Reflectance of small-size optics

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Formulas for reflection coefficient (reflectance) of widespread small-size optics (television, video and photo cameras, binoculars, etc.) are derived, in which the receiver plays a part of the reflector. A generalized Airy formula is derived taking into account the phase shifts due to the surface curvature. It is shown that the front (input) surface of the objective is the main contributor to reflection coefficient of the small-size optics. This surface is usually an air–crown interface with reflectance of approximately 4%. The reflection coefficient of such optics, in its shape, coincides with the typical initial condition for the Gaussian beam. It is shown that the plane wave, incident on the small-size optics, is reflected in the form of a spherical wave.

In connection with the rapid development of optical observation systems, of interest is the question of reflection properties of observation devices themselves, when photoreceivers are considered as reflectors. In this paper, formulas for reflection coefficient of the widespread small-size optics (television, video and photo cameras, binoculars, etc.) are derived. Elements of this optics (objectives, oculars, prisms, etc.) are usually a combination of crown–flint glasses.

Based on the earlier results,[1,2] formulas for the reflection coefficient of actual optical systems, from which flares can be obtained, are analyzed theoretically. Of concern were the visible and, as a more interesting, near IR ranges.

First, let us give some definitions and derive formulas for the reflection coefficient from one interface. Estimate the coefficient of reflection from interface of typical media, used in traditional optical devices, namely air–glass and glass–glass. Then consider formulas for the reflection coefficient of multilayer media. These media have many interfaces in their interior. Obtain numerical estimates for the reflection coefficient of widespread optical systems.

It is well known that if an optical wave is incident on the interface of two homogeneous media with different optical properties, it is split into two waves: refracted (passing to the second medium) and reflected. Denote through θ_1 the angle of wave incidence upon interface from the first medium (numbered by *1*, Fig. 1). The incidence angle is the angle between the normal to the phase front of the wave and the normal to the interface surface. The incidence plane is the plane passing through both normals. In the second medium the wave propagates at the refraction angle θ_2 (angle between the same normals but in medium *2*, Fig. 1). The reflection angle is in the incidence plane and equals the incidence angle. The absolute refraction index n_k of the medium numbered by k is the index of refraction from vacuum to this medium. It is equal to the ratio of the light speed to the phase speed of the wave propagation in the medium k. The absolute refractive index n is connected with the medium dielectric (ε) and magnetic (μ) constants through the Maxwell formula $n^2 = \varepsilon\mu$. For transparent media, the magnetic constant practically does not differ from unity.

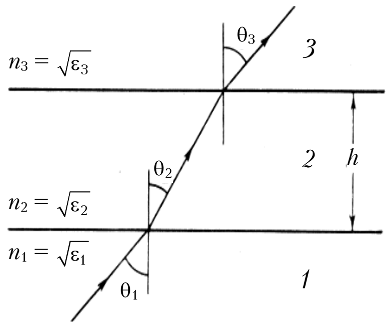

Fig. 1. Optical wave incidence and refraction at plane interfaces.[2]

According to geometric optics laws, the angles of incidence θ_1 and refraction θ_2 are related by the law of refraction (or the Snell's law): $n_1\sin\theta_1 = n_2\sin\theta_2$, where n_1 and n_2 are absolute refractive indices of media *1* and *2*. For small incidence angles θ_1 (when $\sin\theta_1 \approx \theta_1$), the refraction law is simplified to $n_1\theta_1 = n_2\theta_2$, and, correspondingly, for multilayer media $n_1\theta_1 = n_2\theta_2 = \ldots = n_k\theta_k$. When $n_2 > n_1$, the optical depth of the second medium is said to be larger than the optical depth of the first medium. In this case, it follows from the refraction law that $\sin\theta_2 < \sin\theta_1$, since for each incidence angle θ_1 there is a real refraction angle θ_2. However, if the second medium is optically thinner than the first medium ($n_2 < n_1$), then the real value of θ_2 can be obtained only for such incidence angles θ_1, for which $\sin\theta_1 < n_2/n_1$. For instance, if the second medium is air or vacuum ($n_2 = 1$), and the first medium is the glass with a typical refractive index ($n_1 = 1.5$), the real value of θ_2 is obtained for incidence angles $\theta_1 \le \theta_R = \arcsin(n_2/n_1) = 41.8°$. For large angles θ_1 ($\theta_1 > \theta_R$), a so-called total internal reflection takes place.

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Consider formulas for coefficient of reflection from one interface. The electric vector of the incident wave field is decomposed into two components: parallel (‖) and perpendicular (⊥) to the incidence plane. Then, the reflection coefficients with respect to the field (respectively $r_{\parallel}$ and $r_{\perp}$ for these two components) are defined by the Fresnel formulas

$$r_{\parallel} = \frac{n_2\cos\theta_1 - n_1\cos\theta_2}{n_2\cos\theta_1 + n_1\cos\theta_2}, \quad r_{\perp} = \frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2}. \quad (1)$$

In general case, these coefficients are complex. The squared modulus of the reflection coefficient with respect to the radiation intensity is usually called the reflectance. Therefore, for each of two components of the electric vector the reflectances $R_{\parallel}$ and $R_{\perp}$ can be represented in the form $R_{\parallel} = |r_{\parallel}|^2$, $R_{\perp} = |r_{\perp}|^2$. For the normal incidence ($\theta_1 = \theta_2 = 0$), the difference between parallel and perpendicular components disappears, and it follows from the Fresnel formula

$$R_{\parallel} = R_{\perp} = [(n_2 - n_1)/(n_2 + n_1)]^2. \quad (2)$$

It is seen that $R_{\parallel}$, $R_{\perp} \to 0$ at $n_2 \to n_1$. Hence, the less is the difference in the optical density of both media, the less is the energy carried away by the reflected wave.

Using the refraction law, the Fresnel formulas can be rewritten as:

$$r_{\parallel} = \frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)}, \quad r_{\perp} = -\frac{\sin(\theta_1 - \theta_2)}{\sin(\theta_1 + \theta_2)}. \quad (3)$$

The denominators in Eq. (3) are finite except when $\theta_1 + \theta_2 = \pi/2$. Then $\tan(\theta_1 + \theta_2) = \infty$ and, hence, $R_{\parallel} = 0$. In this case, the reflected and refracted beams are perpendicular; and it follows from the refraction law (since $\sin\theta_2 = \sin(\pi/2 - \theta_1) = \cos\theta_1$) that $\tan\theta_1 = n_2/n_1$. The angle defined by this formula is called the Brewster angle or the angle of the total polarization. If light falls at this angle, the electric vector of the reflected wave has no component in the incidence plane ($R_{\parallel} = 0$).

Figure 2 (Ref. 2) shows the dependence of the reflectance of the interface "air – typical glass" ($n_1 = 1$, $n_2 = 1.52$) on the incidence angle θ_1 (θ_2 is the refraction angle). The zero value of $R_{\parallel}$ on curve *3* corresponds to the Brewster angle $\theta_B = \arctan(1.52) = 56°40'$.

If the electric vector of the incident beam forms the angle α with the incidence plane, the total reflectance R of the interface is readily expressed in terms of the reflectance of mutually perpendicular components:

$$R = R_{\parallel}\cos^2\alpha + R_{\perp}\sin^2\alpha. \quad (4)$$

For the natural light emitted by a heated body, as well as for the unpolarized laser radiation, the direction of oscillations in the wave rapidly changes in an irregular, random manner. The corresponding total reflectance R can be obtained by averaging over all directions. Since the means of $\cos^2\alpha$ and $\sin^2\alpha$ are 1/2, we deduce from Eq. (4) that $R = (R_{\parallel} + R_{\perp})/2$. Also shown in Fig. 2 (curve *2*) is the dependence of the total reflectance of glass on the incidence angle. As is seen, the total reflectance of glass practically does not depend on the incidence angle in the considerable interval of the angles θ_1 ($R = 0.04$ from 0 to 40–50°). This interval of the incidence angles is sufficient for estimates of the reflection coefficient of actual optical systems. Angles θ_1 larger 40–50° can be omitted because the reflection angle is equal to the incidence angle (incidence of the wave becomes increasingly sliding). Data of Fig. 2 allow the estimation of R of one interface for typical media. In this case, formula (2) can be used at particular values of the refractive index.

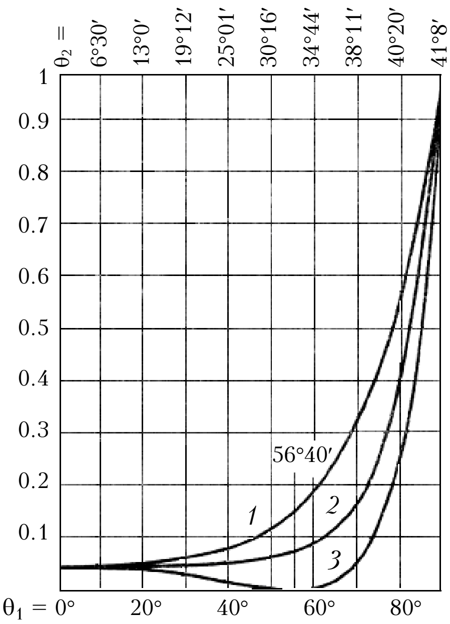

Fig. 2. Dependence of the reflectance R of glass with the refractive index 1.52 on the incidence angle θ_1: $R_{\perp}$ (*1*), $R = (R_{\parallel} + R_{\perp})/2$ (*2*); and $R_{\parallel}$ (*3*).

Table (Ref. 3) and Figure 3 (Ref. 2) present the refractive indices of different optical materials in the near IR region (λ = 0.80–1.1 μm). The dispersion dependence of the refractive index on the wavelength (Fig. 3) is weak for broadband optical receivers usually applied in practice. Traditional optical devices commonly use crown and flint glasses. As it follows from Table, the refractive index of crown glasses is less than that of flint glasses (on the average, 1.53 and 1.67, respectively). Therefore, the average reflectance R of the air–glass interface is 4.4% for crown glasses and 6.3% for flint glasses; correspondingly, minimum (maximum) reflectance is 3.5% (5.5%) for crown glasses and 5.5% (7.2%) for flint glasses. The reflectance is 0.19% for the interface "mean crown – mean flint," 0.24% for the interface "minimal crown – maximal crown" (or "minimal crow – minimal flint"), and 0.72% for the interface "minimal crown – maximal flint." It is clearly seen that the reflectance of the interface "air – glass" is much higher than that of the interface "glass – glass." The flint glasses are on the average much more expensive than the crown ones. Therefore, in production of widespread optical devices, the flint is often replaced by the crown.

Refractive index *n* of optical materials in the near IR region (at $\lambda = 0.80$–$1.1\ \mu m$)

Material	n
Glass:	
crown	1.46–1.61
flint	1.61–1.73
quartz	1.449–1.452
Crystalline quartz	1.53–1.54
Electro-optical materials	1.51–1.52
Iceland spar	1.64–1.65
Optical crystals	1.4–2.4

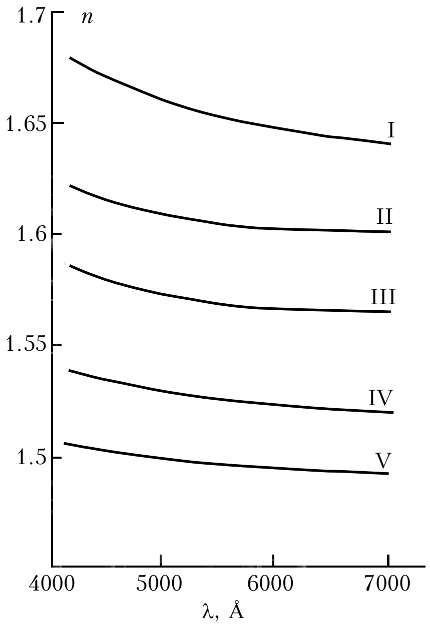

Fig. 3. Typical dispersions for glasses of different grades: heavy flint (I); heavy baric crown (II); light flint (III); heavy crown (IV); and borosilicate crown (V).

Now we can derive formulas for coefficient of reflection from multilayer media. These media contain many interfaces in their interior and are used in traditional optical systems. In practice, of the most use are refracting optical systems (refractors). Reflecting systems (reflectors) are commonly used in specialized devices (such as mirror telescopes) and are rarer in occurrence.

Traditional optical systems comprise: objectives, oculars, condensers, prisms, splitting plates, stops, etc. In turn, these elements themselves are frequently composite. For instance, objectives generally consist of collecting and dispersing lenses. Figures 4 and 5 present schemes of most typical objectives.[2]

The quality of an objective can be improved by reducing its total reflectance in a special way. Usually, this is achieved by spraying thin dielectric films onto its input surface (anti-reflectance optics) or by choosing a combination of lenses (often not thin), fabricated from glasses with different refractive indices. The general rule of decreasing the objective reflectance consists in decreasing abrupt changes in the refractive index when passing from the front boundary to the back one. Both these boundaries are usually in contact with air.

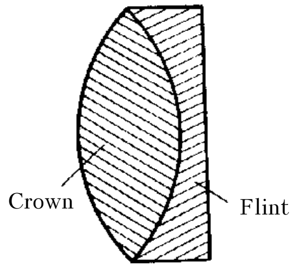

Fig. 4. Objective in the form of glued achromatic doublet.

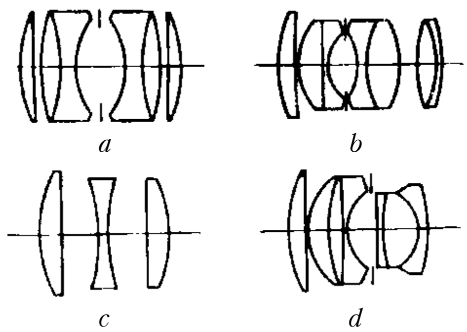

Fig. 5. Types of objectives: Planar (*a*); Zeiss biotar (*b*); Cooke triplet (*c*); Zonnar (*d*).

Jumps of the refractive index inside the objective decrease when the refractive indices of the front and back lenses are less than those of the center lenses. Therefore, the front and back lenses are frequently produced from crown glasses, while the center ones from flint glasses. Simple objectives (see Fig. 4), can be free of back lenses. Elements of the objective are generally glued together by special transparent glue in order to remove air gaps between them, which deteriorate the lens quality. The objectives with air gaps inside are applied, when, for example, it is necessary to widen the field of view due to somewhat longer inter-lens separation or to remove some aberrations. Such objectives are more specialized and used more rarely.

The characteristics of wave reflection from media with many interior interfaces can be estimated in terms of the theory of wave propagation in layered media.[1] This theory uses the notion of the medium impedance (i.e., the wave resistance, defined as the ratio of tangent components of electric and magnetic fields). For instance, if the electric vector of the incident wave field is perpendicular to the incidence plane ($\perp$), the impedance Z_k of the kth medium is defined as $Z_{k\perp} = 1/(n_k \cos\theta_k)$, where θ_k is the angle in the kth medium (see Fig. 6, where $k = n$).

In the case that the electric vector is parallel to the incidence plane ($\|$), then $Z_{k\|} = \cos\theta_k / n_k$. Note that the reflection coefficient with respect to the field of interface between the $(k+1)$th and kth media (wave incidence from the $(k+1)$th medium to the kth medium) can be represented in the form

$$r_{k,k+1} = (Z_k - Z_{k+1})/(Z_k + Z_{k+1}). \qquad (5)$$

Assuming $k = 1$ in Eq. (5) (wave incidence from the medium *2* to the medium *1*), substituting the indicated formulas for impedances $Z_{k\perp}$ and $Z_{k\parallel}$ in this equality, and taking into account the opposite ordering of layers in Figs. 1 and 6, we obtain Fresnel formulas (1), where $r = r_{1,2}$.

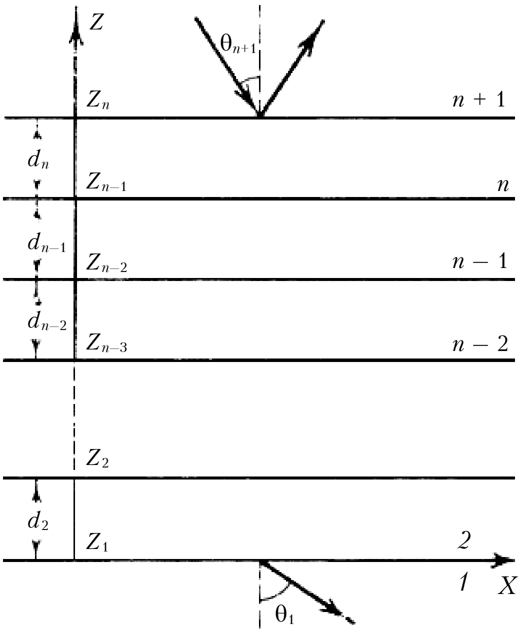

Fig. 6. Scheme of reflections and refractions in a system of plane layers.

The notion of impedance permits a substantial simplification of the formulas obtained for the reflection coefficient of the system of plane layers. Thus, if there is one reflecting layer ($n = 2$ in Fig. 6; in the general case, $n + 1$ is the total number of media, n is the number of interfaces, $n - 1$ is the number of reflecting layers, and the media 1 and $n + 1$ extend to infinity), the reflection coefficient with respect to the field r_1^L is expressed through the impedance Z_3 and input impedance of one layer $Z_{\text{in}}^{(2)}$:

$$r_1^L = \frac{Z_{\text{in}}^{(2)} - Z_3}{Z_{\text{in}}^{(2)} + Z_3}; \quad Z_{\text{in}}^{(2)} = \frac{Z_1 - iZ_2\tan\beta_2}{Z_2 - iZ_1\tan\beta_2} Z_2.$$

Here, $\beta_2 = k_0 n_2 d_2 \cos\theta_2$, where $k_0 = 2\pi/\lambda$ is the wavenumber in vacuum; d_2 is the layer thickness (thickness of the second medium, Fig. 6); n_2 and θ_2 are the refractive index and the angle in medium *2*. In the case, when the refractive indices of the media *1* and *2* are identical ($Z_1 = Z_2$, $Z_{\text{in}}^{(2)} = Z_2$), the layer defined by the medium *2* can be considered absent. Therefore, following Eq. (5), we have that $r_1^L = r_{2,3}$, i.e., the reflection coefficient of the absent layer coincides with the reflection coefficient of one interface. Reducing the number of interfaces by one ($n = 1$) and denoting through r_0^L the reflection coefficient for the no-layer case we have $r_0^L = r_{1,2}$.

The coefficient of reflection from $n - 1$ layers r_{n-1}^L is expressed via the impedance Z_{n+1} and the input impedance of $n - 1$ layers $Z_{\text{in}}^{(n)}$:

$$r_{n-1}^L = \frac{Z_{\text{in}}^{(n)} - Z_{n+1}}{Z_{\text{in}}^{(n)} + Z_{n+1}}; \quad Z_{\text{in}}^{(n)} = \frac{Z_{\text{in}}^{(n-1)} - iZ_n\tan\beta_n}{Z_n - iZ_{\text{in}}^{(n-1)}\tan\beta_n} Z_n, \qquad (6)$$

where $\beta_n = k_0 n_n d_n \cos\theta_n$. Formula (6) for the reflection coefficient of $n - 1$ layers r_{n-1}^L coincides in the form with formula (5) for the reflection coefficient of the interface $r_{n,n+1}$ between media n and $n + 1$ provided that in Eq. (5) the impedance Z_n is replaced by the input impedance $Z_{\text{in}}^{(n)}$. Applying in formulas (6) the equality following from the definition of tan, and taking into consideration Eq. (5), we finally obtain

$$r_{n-1}^L = \frac{r_{n,n+1} + r_{n-2}^L \exp(i2\beta_n)}{1 + r_{n,n+1} r_{n-2}^L \exp(i2\beta_n)}. \qquad (7)$$

Formula (7) is called the Airy formula; it is a recurrence relation, making it possible, at the known reflection coefficient of the interface between $n + 1$ and n media ($r_{n,n+1}$), to express the reflection coefficient of $n - 1$ layers (r_{n-1}^L) through the reflection coefficient of the less by one number of layers ($n - 2$ layers, r_{n-2}^L). The Airy formula should be supplemented with the initial condition $r_0^L = r_{1,2}$, corresponding to $r_{-1}^L = 0$.

The reflectance of the system of $n - 1$ layers R_{n-1}^L (or the reflection coefficient with respect to intensity), like for one interface, is the squared modulus of the reflection coefficient over the field. Based on the Airy formula, we can find the reflectance of a single layer R_1^L. Assuming $n = 2$ in Eq. (7), and taking into account that $r_0^L = r_{1,2}$, we have

$$R_1^L = \frac{r_{1,2}^2 + r_{2,3}^2 + 2r_{1,2}r_{2,3}\cos(2\beta_2)}{1 + r_{1,2}^2 r_{2,3}^2 2r_{1,2}r_{2,3}\cos(2\beta_2)}. \qquad (8)$$

This formula corresponds to any single component (either $\perp$, or $\parallel$) of the field electric vector. The choice of a particular component is determined by particular values of impedances (either $Z_{k\perp}$, or $Z_{k\parallel}$).

The oscillating character of the layer reflectance is seen from formula (8) at variations of $\beta_2 = k_0 n_2 d_2 \cos\theta_2$, which is a periodic function of the layer thickness; this phenomenon is confirmed experimentally and serves as a basis for optics anti-reflectance.

Figure 7 (Ref. 2) shows the reflectance of a dielectric film (one layer with refractive index n_2 and thickness $h = d_2$) as a function of its optical depth $n_2 h$ for normal incidence ($\theta_3 = \theta_2 = \theta_1 = 0$). The wave falls from air, passes through the film, and penetrates the glass with a refractive index of 1.5 (following the numbering in Fig. 6, $n_3 = 1$, $n_1 = 1.5$). It is seen that for the film with optical thickness of $\lambda/4$, $3\lambda/4$, $5\lambda/4$, ..., the reflectance reaches its maximum (minimum) depending on whether the refractive index of the film is larger (less) than that of the last medium. For the film with optical thickness of $\lambda/2$, $2\lambda/2$, $3\lambda/2$, ..., the converse is true. Quarter-wavelength film (with $n_2 < n_1$) is commonly used as an anti-reflection one. Figure 8 (Ref. 1) presents the wavelength dependence (for normal incidence, with the last medium being a flat glass) of the reflectance of typical multilayer anti-reflective coatings. As follows from Fig. 8, the reflectance of the anti-reflective coatings in the visible range ($\lambda = 0.45$–$0.6\ \mu$m) is minimal. When passing to the near IR ($\lambda > 0.6\ \mu$m) it grows (anti-reflectance disappears). However, even in the case of total loss of the anti-reflectance, the reflectance at the boundary of IR range ($\lambda = 0.7\ \mu$m) does not exceed 4%. The largest reflection of good (two- and three-layer) anti-reflective coatings is observed on the border with UV region ($\lambda < 0.43\ \mu$m). For this reason, as an example, the objectives of high-quality anti-reflection photo cameras have a violet tinge.

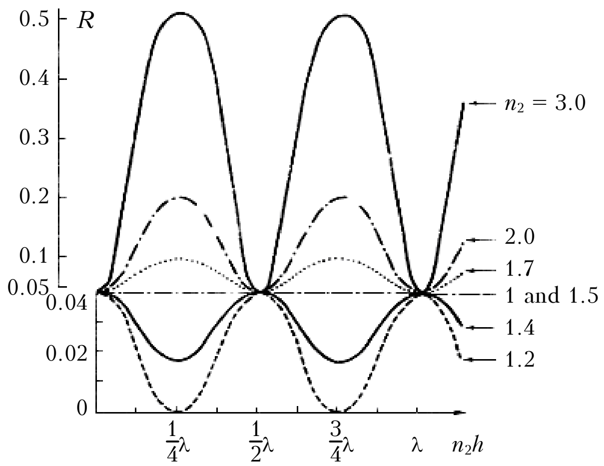

Fig. 7. Reflectance ($R = R^L_{1\perp}$) of dielectric film with refractive index n_2 as a function of its optical depth n_2h: $\theta_3 = 0$, $n_3 = 1$, $n_1 = 1.5$.

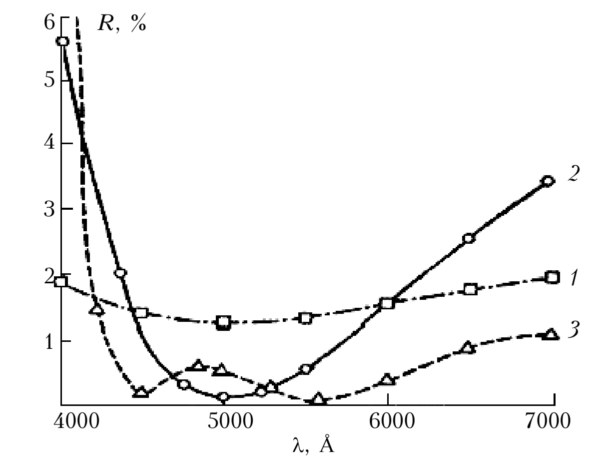

Fig. 8. Reflectance of typical anti-reflective coatings: one- (*1*), two- (*2*), and three- (*3*) layer coatings.

Demonstrate how the Airy formula (7) can be used to obtain simple estimates of the reflectance of multilayer media contained in traditional optical systems. Considering that the reflectance of the interfaces air – glass and glass – glass, as was shown above, makes a few percent, and that $r^L_{n-2}\exp(i2\beta_n)$ does not exceed unity in the absolute value, let us expand the denominator in formula (7) in powers of $r_{n,n+1}r^L_{n-2}\exp(i2\beta_n)$. Retaining only the first term of the expansion, we obtain

$$r^L_{n-1} = r_{n,n+1} + r^L_{n-2}\exp(i2\beta_n) + O(r^L_{n-2}r_{n,n+1}{}^2) + O(r_{n,n+1}r^L_{n-2}{}^2). \qquad (9)$$

When $|r^L_{n-2}| \to 1$, the omitted terms $O(r_{n,n+1}r^L_{n-2}{}^2)$ in Eq. (9) may be of the same order as the first term $r_{n,n+1}$ retained in Eq. (9). However, in this case, as is seen from Eq. (9), $r_{n,n+1}$ itself plays a role of small corrector. Hence, the terms $O(r_{n,n+1}r^L_{n-2}{}^2)$ can be disregarded.

Let us estimate the reflectivity of actual optical systems by the example of a typical model system containing an objective and an ocular. Considering this model system, it is possible to gain an impression of features of the reflection observed in many optical devices.

The system consisting of the objective and ocular has no less than four reflecting layers. For instance, the first two layers are the objective of the achromatic doublet type (see Fig. 4), the third layer is the air gap, and the fourth one is the ocular. The refractive index of the first layer is the mean value of the refractive index for crown glasses (1.53), and that of the second layer is the mean value for the flint glasses (1.67). The fourth layer (ocular) is assumed to be crown (1.53). The air media have a unit refractive index. As the thicknesses of layers (d_n in $\beta_n = k_0 n_n d_n \cos\theta_n$), their typical values are taken: $d_5 = 1$ cm (crown in the objective), $d_4 = 2$ cm (flint in the objective), $d_3 = 15$ cm (the distance between the objective and ocular), $d_2 = 1$ cm (crown in the ocular).

Take $n = 5$ in Eq. (9), which corresponds to four reflecting layers, five reflecting interfaces, and six media. The medium emitting the radiation is air. The medium number is 6 (Fig. 6). Applying sequentially the recurrence formula (9) and taking into consideration that $r^L_0 = r_{1,2}$, we have

$$r^L_4 = r_{5,6} + r_{4,5}\exp[i2\beta_5] + r_{3,4}\exp[i2(\beta_4 + \beta_5)] + r_{2,3}\exp[i2(\beta_3 + \beta_4 + \beta_5)] + r_{1,2}\exp[i2(\beta_2 + \beta_3 + \beta_4 + \beta_5)]. \qquad (10)$$

Thus, the reflection coefficient over the field for four reflecting plane layers is expressed as the sum (multiplied by oscillating factors) of reflection coefficients of five interfaces. Following Eq. (10), the reflectance of such four-layer medium R^L_4 (for any of the components: $\perp$ or $\|$) is written as follows:

$$R^L_4 = R_{5,6} + R_{4,5} + R_{3,4} + R_{2,3} + R_{1,2} + 2A; \qquad (11)$$

$$A = r_{5,6}r_{4,5}\cos(2\beta_5) + r_{4,5}r_{3,4}\cos(2\beta_4) + r_{3,4}r_{2,3}\cos(2\beta_3) + r_{2,3}r_{1,2}\cos(2\beta_2) + r_{5,6}r_{3,4}\cos[2(\beta_5 + \beta_4)] + r_{4,5}r_{2,3}\cos[2(\beta_4 + \beta_3)] +$$

$$+ r_{3,4}r_{1,2}\cos[2(\beta_3+\beta_2)] + r_{5,6}r_{2,3}\cos[2(\beta_5+\beta_4+\beta_3)] + \\ + r_{4,5}r_{1,2}\cos[2(\beta_4+\beta_3+\beta_2)] + \\ + r_{5,6}r_{1,2}\cos[2(\beta_5+\beta_4+\beta_3+\beta_2)].$$

Here, we took into account that, in the absence of total internal reflection (when the angles θ_k, $k = 1, ..., 6$, fall in the range 0–40°), the reflection coefficients of interfaces of nonabsorbing media are real.

Figures 9 and 10 show the total reflectance of the considered model optical system in the near IR range, calculated by formula (11).

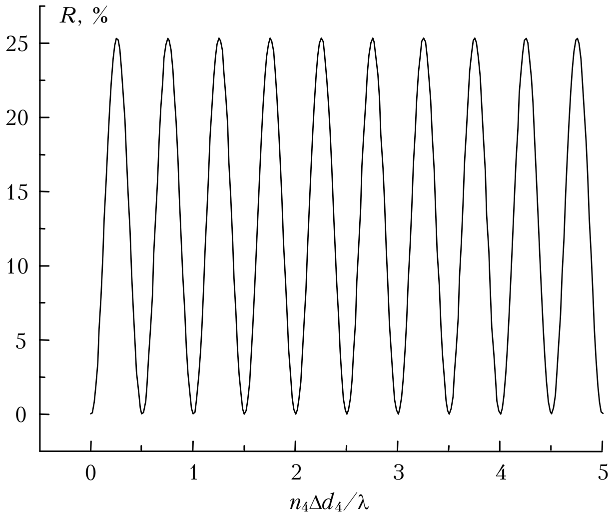

Fig. 9. Reflectance of the model optical system for the normal incidence: $\lambda = 1$ μm; $n_6 = n_3 = n_1 = 1$, $n_5 = n_2 = 1.53$, $n_4 = 1.67$; $d_2 = d_5 = 1$ cm, $d_3 = 15$ cm, $d_4 = 2$ cm; Δd_4 is deviation of the thickness of the fourth layer d_4.

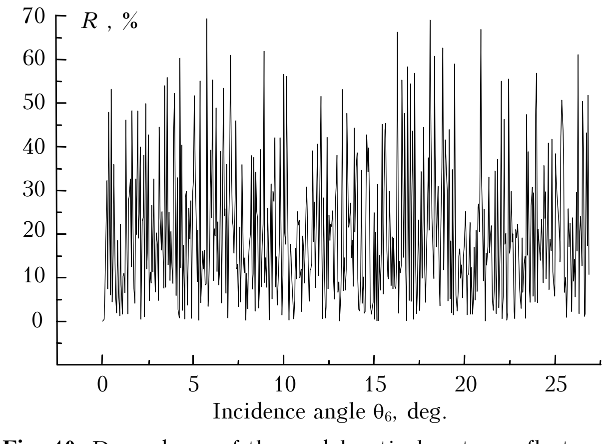

Fig. 10. Dependence of the model optical system reflectance on the incidence angle θ_6: $\lambda = 1$ μm; $n_6 = n_3 = n_1 = 1$, $n_5 = n_2 = 1.53$, $n_4 = 1.67$; $d_2 = d_5 = 1$ cm, $d_3 = 15$ cm, $d_4 = 2$ cm.

Figure 9 corresponds to variations of the optical depth of the medium 4 (flint, Δd_4 is the deviation of the layer thickness from the initial value) for the normal incidence ($\theta_k = 0$, $k = 1, ..., 6$), and Fig. 10 corresponds to variations of only input incidence angle θ_6 (all other angles also change because they are related to θ_6 by the refraction law).

It follows from Fig. 9 that at $\lambda/4$, $3\lambda/4$, $5\lambda/4$, ... increments of the flint glass optical depth ($n_4\Delta d_4$) the reflectance of the entire multilayer system is maximal, while at $\lambda/2$, $2\lambda/2$, $3\lambda/2$, ... increments it is minimal (anti-reflectance). This result agrees with the behavior of the reflectance of a single layer at variations of its optical depth, when the refractive index of the layer exceeds the refractive indices of other media (see Fig. 7). The dependence of the multilayer optical system reflectance on the incidence angle (see Fig. 10) is more complex, primarily because of the nonlinear variations of β_n (at increasing θ_n) in formula (11).

As shows from Figs. 9 and 10, the reflection coefficient of the multilayer optical system oscillates at optical frequencies (the frequencies are high due to high wavenumber k_0). Therefore, the slow time variation of the input incidence angle θ_6 (for example, at the entire system rotation with some angular velocity ω) leads to rapid change of β_n in formula (11).

Let equation (11) be time-averaged over the time interval T. This corresponds, for instance, to averaging of the received reflected energy by the photoreceiver. If $\theta_6 = \omega t$, the averaging of the first term in A in Eq. (11) at $T\omega\alpha_5 \gg 1$ gives

$$\frac{1}{T}\int_0^T \mathrm{d}t\cos[2\beta_5(t)] = \cos\left(2\alpha_5 + \frac{\pi}{4}\right)\frac{\sqrt{\pi}/2}{T\omega\alpha_5},\ \alpha_5 = k_0 n_5 d_5.$$

The analogous result is also obtained after averaging other terms in A. At $\lambda = 1$ μm, $d_5 = 1$ cm, and $n_5 = 1.53$, the inequality $T\omega\alpha_5 \gg 1$ can be rewritten as $\omega \gg 10^{-5}/T$. If the eye inertia constant $T = 1/25$ s is taken as the averaging interval, then, when observing the reflector by the eye (in the visible range immediately and in the IR range – on the display), this inequality holds provided the relative angular velocity of rotation of the system "eye – reflector" ω exceeds a few angular minutes per second. In this case, the oscillating terms in formula (11) are small, they can be neglected, and $A = 0$. Then the averaged total reflectance $<R_4^L>$ of the four-layer plane system is approximately equal to the summarized reflectance of all interfaces:

$$<R_4^L> = R_{5,6} + R_{4,5} + R_{3,4} + R_{2,3} + R_{1,2}. \quad (12)$$

For the considered model system, $<R_4^L> = 19.6\%$ ($R_{5,6} = R_{2,3} = R_{1,2} = 4.39\%$, $R_{4,5} = 0.19\%$, $R_{3,4} = 6.29\%$). Therefore, at the above rotation velocity ω the eye does not resolve variations in the level of the reflected energy, and this energy for eye is constant and equal to 19.6% of the energy incident upon the multilayer system. At a somewhat slower rotation, the eye will resolve changes in the reflected energy as a variable reflector brightness. In this case, as shows from Fig. 10, the reflection coefficient R can reach 70%.

Obviously, for the system with some arbitrary number of layers the formulas are similar to Eqs. (11) and (12). Therefore, the averaged reflectance of any multilayer system of the plane layers approximately is equal to summarized reflectance of all interfaces.

These results correspond to the system of plane layers. However, in actual optical systems the layers can be considered plane when dealing with sufficiently large-size optics with (on the average) large focal distances. In most widespread small-size optical devices, the curvature radii of surfaces are small, and the surfaces themselves have a close-to-spherical shapes.

In general, the interface curvature influence is quite complicated and this problem can be solved applying the corresponding equations describing the wave diffraction in the system of enclosed media. However, in the case of our interest, when the reflectances of interfaces are not large, the accounting for the effect of their curvature can be made in a standard way, i.e., by the calculations similar to those in Refs. 1 and 2.

Assuming that the wave field, incident on the multilayer system, is plane, we can determine the field, reflected from a certain chosen interface, and the field, having passed through it. The reflected field is then considered as the initial one and can be recalculated in the opposite direction. It is shown in Ref. 1 that if a spherical wave falls on the plane interface, the reflection coefficient remains almost the same and follows the Fresnel formulas. This means that for a small section of the curved surface the reflection coefficient remains the same. Locally, it is the same as for the plane interface. However, in contrast to the plane interface, the reflection coefficient of the close-to-spherical surface has the phase factor, which accounts for the phase shift due to the surface curvature. Thus, if $\rho = (x,y)$ are the transverse coordinates and $u_0(\rho)$ is the field incident on the interface $k+1 \to k$ (wave incidence from the medium $k+1$ to the medium k), then the field of the wave, passed through the interface and reflected from it, is represented as

$$u_0(\rho) w_{k,k+1} \exp\left\{-\frac{ik_0\rho^2}{2f_{k,k+1}}\right\}; \quad F^{\mathrm{T}}_{k,k+1} = \frac{F_{k,k+1}}{n_{k+1} - n_k};$$
$$F^{\mathrm{R}}_{k,k+1} = \frac{F_{k,k+1}}{2n_{k+1}}. \qquad (13)$$

Here $f = F^{\mathrm{R}}$, $w = r$ for the reflected wave; $f = F^{\mathrm{T}}$, $w = 1 + r$ for the transmitted wave (due to phase jump by π upon reflection[1]; the field coefficients of reflection r and transmission t are related by $t = 1 + r$); $r = r_{k,k+1}$ is the coefficient of reflection from the plane surface; $F_{k,k+1}$ is the curvature radius of the surface; n_{k+1} and n_k are refractive indices of the media $k + 1$ and k. If the interface is the collecting surface (convexity is directed toward the surface), then $F_{k,k+1}$ is negative; otherwise, if the surface is dispersing (the convexity coincides with the direction of wave propagation), it is positive. For the plane wave, $F_{k,k+1} = \infty$. When changing the wave propagation direction to the opposite one, the collecting surface becomes dispersing and the curvature radius changes its sign, i.e., $F_{k,k+1} = -F_{k+1,k}$. The difference $n_{k+1} - n_k$ also changes the sign, hence $F^{\mathrm{T}}_{k,k+1} = F^{\mathrm{T}}_{k+1,k}$.

It follows from representations (13) that, when the plane wave ($u_0(\rho) = u_0 = \mathrm{const}$) propagates from a thinner medium to a denser one ($n_{k+1} < n_k$), there appears focusing for the collecting interface ($F_{k,k+1} < 0$, $F^{\mathrm{T}}_{k,k+1} > 0$) and defocusing for the dispersing interface ($F_{k,k+1} > 0$, $F^{\mathrm{T}}_{k,k+1} < 0$). On the contrary, for the reflected wave the collecting surface gives defocusing ($F_{k,k+1} < 0$, $F^{\mathrm{R}}_{k,k+1} < 0$), and dispersing surface gives focusing ($F_{k,k+1} > 0$, $F^{\mathrm{R}}_{k,k+1} > 0$).

Accounting for formulas (13) in calculation of the field, reflected from each interface, shows that the Airy formula (10) is valid. For the four-layer medium, it can be written in the form

$$r_4^L = a_{5,6} r_{5,6} + a_{4,5} r_{4,5} \exp[i2\beta_5] +$$
$$+ a_{3,4} r_{3,4} \exp[i2(\beta_4 + \beta_5)] +$$
$$+ a_{2,3} r_{2,3} \exp[i2(\beta_3 + \beta_4 + \beta_5)] +$$
$$+ a_{1,2} r_{1,2} \exp[i2(\beta_2 + \beta_3 + \beta_4 + \beta_5)]. \qquad (14)$$

Here, as in Eq. (10), $r_{k,k+1}$ is the coefficient of reflection from the plane surface; $\beta_k = k_0 n_k d_k \cos\theta_k$; $a_{k,k+1}$ is the coefficient accounting for the surface curvature.

In the general case, the coefficients $a_{k,k+1}$ depend on the thicknesses of the layers and curvature radii of the surface, through which the incident wave has passed in forward and backward directions (to and from the boundary $k+1 \to k$). In the traditional small-size reflecting optics, the layer thicknesses in the objectives are much less than the curvature radii of their interfaces. This makes it possible to considerably simplify the formulas for the coefficients $a_{k,k+1}$. For multilayer system, containing N interfaces, after expressing the curvature radii of all interfaces through the curvature radius of the input surface $F_{N,N+1}$, we obtain

$$a_{k,k+1} = \exp\left\{-\frac{ik_0\rho^2 \nu_{k,k+1}}{F_{N,N+1}}\right\}, \quad 1 \le k \le N,$$
$$F_{k,k+1} = m_{k,k+1} F_{N,N+1}, \quad m_{N,N+1} = 1, \qquad (15)$$

where $\nu_{k,k+1}$ are calculated using the recurrent relation

$$\nu_{k-1,k} = \nu_{k,k+1} + n_k\left(\frac{1}{m_{k-1,k}} - \frac{1}{m_{k,k+1}}\right),$$
$$\nu_{N,N+1} = n_{N+1}, \quad 2 \le k \le N. \qquad (16)$$

Moreover, the effective focal distance F_j^S of the subsystem containing j interfaces (with j counted of from the input interface $N+1 \to N$) also depends on $\nu_{k,k+1}$:

$$F_j^S = F_{N,N+1}/[\nu_{S,S+1} - n_S/m_{S,S+1}],$$
$$S = N + 1 - j, \quad 1 \le j \le N. \qquad (17)$$

The focal distance F_j^S is positive (negative) for the focusing (defocusing) subsystem.

Let us estimate $\nu_{k,k+1}$ [and, hence, $a_{k,k+1}$ in Eq. (14)], corresponding to the objective in the considered model system. In this case, it is necessary to know the curvature radii of interfaces. As is seen from Figs. 4 and 5, the front surface in the objectives (input interface $N+1 \to N$, $N=5$, $F_{5,6}$) is the collecting surface, and the back surface is usually the dispersing one. The front layer (numbered N) usually is a symmetrical biconvex lens. Therefore, we can consider that $F_{4,5} = -F_{5,6}$ (or equivalently that $m_{4,5} = -1$). Then, $\nu_{5,6} = n_6 = 1$, $\nu_{4,5} = n_6 + n_5\ (1/m_{4,5} - 1) = n_6 - 2n_5 = -2.06$, where we have used particular values of refractive indices of the layers, assumed in the model system. Since the objective in the system consists of two layers (three interfaces), its effective focal distance F_3^S, following Eq. (17) is represented as

$$F_3^S = F_{5,6}/B,\ B = n_6 + n_5(1/m_{4,5} - 1) + n_4(1/m_{3,4} - 1/m_{4,5}) - n_3/m_{3,4}.$$

With known $m_{4,5}$ and B, this expression can be used for finding $m_{3,4}$ (and, hence, the curvature radius of the back surface of the objective $F_{3,4} = m_{3,4}F_{5,6}$). Substitution of the obtained expression for $m_{3,4}$ into definition of $\nu_{3,4}$ gives

$$\nu_{3,4} = [Bn_4 - n_3(n_6 - 2n_5 + n_4)]/(n_4 - n_3).$$

The objective is the focusing subsystem, therefore $F_3^S > 0$. Consequently, at $F_{5,6} < 0$ (the collecting input surface), $B < 0$. In the widespread optical systems, the focal distances of the objectives are usually less than the curvature radius of the front surface ($F_3^S < |F_{5,6}|$, $|B| \geq 1$) and much less than for the systems with ocular ($|B| \gg 1$). The calculation of $\nu_{3,4}$ (for assumed refractive indices) shows that at $B = -1, -2, -5, -10$ $\nu_{3,4} = -1.9, -4.4, -11.9, -24.3$, respectively.

Taking into account the obtained values of $\nu_{5,6}$, $\nu_{4,5}$, $\nu_{3,4}$, estimate now relative contribution of each term to the reflection coefficient of the four-layer medium (14). In the region where the directional pattern of the multilayer reflector has been formed (at a distance before the inlet pupil of the reflector, substantially exceeding both the length of the reflector itself and the largest radius from all curvature radii in the reflector), we obtain

$$|a_{4,5}/a_{5,6}| = |\nu_{5,6}/\nu_{4,5}| = 0.48;$$

$$|a_{3,4}/a_{5,6}| = |\nu_{5,6}/\nu_{3,4}| = 0.52\text{–}0.04.$$

For the reflectance, which is the squared reflection coefficient with respect to the field, the contribution of the second and third terms in Eq. (14) (due to $a_{k,k+1}$) are 23 and 27–0.16%, respectively, of the contribution of the first term. Hence, the main contributor to the reflection coefficient of the objective is the front interface. In contrast to the system of plane layers Eq. (12), this phenomenon is associated with the influence of the curvature of interfaces.

As the analogous analysis shows, deeper interfaces ($F_{2,3}$, $F_{1,2}$) make still less relative contribution to the reflectance. For instance, at $B = -2$, $m_{2,3} = -m_{3,4}/2$, $m_{1,2} = -m_{2,3}$, we have

$$|a_{2,3}/a_{5,6}|^2 = 12.7\%,\ |a_{1,2}/a_{5,6}|^2 = 0.7\%.$$

Moreover, the wave reflected from the deeper surfaces is subject to vignetting (cutting the beam part by the cylindrical mounting of optics). As a result, the reflection coefficient of such surfaces additionally decreases. Application of protective blinds to the objectives enhances the vignetting of all reflecting surfaces.

Taking into account the main contribution of the input surface, we obtain from formulas (14) and (15) the approximate expression for the reflection coefficient $r(\rho)$ of the small-size optics

$$r(\rho) = u_r \exp\left\{-\frac{\rho^2}{2a_r^2} - \frac{ik_0\rho^2}{2F_r}\right\},\ u_r = r_{N,N+1},$$
$$F_r = F_{N,N+1}/2 < 0, \tag{18}$$

where u_r, F_r, and a_r are the reflector parameters, namely, the reflection coefficient, focal distance, and radius; $r_{N,N+1}$ and $F_{N,N+1}$ are the reflection coefficient and curvature radius of the input collecting surface. The reflection coefficient (18) coincides in the form with the typical initial condition for the Gaussian beam.

As is seen from Eq. (18), the plane wave incident on the small-size optics is reflected in the form of a spherical wave, whose source is located at the distance $|F_{N,N+1}|/2$ behind the objective front surface. All other conditions the same, the cross-sectional area of the section, reflecting the radiation, which then comes to the photoreceiver, is less for collecting plane as compared to the plane one (the equivalent radius a_{rp} of the plane section, corresponding in energy to the reflection from the collecting surface, can be determined from the relation $k_0 a_{rp}^2 = F_r$). Therefore, the effective reflectance, determining the energy of different reflectors, is less for small-size optics than for a single plane surface.

Thus, in the traditional small-size reflecting optics with the close-to-spherical interface, the front input surface is the main contributor. This surface is most often the air–crown interface with a mean reflectance of about 4%. The reflectance of high-quality anti-reflective objectives in the IR range does not exceed 4%; therefore the reflection coefficient of the widespread optics does not exceed 4% on the average. On the whole, the reflection coefficient of small-size optical devices is much less than that of large-size optics with large focal distances and reflectance of 20% and larger (at the observation distances less than the focal distance, the large-size optics can be considered as a system of plane layers). The effective reflectance of small-size optics appears to be even less than that of a single plane surface.

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