# ON THE BACKSCATTERING INTENSIFICATION EFFECTS IN LASER SENSING THROUGH THE RANDOM BOUNDARY OF A RANDOMLY INHOMOGENEOUS MEDIUM

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*Simultaneous effect of the random interface and of the large–scale volume inhomogeneities of the medium on the backscattering intensification in laser sensing through the interface is considered. The approximate relations for the radiation pattern of the bounded beam, reflected from the thin diffusely scattering layer lying behind the random interface are derived.*

**1. Introduction.** In sensing the randomly inhomogeneous media, the description of the signal reception by collocated transmitting and receiving systems is difficult because it is necessary to take into account the coherent backscattering intensification effects, being outside the scope of the classical theory of radiative transfer. At present much attention is devoted to these effects (see, for example, Refs. 1–5) caused by various mechanisms.[6]

In this paper we consider the simultaneous effect of two factors, i.e., refraction on the random interface and scattering by the large–scale volume inhomogeneities of the medium. Such a problem can arise in many applications. It is sufficient to point out lidar sensing of the upper layer of the ocean[7,8] as well as sensing through glacial surfaces and snow covers. Here we will confine ourselves to the case of the large–scale media which allows a "causal" description within the frameworks of the parabolic approximation and the Huygens––Kirchhoff phase approximation[9,10] adjacent to it.[9,10] We take as a reflector the diffusely reflecting thin layer lying behind the random interface in a randomly inhomogeneous medium. We concentrate our attention on the description of radiation pattern in terms of the generalized brightness.[11] At present this notion is widely used in theoretical works and hardly ever in practical calculations. The demonstration of the convenience of this notion is one of the aims of this paper.

**2. Formulation and formal solution of the problem.** Let the source and the receiver be located in the plane $z = -d$ and sensing be carried out though the random interface at $z = \xi(\rho)$, where $\rho = (x, y, 0)$ ($\xi(\rho)$ is the random function with zero mean $\langle\xi\rangle = 0$), between the media with permittivities $\varepsilon_1 = 1$ and $\varepsilon_2 = \bar{\varepsilon}_2 + \tilde{\varepsilon}(\mathbf{r})$, where $\bar{\varepsilon}_2$ is the mean value, $\tilde{\varepsilon}_2(\mathbf{r})$ describes the weak fluctuations, $\langle\tilde{\varepsilon}_2\rangle = 0$, and $\langle\tilde{\varepsilon}_2^2\rangle \ll \bar{\varepsilon}_2^2$ (Fig. 1).

Assuming all inhomogeneities to be large–scaled, we employ the approximation of parabolic equation for the description of the propagation of narrow beams in each medium. The propagation through the random interface we also describe approximately, with the rough boundary being replaced by the plane phase screen positioned at $z = 0$ with the run–on of the phase $\psi(\rho) = (k_2 - k_1)\, \xi(\rho)$, where $k_{1,2} = k_0\sqrt{\bar{\varepsilon}_{1,2}}$ and $k_0$ is the wave number in free space.

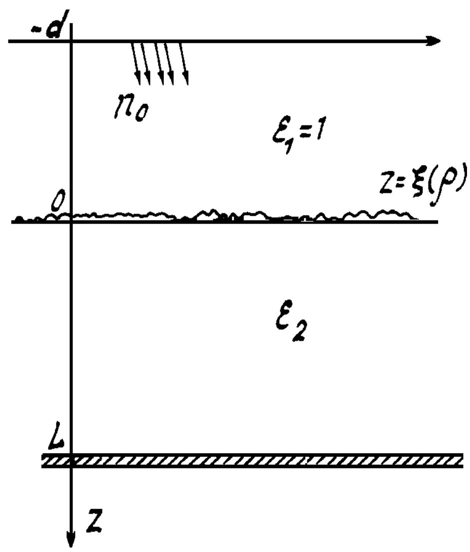


*FIG. 1. Geometrical diagram of the problem.*

In addition, we will assume that the incident wave is reflected in the plane $z = L$ from the thin layer of the small–scale scatterers described by the delta–correlated random reflection coefficient $r(\rho)$ ($\langle r\rangle = 0$, $\langle r(\rho)\, r(0)\rangle = R\delta(\rho)$, where $R \geq 0$ means the energetic reflection coefficient). For the bounded beam of radius $a$ the assumption on delta–correlated $r(\rho)$ is applicable only for not too short paths $L \gg kal_k$, where $k$ is the wave–number and $l_k$ is the characteristic size or the correlation length of the scatterers in the so–called statistically far zone of diffraction.[12]

Under these assumptions the propagation of the radiation along the section of the path $(-d, 0)$ is reduced to the problem of free diffraction of the partially coherent light. This problem has been studied in detail. As to the second moment of the radiation field, the description of the free propagation of radiation has the simplest form in terms of generalized brightness[11]

$$I(\mathbf{R}, \mathbf{n}) \equiv I(\mathbf{R}, \mathbf{n}, \mathbf{z}) =$$

$$= \left(\frac{k_0}{2\pi}\right)^2 \int \left\langle u\left(\mathbf{R} + \frac{\rho}{2}, \mathbf{z}\right) u^*\left(\mathbf{R} - \frac{\rho}{2}, \mathbf{z}\right)\right\rangle \times$$

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$$\times \exp(-ik_0\mathbf{n}\rho)\ d^2\rho\ , \qquad (1)$$

for which the transition from $z = -d$ to $z = 0$ is reduced to the translation of the argument

$$I(\mathbf{R}, \mathbf{n}, 0) = I(R - \mathbf{n}d, \mathbf{n} - d)\ , \qquad (2)$$

i.e., it is quite trivial. Taking this into account, we see that it is sufficient to examine the characteristics of the reflected beam in the plane $z = 0$, expressing them in terms of the analogous characteristics of the incident beam in the same plane, i.e., to describe the propagation of radiation along the path $0 \to L \to 0$. In the diagram notation this relation has the form

$$u^{(-)} = \times\!-\!0\!-\!\times\ u^{(+)}\ . \qquad (3)$$

Here $u^{(+)}$ and $u^{(-)}$ are the amplitudes of incident and reflected waves in the plane $z = 0$, the propagation lines correspond to the Green's operators

$$- = G^{(\pm)}_{z,\, z_0}\ , \qquad (4)$$

describing the propagation of radiation in the directions $\pm z$ and satisfying the parabolic equations, the symbol $t_\pm$ describes the propagation though the interface

$$\times = t_\pm e^{i\Psi}\ ,$$

$t_\pm$ are Fresnel's coefficients, characterizing the passage through the interface in the directions $\pm z$, which are taken for the normal incidence of the planar wave, and the symbol $0 = r(\rho)$ denotes the reflectance.

It follows from Eq. (3) that the moments of the $n$th order of the reflected wave amplitude are expressed in terms of analogous moments of the 2 $n$th order of the Green's function describing the propagation in the forward direction. Thus, for example, the coherence function of the reflected radiation field in the composite algebraic–diagram notation can be written as

$$\Gamma = \left\langle \begin{matrix} u(-) \\ u(-) \end{matrix} \right\rangle = \left\langle \begin{matrix} \times - 0 - \times \\ \times - 0 - \times \end{matrix} \right\rangle \Gamma_0\ , \qquad (5)$$

where the functions in the lower row are complex conjugate. Four propagation lines (—), i.e., the fourth moment of the Green's function, enter into relation (5).

Although relation (3) is formally the complete solution of the problem, the explicit form of the operators $G^{(\pm)}_{z,\, z_0}$ entering into it is unknown. Using Eq. (3) we can obtain the equations of the Markovian approximation for the arbitrary moments of the amplitude $u^{(-)}$ (Refs. 13–16 and 10) whose rigorous solution with the exception of the equations for the first moment $<u^{(-)}>$, is unknown either. Below we make use of the simpler Huygens–Kirchhoff phase approximation, first proposed in Ref. 9, which enables us to express arbitrary statistical moments of the reflected wave field in quadratures. In this approximation the unperturbed Green's function is multiplied by the phase factor $\exp(i\psi_\varepsilon(\rho, z))$, where $\psi_\varepsilon(\rho, z)$ is the run–on of the phase in the direction of the ineident beam, joining the points (0, 0) and $(\rho, z)$.

The use of the above–indicated approximation for the description of the volume inhomogeneities enables us to write easily relation (5) for $\Gamma$ in the explicit form

$$\Gamma(\mathbf{R}, \rho) = C \int \exp(2i\,\kappa\rho\,\delta - \varphi(\rho, \delta))\ \Gamma_0(\mathbf{R} - \delta, -\rho)\ d^2\delta\ , \quad (6)$$

where $C = \left| \frac{k}{\pi} t_+ t_- \right|^2 R$ and $\kappa = \frac{k^2}{2L}$. Analogous relation for the generalised brightness has the form

$$I(\mathbf{R}, \mathbf{n}) = \left| \frac{k_0}{2\pi} \right|^2 C \int\int\int \exp(2i\,\kappa\rho\delta - \varphi(\rho, \delta) -$$

$$- ik_0(\mathbf{n} + \mathbf{n}')\rho)\ I^0(\mathbf{R} - \delta, \mathbf{n}')\ d^2\delta\ d^2\rho\ d^2n'\ . \qquad (7)$$

Here $\Gamma_0$ and $I_0$ are the coherence function and brightness of the incident wave $u^{(+)}(\mathbf{R}, 0)$, the function $\varphi(\rho, \delta) = \varphi_\xi(\rho, \delta) + \varphi_\varepsilon(\rho, \delta)$ describes the simultaneous effect of the interface fluctuations $\xi$ and volume fluctuations $\varepsilon$,

$$\varphi_{\xi,\varepsilon}(\rho, \delta) = D_{\perp\xi,\varepsilon}(\rho) + D_{\perp\xi,\varepsilon}(\delta) - \frac{1}{2}(D_{\perp\xi,\varepsilon}(\rho + \delta) +$$

$$+ D_{\perp\xi,\varepsilon}(\rho - \delta))\ , \qquad (8)$$

where

$$D_{\perp\xi}(\rho) = <(\psi(\rho) - \psi(0))^2> \text{ and } D_{\perp\varepsilon}(\rho) = <(\psi_\varepsilon(\rho, 0) - \psi_\varepsilon(0, 0))^2> \qquad (9)$$

are the structure functions of corresponding runs–on of the phase. The fluctuations $\xi$ and $\varepsilon$ are assumed to be independent, Gaussian, and statistically uniform.

By integrating Eq. (7) over **R** we obtain the following relation between the radiation patterns of the reflected $J(\mathbf{n})$ and incident $J_0(\mathbf{n})$ beams:

$$J(\mathbf{n}) \equiv \int I(\mathbf{R}, \mathbf{n}) d^2R = \left| \frac{k_0}{2p} \right|^2 C \int\int\int \exp(2i\,\kappa\rho\delta -$$

$$- \varphi(\rho, \delta) - ik_0(\mathbf{n} + \mathbf{n}')\rho)\ J_0(\mathbf{n}')\ d^2\delta\ d^2\rho\ d^2\delta\ d^2n'\ . \qquad (10)$$

Relations (6) and (7) are equivalent and describe the problem in $(\mathbf{R}, \rho)$ and $(\mathbf{R}, \mathbf{n})$ representations, respectively.

It is convenient to calculate the reflected wave intensity $<|u^{(-)}(\mathbf{R})|^2> = \Gamma(\mathbf{R}, 0)$ in the plane $z = 0$ with the use of Eq. (6), while Eq. (10) yields the radiation pattern of the reflected beam.

**3. Unperturbed problem.** In order to estimate the contribution of fluctuations in the medium, let us first consider the unperturbed problem, i.e., without fluctuations. In this case $\varphi(\rho, \delta) \equiv 0$, so that relations (6), (7), and (13) take the form

$$\Gamma(\mathbf{R}, \rho) = C \int \exp(2i\kappa\rho\delta)\ \Gamma_0(\mathbf{R} - \delta, -\rho)\ d^2\delta\ ,$$

$$I(\mathbf{R}, \mathbf{n}) = C_0 \int I_0(\mathbf{R} - L(\mathbf{n} + \mathbf{n}')/\sqrt{\bar{\varepsilon}_2}, \mathbf{n}')\ d^2n'\ , \qquad (11)$$

$$J(\mathbf{n}) = C_0 \int J_0(\mathbf{n}')\ d^2n'\ ,$$

where $C_0 = \left| \frac{\kappa_0}{2\pi} t_+ t_- \right|^2 R$. In the simplest case of a collimated beam without reflection

$$I(\mathbf{R}, \mathbf{n}, 0) = I_0\,\Theta(\mathbf{R})\,\delta(\mathbf{n} - \mathbf{n}_0)\ , \qquad (12)$$

where $\Theta(\mathbf{R})$ describes the envelope ($\Theta(0) = 1$), we have

$$I(\mathbf{R}, \mathbf{n}) = C_0 I_0\, \Theta\, (\mathbf{R} - L(\mathbf{n} + \mathbf{n}_0)) ,$$

$$J(\mathbf{n}) = C_0 I_0\, \Sigma. \tag{13}$$

Here $\Sigma = \int \Theta(\mathbf{R})\, d^2R$ (14)

is the effective cross–sectional area of the beam. The physical sense of these relations is quite apparent, i.e., the collimated beam is incident at an angle to the reflecting plane located at $z = L$, as shown in Fig. 2.

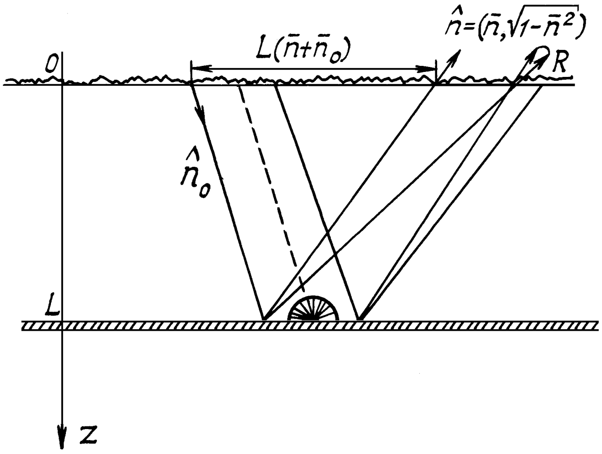



*FIG. 2. Reflection of the paraxial beam from the layer of the diffuse scatterers in the homogeneous medium: Unperturbed porblem.*

After reflection the beam becomes delta–correlated, i.e., isotropic, so that the radiation observed in the direction $\mathbf{n}$ can be seen as an illuminated spot in the plane $z = 0$ being displaced with respect to the initial spot along the vector $L(\mathbf{n} + \mathbf{n}_0)$. In addition, the angular width of the scattering phase function of the reflected radiation observed from the point $\mathbf{R}$ is determined by the observation angle of illuminated spot, as it is shown in Fig. 2, while the total brightness is isotropic. This simple geometry is described by relations (13).

**4. Effect of the fluctuations in the medium: the Gaussian approximation.** In evaluating the contribution of fluctuations in the medium to the backscattering intensification effect, we confine ourselves for simplicity to the case of a collimated beam (Eq. (12)). Then in accordance with Eq. (7)

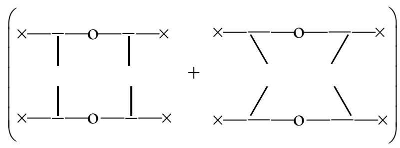
$$I(\mathbf{R}, \mathbf{n}) = \left| \frac{k_0}{2\pi} \right|^2 C I_0 \int\!\!\int \exp(2i\, \kappa\rho\delta - \varphi(\rho, \delta)) -$$

$$- ik_0(\mathbf{n} + \mathbf{n}_0)\rho)\, \Theta\, (\mathbf{R} - \delta)\, d^2\rho\, d^2\delta\ . \tag{15}$$

The value $\boldsymbol{\theta} = \mathbf{n} + \mathbf{n}_0$ entering into the integrand of Eq. (15) describes the angular deflection from the backscattering direction, so that for scattering in the backward direction the angle $|\boldsymbol{\theta}| = |\mathbf{n} + \mathbf{n}_0| = 0$. In addition, in the case of unbounded beam ($\Theta(\mathbf{R}) = 1$) the integral entering into Eq. (15) agrees to within a factor with the fourth moment (average squared intensity) for the planar incident wave. This agreement is due to the double passage of radiation through the same inhomogeneities of the medium. As a result, the calculation of the second moment of the reflected–wave field is equivalent to the calculation of the fourth moment of the wave propagating in the forward direction.

Significant difficulties arise when one attempts to estimate the multiple integral entering in to Eq. (15). In general, this estimate can be obtained only in the limiting cases of weak and strong fluctuations. In the first case various approaches of the perturbation theory may be employed, while in the second case the fluctuations of radiation incident on the reflecting plane are close to the Gaussian and, therefore, the calculation of the fourth moment of the radiation field may be reduced to the calculation of the second moments of this field. Let us first consider this simple case.

It is well known that the fluctuations of the wave incident on the reflecting plane are close to the Gaussian on the long path in the regime of multiple ray propagation (or in the saturation region), where many rays arrive at each point. For this region we can write down in the diagram notation of Eq. (5)

$\Gamma \simeq$  $\Gamma_0$ , (16)

where dashed lines denote pairwise averaging of joined multiplyers ×– and –×, i.e., an account of the Green's function correlations.

The correlation between forward and backward waves is neglected in the first term of Eq. (16), whereas in the second term this correlation is taken into account. Note that the Gaussian approximation (16), generally speaking, does not employ the Huygens–Kirchhoff phase approximation.

In accordance with Eq. (16), formula (15) can be approximately disintegated into two terms, corresponding to two terms in Eq. (16):

$$I(\mathbf{R}, \mathbf{n}) = I_1(\mathbf{R}, \mathbf{n}) + I_2(\mathbf{R}, \mathbf{n})\ . \tag{17}$$

Here $I_1$ and $I_2$ are given by Eq. (15), in which $\varphi(\rho, \delta)$ must be replaced by $\varphi_\rho \equiv \varphi(\rho) \equiv D_{\perp\xi}(\rho) + D_{\perp\varepsilon}(\rho)$ and by $\varphi_\delta = \varphi(\delta)$, respectively. Thus we have for $I_1$ the integral

$$I_1 = \left| \frac{k_0}{2\pi} \right|^2 C\, I_0 \int\!\!\int \exp\, (2i\kappa\rho\delta - \varphi_\rho -$$

$$- ik_0(\mathbf{n} + \mathbf{n}_0)\rho)\, \Theta\, (\mathbf{R} - \delta)\, d^2\rho\, d^2\delta\ , \tag{18}$$

which can be simplified in the case of the Gaussian envelope $\Theta(\mathbf{R}) = e^{-\alpha R^2}$

$$I_1 = \left| \frac{k_0}{2\pi} \right|^2 C\, \frac{\pi}{\alpha} I_0\, \times$$

$$\times \int \exp\left[\, 2i\, \kappa\rho(\mathbf{R} - L(\mathbf{n} + \mathbf{n}_0)) - \varphi_\rho - \frac{k^2\rho^2}{\alpha} \right] d^2\rho\ , \tag{19}$$

while $I_2$ can be calculated in the explicit form

$$I_2 = C_0 I_0\, \Theta\,(\mathbf{R} - L(\mathbf{n} + \mathbf{n}_0))\, \exp\, [\, - \varphi((\mathbf{n} + \mathbf{n}_0)\, L)]\,, \qquad (20)$$

and differs from the corresponding value in the case of free propagation described by Eq. (13) only in the exponential factor alone.

Integrating both sides of Eq. (17) over **R** we obtain for the total brightness the representation $J(\mathbf{n}) = J_1(\mathbf{n}) + J_2(\mathbf{n})$ in analogy with Eq. (20), in which the value

$$J_1(\mathbf{n}) = C_0\, \Sigma \qquad (21)$$

is isotropic, i.e., is independent of **n**, while the value

$$J_2(\mathbf{n}) = C_0\, \Sigma\, \exp\, [\, - \varphi((\mathbf{n} + \mathbf{n}_0)L)] \qquad (22)$$

is maximum in the backscattering direction.

Let us analyze the physical meaning of the obtained relations. The value $I_1$ describes the brightness distribution in the transverse cross section of the reflected beam disregarding the correlation between the forward and backward waves and $I_2$ describes this correlation in the Gaussian approximation. The meaning of $I_1$ can be easy understood from the step–by–step analysis of the wave propagation along the path $0 \to L \to 0$. During the passage of the section $0 \to L$, the incident collimated beam is broadened due to small–angle scattering, so that the illuminated spot in the plane $z = 0$, analogous to the unperturbed case, displays at the distance of the order of $L\mathbf{n}_0$, but has a larger diameter (Fig. 3). Radiation reflected from the plane $z = L$ is assumed to be isotropic and, in addition, during its propagation in the backward direction $L \to 0$ the reflected beam undergoes additional broadening due to small–angle scattering. As a result, the brightness distribution of the reflected beam $I_1$ is a function of the angle of observation of the illuminated spot in the plane $z = L$ and of the fluctuations in the medium. This can be seen from Eq. (18). In addition, in accordance with Eq. (21), the scattering phase function of the reflected beam as a whole remains isotropic.

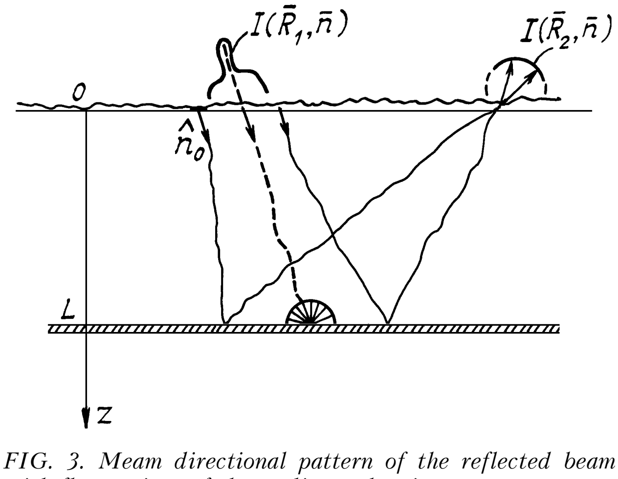



*FIG. 3. Meam directional pattern of the reflected beam with fluctuations of the medium taken into account.*

These considerations disregard the correlation between the forward and backward waves. These correlations are described by relations (20) and (22) and are the result of double passage of radiation through the same inhomogeneities of the medium. Since these inhomogeneities, generally speaking, have finite dimensions, the corresponding brightness $I_2$ given by Eq. (20), irrespective of the diameter of the illuminated spot in the reflection plane, is concentrated near the spot of the incident radiation at the distance $(\mathbf{n} + \mathbf{n}_0)L$, which is determined by the exponential factor in Eq. (20). The value of the maximum distance from the incident beam $\rho_0 = (\mathbf{n} + \mathbf{n}_0)L$, at which the intensification of backscattering is pronounced (i.e., the function $I_2$ differs strongly from zero), is estimated from the condition $\varphi(\rho_0) = 1$, so that $\rho_0$ coincides with the coherence length for the spherical wave after it has passed the path $(0, L)$.

The total intensity $I_2$ given by Eq. (22) and associated with the intensification effect is anisotropic. It is concentrated near the backscattering direction within the intensification angle $|\mathbf{n} + \mathbf{n}_0| = \theta \sim \rho_0/L$. For the strictly backward scattering direction $\theta = 0$ the contributions given by Eqs. (21) and (22) are identical, so that the intensification results in a twofold increase in the backscattering intensity.

The distribution of the generalized brightness in the cross section of the reflected beam is illustrated in Fig. 3. The distributions $I(\mathbf{R}, \mathbf{n})$ are shown for two points, that is, for $\mathbf{R}_1$ lying inside the illuminated spot where $I = I_1 + I_2$ has the maximum in the direction of backscattering, and for $\mathbf{R}_2$ lying outside the illuminated spot, where $I_2 = 0$ and the brightness of the reflected beam $I = I_1$ are practically isotropic.

**5. Simultaneous effect of the volume and interface fluctuations in the medium in the region of saturation.** In the region of the saturated intensity fluctuations the solution of problem (16) is expressed in terms of the second moments of the Green's function. Therefore, it is sufficient to consider the simultaneous effects of the volume and interface inhomogeneities on the second moments rather than on the fourth moments as in the general case described by Eq. (5).

Let us restrict ourselves to the description of the total brightness of the reflected beam $J = J_1 + J_2$ and define the amplification factor $\gamma$ as the ratio of the total brightness to its isotropic component, that is, $\gamma = (J_1 + J_2)/J_1$. In accordance with Eqs. (21) and (22), we have

$$\gamma - 1 = \exp\, (\, - \varphi(\theta L)) = \gamma_\xi(\theta L)\, \gamma_\varepsilon(\theta L)\,, \qquad (23)$$

where $\gamma_{\xi,\varepsilon} = \exp(\, - D_{\perp\xi,\varepsilon})$ and $\boldsymbol{\theta} = \mathbf{n} + \mathbf{n}_0$ is the angle of deflection from the backscattering direction.

According to Eq. (23), the angular dependence of the amplification coefficient is determined by the product of two factors associated with the interface fluctuations $\xi$ and volume inhomogeneities $\varepsilon$. In addition, in the Gaussian approximation the rate of decay of the amplification coefficient from the value $\gamma = 2$ for the backscattering direction to the value $\gamma = 1$ for larger angles is determined by the behavior of the structure phase functions $D_{\perp\xi}$ and $D_{\perp\varepsilon}$. In accordance with Eq. (9), these functions are expressed as follows:

$$D_{\perp\xi}(\rho) = k_0^2\, (\sqrt{\varepsilon} - 1)^2\, D_\xi(\rho)\,,$$

$$D_{\perp\varepsilon}(\rho) = \frac{k_0^2}{4}\, L \int_0^1 ds \int d\xi\, [D_\varepsilon(\rho s,\, \xi) - D_\varepsilon(0,\, \xi)]\,. \qquad (24)$$

Here $D_\xi(\rho) = \langle(\xi(\rho) - \xi(0))^2\rangle$ and $D_\varepsilon(\rho) = \langle(\varepsilon(\rho) - \varepsilon(0))^2\rangle$ are the structure phase functions of $\xi$ and $\varepsilon$.

It can be seen from Eq. (23) that the typical intensification angle $\theta_i$ near the backscattering direction, within which the intensification effect is strong, is of the order of $\theta_i = \rho_c/L$, where $\rho_c$ is the coherence length for the spherical wave transmitted through the path $(0, L)$, determined from the condition

$$D_{\perp\xi}(\rho_c) + D_{\perp\varepsilon}(\rho_c) = 1 . \qquad (25)$$

In the case of large run–on of the phase, the structure functions $D_{\perp\xi}$ and $D_{\perp\varepsilon}$ can be often approximated by the power–law functions assuming $D_{\perp\xi,\varepsilon}(\rho) = (\rho/\rho_{\xi,\varepsilon})^{\nu}_{\xi,\varepsilon}$ (for the one–scale nature of fluctuations $\gamma_{\xi,\varepsilon} = 2$ and for Kolmogorov's spectrum of turbulent fluctuations $\gamma_{\xi} = 5/3$). It can be easy seen that the parameters $\rho_\xi$ and $\rho_\varepsilon$ mean the coherence lengths corresponding to the fluctuations $\xi$ and $\varepsilon$, taken separately, and Eq. (25) can be considered as "nonlinear law of addition" of coherence lengths[17]

$$\left(\frac{\rho_c}{\rho_\xi}\right)^{\nu_\xi} + \left(\frac{\rho_\kappa}{\rho_\varepsilon}\right)^{\nu_\varepsilon} = 1 . \qquad (26)$$

In general this equation is transcendental, but in the case in which the values $D_{\perp\xi}$ and $D_{\perp\varepsilon}$ change similarly, the solution of the equation for $\nu_\xi = \nu_\varepsilon = \nu$ is trivial: $\rho_c = \rho_\xi\rho_\varepsilon/(\rho_\xi^\nu + \rho_\varepsilon^\nu)^{1/\nu}$.

The above–considered case corresponds to that when only the phase fluctuations are present in the medium. It can be shown that in the case in which the medium contains the particles (in general, the absorbing particles) being large in comparison with the wavelength, then in the right side of Eq. (23) the additional factor appears

$$\gamma_p(\theta L) = \exp\left[\, - D_p(\theta L)\right] , \qquad (27)$$

where the explicit form of $D_p(\rho)$ depends on the model of the particles. For example, for optically soft and uncorrelated particles we have[18]

$$D_p(\rho) = cL \int \langle \exp\left[i(l(\rho') - l^*(\rho + \rho'))\right] - 1\rangle\, d^2\rho' . \qquad (28)$$

Here $c$ is the particle number density and $l(\rho)$ is the additional run–on of the phase caused by the particle when the beam penetrates through it at the point $\rho$. In the case of strongly absorbing particles $\mathrm{Im}\, l(\rho) \gg 1$ Eq. (28) transforms into

$$D_p(\rho) = \int \langle 2\eta(\rho') - \eta(\rho')\, \eta(\rho - \rho')\rangle\, d^2\rho' , \qquad (29)$$

where $\eta(\rho)$ is the characteristic function which is equal to unity in the region of the shadow from the particle and to zero outside of the shadow, in accordance with the black screen model.[19]

**6. The effect of the fluctuations in the medium: the general case.** In general, in analogy with Gaussian approximation (16), the contribution of the fluctuations in the medium strongly depends on the shapes of spectra of $\xi$ and $\varepsilon$. The detailed study of Eqs. (6) and (7) providing the approximate solution of the problem, is rather a complicated problem and remains outside the scope of this paper. We only make some comments. First of all, note that in the case in which the observation point in the plane $z = 0$ is far from the illuminated spot, the direct and reflected waves propagates through the different inhomogeneities of the medium and, as a result, the correlation between them is negligible. Therefore, far from the illuminated spot the generalized brightness may be given by formula (19), in which this correlation is neglected. This formula takes into account the first term in the right side of Eq. (16).

Further, in general, in contrast to the case of the Gaussian approximation, the generalized brightness of the reflected wave is a function of the fourth moments rather than the second moments of the Green's function, so that the basic characteristic of the second moment, that is, the coherence length $\rho_c$, is no longer the basic characteristic parameter of the problem. The quantities, such as the characteristic curvature of the phase front (or the length of focusing) $F$, as well as the radius of the statistical Fresnel zone $\rho_\varphi$[20] which is related with $F$ by the equation $F = k\, \rho_\varphi^2$, start to play an important role along with $\rho_c$.

In order to illustrate the above discussion, we consider the case of the planar incident wave $I_0(\mathbf{R}, \mathbf{n}) = \delta(\mathbf{n} - \mathbf{n}_0)$. In this case for the backscattering direction $\mathbf{n} = -\,\mathbf{n}_0$ from Eq. (7) we have

$$\mathrm{I}(\mathbf{R}, -\mathbf{n}_0) = \left|\, \frac{k_0}{2\pi} \right|^2 C \int\!\!\int (2i\, \kappa\rho\delta - \varphi(\rho, \delta))\, d^2\rho\, d^2\delta \qquad (30)$$

(the same equation can be derived for the total intensity $J(-\,\mathbf{n}_0)$ of the bounded colimated beam given by Eq. (13)).

It is natural that relation (30) is independent of the choice of the point $\mathbf{R}$. It is of interest to examine Eq. (30) as a function of the reflecting layer depth $L$.

The integral appearing in Eq. (30) agrees to within the factor with the relation for the mean squared intensity of the radiation propagating behind the generalized phase screen.[20] The asymptotic behavior of these integrals in the case of strong fluctuations in the run–on of the phase was studied in the literature in detail (see Ref. 20). The typical behavior of Eq. (30) as a function of the path length $L$ in the case of the one–scale nature of fluctuations $\xi$ and $\varepsilon$ is wellknown. Starting from its initial value at $z = 0$, corresponding to the case without fluctuations, the right side of Eq. (30) increases up to its maximum value in the region of focusing $z \simeq F$, and then decreases down to the value being two times larger than the initial one in the saturation region. In the case of fluctuations with power–law spectrum the maximum of Eq. (30) is usually pronounced much weaker than in the case of the one–scale nature of fluctuations and may even be absent at all. The investigation of the reflected beam brightness in ample detail is beyond the scope of phase approximation (30) as well as the study of the behavior of the fourth moments of the Green's function which may be based, for example, on the well known equations of the Markovian approach.[12]

**7. On the possibility to compensate for the effect of the rough interface in laser sensing of the upper layer of the ocean.** The above–considered effects result in strong fluctuations in the pulse shape of the reflected beam in laser sensing of the upper layer of the ocean. The inhomogeneities are primarily related to the refraction at the water–air rough interface so that the pulse shape decay is no longer exponential and the spikes and fluctuations arise which may be especially strong in the region of focusing. For adequate description of the statistical properties of these fluctuations the characteristics of the surface roughness, including standard deviation of the heights of the surface roughness $\sigma_\xi$, its slopes $\sigma_\theta$ and curvature $\sigma_\nu$, are needed, in addition, the surface roughness curvature has the strongest effect on focusing.

In the case of the one–scale nature of fluctuations the curvature $\sigma_v$ is related with $\sigma_\theta$ and $\sigma_\xi$ by the expression $\sigma_v \sim \sigma_\theta^2/\sigma_\xi$. Thus, first of all it is necessary to determine two independent parameters. In order to estimate these parameters, the measurements of the parameters of the signal reflected from the interface may be used.

It is well known that in the case of large heights of the surface roughness the reflected radiation intensity is determined primarily by slopes, i.e., it is a function of $\sigma_\theta$, which can be retrieved from the data on the angular dependence of the backscattered signal. The second necessary parameter can be retrieved from any additional measurement, for example, of the radiation field correlation or of the correlation of reflected radiation intensity. After that, employing the model of surface roughness and using the above–given relations, the shape of the pulse averaged over the ensemble of realizations can be estimated, which in accordance with the above–indicated facts, generally speaking, is nonmonotonic and has the maximum in the region of random focusing.

**8. On other mechanisms of backscattering intensification effect.** Above we have considered the case of large–scale scattering media and random interfaces allowing the causal description. In the foregoing formulation of the problem, the solution has the relatively simple form because the radiation during its propagation undergoes only single backscattering. Meanwhile, the intensification effects can be observed in many other cases under conditions of both single backscattering and multiple backscattering. In practice we have the variety of different physical mechanisms of the intensification which are not considered in the paper. The great number of these mechanisms were summarized in Ref. 6, in which many problems were mentioned, which were responsible for the intensification, such as various cases of scattering by the discrete and continuous inhomogeneities and by the transparent phase and absorbing scatterers, of propagation through the random interfaces, and of reflection from the rough surfaces in the regimes of near – and far–difraction zones with respect to the reflectors. In some cases an account of the intensification effect may result in only small corrections, whereas in the others the intensification effect produces a significant change in the averagely reflected radiation intensity.

Thus, when describing backscattering in the randomly inhomogeneous media, one must take into account the backscattering intensification effect whose ignorance may lead to large errors.

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