are polar coordinates of vector $\boldsymbol{\rho}$.

The corresponding one-dimensional integral equations for the radial K-L functions K_p^q are given by:

$$\int\limits_0^R \mathrm{d}\rho_1\, \rho_2\, B^q(\rho_1, \rho_2)\, K_p^q(\rho_1) = \lambda_{pq}^2\, K_p^q(\rho_2), \tag{4}$$

where R is the aperture radius, and the kernels $B^q(\rho_1, \rho_2)$ are expressed in terms of the log-amplitude correlation function B_χ as

$$B^q(\rho_1, \rho_2) = \int\limits_0^{2\pi} \mathrm{d}\psi\, B_\chi\left(\sqrt{\rho_1^2 + \rho_2^2 \$ 2\rho_1\rho_2 \cos\psi}\right) \cos(q\psi). \tag{5}$$

In what follows we assume that the log-amplitude fluctuations to be simulated are produced by the weak turbulence on the vertical propagation path. Under

0235-6880/96/11 957-02 $02.00

this condition, the log-amplitude correlation function B_χ can be written as[10]

$$B_\chi(x) = 2\pi^2\, k^2 \times 0.033 \int\limits_0^\infty \mathrm{d}z\, C_n^2(z) \times$$

$$\times \int\limits_0^\infty \mathrm{d}\xi\, \xi^{-8/3} J_0(\xi x) \left[1 \, \$ \, \cos\left(\frac{z\xi^2}{k}\right)\right], \tag{6}$$

where $C_n^2(z)$ is the vertical profile of the refractive index structure characteristic, J_0 is the Bessel function, and k is the wave number.

In order to get the final results in closed form, we apply the Hufnagel model[11] for $C_n^2(z)$. In r_0 parametrization (r_0 is the Fried parameter[12]), this profile can be expressed as:

$$C_n^2(z) = C_0\, r_0^{-5/3} k^{-2} \left[\left(\frac{z}{z_0}\right)^{10} \exp\left\{ \$ \frac{z}{z_1}\right\} + \exp\left\{ \$ \frac{z}{z_2}\right\}\right], \tag{7}$$

where $C_0 = 1.027 \cdot 10^{-3}$ m^{-1}, $z_0 = 4.632 \cdot 10^3$ m, $z_1 = 10^3$ m, and $z_2 = 1.5 \cdot 10^3$ m.

In Eqs. (6) and (7), the altitude z and the Fried parameter r_0 are given in meters, the wave number k is given in inverse meters, and the units of C_n^2 are meters to power $2/3.

The inner integral in Eq. (6) is expressed in terms of generalized hypergeometric function, and after that the outer integral is easily evaluated by means of the Gauss-Laguerre quadrature. The correlation function B_χ is plotted in Fig. 1 by the solid curve for r_0=0.1 m.

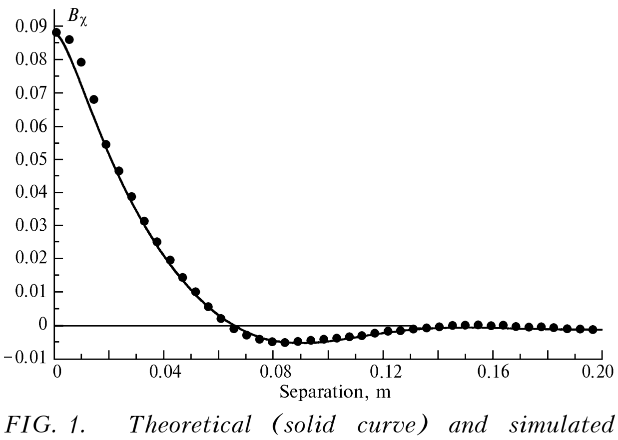

FIG. 1. Theoretical (solid curve) and simulated (points) log-amplitude correlation functions for r_0=0.1 m.

As soon as the function B_χ is calculated, the integral equations (4) can be solved numerically after symmetrization of the kernels applying the following variables[7]:

$$r_1 = \rho_1^2, \quad r_2 = \rho_2^2.$$

3. RESULTS OF SIMULATIONS

In previous Section, we have outlined the scheme of calculation of the K-L functions associated with log-amplitude distortions. Taking their superposition with random coefficients (the standards of generated coefficients have to be equal to the corresponding eigenvalues λ_{pq}), we obtain the samplings of interest. A number of samplings have been simulated and some statistical characteristics obtained making use of these samplings have been compared to the theoretical ones to check up the validity of simulations. The results are presented below.

Figure 1 shows the theoretical (solid lines) and experimental (points) log-amplitude correlation functions for the Fried parameter r_0=0.1 m. The experimental curves have been obtained applying an average over 10^4 samplings. One can see that the theoretical and simulated functions are very close to one another.

Let us discuss now some possible applications of the simulator presented. One application may be an analysis of the role of amplitude fluctuations in adaptive optics (in other words, how great the errors of wavefront measurements dealing with the influence of these fluctuations are). For instance, a pronounced influence may appear only if a scale of fluctuations is compatible with or greater than a zone of analysis. Such a situation takes place in modern adaptive systems with high-order wavefront correction where multi-zones schemes are used. Another interesting result may be obtained considering an effect of amplitude fluctuations on the formation of a speckle pattern produced by the atmospheric turbulence.

REFERENCES

1. B.L. McGlamery, Proc. SPIE **74**, 225$233 (1976).
2. N. Roddier, Proc. SPIE **1237**, 668$679 (1990).
3. N. Roddier, Opt. Eng. **29**, 1174$1180 (1990).
4. N. Takato, M. Iye, and I. Yamaguchi, Publications of the Astronomical Society of the Pacific **106**, 182$188 (1994).
5. M.C. Roggermann, B.M. Welsh, D. Montera, and T.A. Rhoadarmer, Appl. Opt. **34**, 4037$4051 (1995).
6. D. Kouznetsov, and R. Ortega-Martinez, Revista Mexicana de Fisica **41**, 563$571 (1995).
7. R.C. Cannon, J. Opt. Soc. Am. **A13**, 862$867 (1996).
8. H. Jakobsson, Appl. Opt. **35**, 1561$1565 (1996).
9. J.Y. Wang, and J.K. Markey, J. Opt. Soc. Am. **68**, 78$87 (1978).
10. V.I. Tatarski, *Wave Propagation in a Turbulent Medium* (McGraw-Hill, NewYork, 1961), 386 pp.
11. R.E. Hufnagel, in: *Optical Propagation through Turbulence*, OSA Technical Digest Series (OSA, Washington, D.C., 1974), pp. WA1-1$WA1-4.
12. D.L. Fried, J. Opt. Soc. Am. **56**, 1372$1379 (1966).