PROBLEM ON RECONSTRUCTION OF THE WAVE FIELD FROM A 3D-IMAGE

S.M. Chernyavskii

A.N. Tupolev State Technical University, Kazan'
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It is proved using the momentum method that under some limitations the pupil function can be unambiguously determined, accurate to insignificant complex constant, from the intensity distribution over the volume containing focal plane. New momentum relationships are obtained that allow identification of the phase distortions by means of an adaptive mirror.

One of the central problems in adaptive optics is the problem on determining the wave front (WF) that forms image in an optical system. Many methods have been proposed for its solving, and among them the method that uses only the information on the intensity distribution $I(x, y)$ in the plane of image recording.

Its main advantage is simple performance. If one describes the wave field by the pupil function[1] $G(\xi, \eta) = P(\xi, \eta)\exp(i\phi(\xi, \eta))$, where the function $P(\xi, \eta)$ describes the field amplitude attenuation in the output pupil Ω, and the function $\phi(\xi, \eta)$ is the phase distortion of the field, then the problem is reduced to determining $\phi(\xi, \eta)$ from $I(x, y)$ with known or unknown function $P(\xi, \eta)$. Correspondingly this is called the phase (PP) or wave (WP) problem of optics.

Two groups of papers can be isolated here. The first one is devoted to solving PP using $I(x, y)$ from a point source. Authors of Ref. 2 have studied the solution of PP for one-dimensional case and have shown that it has no a unique solution. Iteration algorithm for solving PP for a two-dimensional case is proposed in Ref. 3, and it is shown that it converges (although slowly) for quite a wide set of functions $\phi(\xi, \eta)$ except for some subset of the zero measure. Then the qualitative question on whether is the information on phase $\phi(\xi, \eta)$ contained in the distribution $I(x, y)$ sufficient or not in PP is answered for the two-dimensional case important for applications.

The approach to solving PP connected with the representation of the phase by the finite part of the series over a system of basis functions seems to be natural. Then PP is reduced to determining the coefficients of the series $I(x, y)$. The coefficients are determined based on the condition of minimization of the discrepancy between the measured intensity and calculated by means of the diffraction integral. However, the numerical simulation of solution of PP in such a statement gives quite good result,[4] but only at small ϕ. An attempt to extend the applicability of such an approach to moderate values of ϕ has led to solving PP using the intensity distribution not over one but over several parallel planes,[5] i.e., one can say about solving PP using the three-dimensional image containing the focal plane.

The second group of papers deals with solution of WP using the distribution of intensity from an arbitrary extended source. Different approaches to solving it were discussed in Ref. 6. Let us note the momentum method,[7,8] where the intensity distribution $I(x, y, z)$ over the image space is replaced by the distribution of the planar momenta:

$$M_{st}(z) = \iint I(x, y, z)\, x^s\, y^t\, \mathrm{d}x\, \mathrm{d}y\ . \qquad (1)$$

Although it can happen that the momenta of the high order do not exist, some of their derivatives $\mathrm{d}^n M_{st}(0)/\mathrm{d}z^n$ exist[18] and are closely related to the phase ϕ gradient.

Based on the momentum method, we prove in this paper that the volume intensity distribution $I(x, y, z)$ unambiguously solves WP, and we obtain the momentum relationships based on the Parseval equality in a simpler way than in Ref. 8. In addition, new momentum relationships are obtained that can be used, together with the active methods of reconstruction of the phase,[6] for solving WP.

MOMENTUM RELATIONSHIPS

An invariant imaging optical system is described for the incoherent radiation by the convolution[1]:

$$I(x, y) = \iint h(x - x_0, y - y_0)\, I_0(x_0, y_0)\, \mathrm{d}x_0\, \mathrm{d}y_0\ , \qquad (2)$$

where $I(x, y)$ is the intensity at the point (x, y) of the image (measurement) plane; $I(x_0, y_0)$ is the intensity at the object plane point, which has a paraxial image at the point (x_0, y_0); $h(x, y)$ is the point spread function. If (x, y) are the optical coordinates in the image plane, and (ξ, η) are the relative coordinates of the point in the output pupil plane, then[1]

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$$h(x,y) = |g(x,y)|^2,$$
$$g(x,y) = \iint_\Omega G(\xi,\eta) \exp(-i(x\xi + y\eta))\, d\xi\, d\eta.$$

Let us suppose that $P(\xi,\eta) > 0$ is a function limited on Ω, and the function $\phi(\xi,\eta) \in W_2^1(\Omega)$, the Sobolev function space[9] on Ω.

Let us derive the momentum relationships by passing to the limit and supposing for the beginning that P and ϕ are quite smooth functions, for which the momenta considered below exist. Let us substitute Eq. (2) for I into Eq. (1), then

$$M_{st} = \iint x^s y^t \times$$
$$\times \left(\iint h(x - x_0, y - y_0)\, I_0(x_0, y_0)\, dx_0\, dy_0 \right) dx\, dy,$$

and after the substitution $x = (x - x_0) + x_0$, $y = (y - y_0) + y_0$, we obtain

$$M_{st} = \sum_{k=0}^{s} \sum_{l=0}^{t} C_s^k C_t^l \times$$
$$\times \left(\iint (x - x_0)^{s-k} (y - y_0)^{t-l} h(x - x_0, y - y_0)\, dx\, dy \right) \times$$
$$\times \left(\iint x_0^{s-k} y_0^{t-l} I_0(x_0, y_0)\, dx_0\, dy_0 \right) =$$
$$= s!\, t! \sum_{k=0}^{s} \sum_{l=0}^{t} \frac{H_{s-k,\, t-l}}{(s-k)!\,(t-l)!} \frac{N_{kl}}{k!\, l!},$$

where $H_{st} = \iint x^s y^t h(x, y)\, dx\, dy$;

$N_{st} = \iint x_0^s y_0^t I_0(x_0, y_0)\, dx_0\, dy_0$ are the momenta of the functions $h(x, y)$ and $I_0(x_0, y_0)$. In particular, $M_{00} = H_{00} N_{00}$.

Assuming that $m_{st} = \dfrac{1}{s!\, t!} \dfrac{M_{st}}{M_{00}}$, $h_{st} = \dfrac{1}{s!\, t!} \dfrac{H_{st}}{H_{00}}$, $n_{st} = \dfrac{1}{s!\, t!} \dfrac{N_{st}}{N_{00}}$, we obtain the momentum model of the image formation

$$m_{st} = \sum_{k=0}^{s} \sum_{l=0}^{t} h_{s-k,\, t-l}\, n_{kl}. \tag{3}$$

Let us express the momenta h_{st} in terms of the pupil function G. From the Parceval equality, we have

$$H_{st} = \iint x^s y^t |g(x, y)|^2\, dx\, dy = \iint (x^s g(x, y)) \times$$
$$\times (y^t g(x, y))^*\, dx\, dy = \frac{4\pi^2 (-1)^t}{i^{s+t}} \times$$
$$\times \iint (G G_0)^{(s)} (G G_0)^{*(t)}\, d\xi\, d\eta,$$

where asterisk denotes the complex conjugation; (s) and (t) are the signs of the order of derivatives with respect to ξ and η, respectively; $G_0 = \exp(-iu(\xi^2 + \eta^2)/2)$ is the phase factor that has been introduced here to take into account the influence of the optical coordinate u, proportional to the coordinate z, on the h_{st} and m_{st} moment distributions. Then

$$h_{st} = \frac{1}{s!\, t!} \frac{(-1)^t}{i^{s+t}} \times$$
$$\times \iint (G G_0)^{(s)} (G G_0)^{*(t)}\, d\xi\, d\eta \Big/ \left(\iint P^2\, d\xi\, d\eta \right). \tag{4}$$

It is easy to check the validity of expressions for partial derivatives with respect to ξ:

$$G_0' = \gamma\, \xi\, G_0,\quad G_0'' = (\gamma^2 \xi^2 + \gamma)\, G_0,$$
$$G_0^{(n)} = (\gamma^n \xi^n + C_n^2 \gamma^{n-1} \xi^{n-2} +$$
$$+ C_n \gamma^{n-2} \xi^{n-4} + \ldots)\, G_0, \tag{5}$$

where $\gamma = -iu$; C_n^2 is the number of combinations; C_n is the coefficient determined by the recurrence formula: $C_3 = 0$, $C_n = C_{n-1} + 3\, C_{n-1}^3$ for $n \geq 4$; ellipses designate the terms proportional to γ to lower powers;

$$G' = (P' - iP\, \phi')\, \exp(i\phi),$$
$$G'' = [(P'' - P\, \phi'^2) + i\,(2\, P'\, \phi' + P\, \phi'')]\, \exp(i\phi). \tag{6}$$

Analogous expressions can be obtained for the partial derivatives with respect to η.

Taking into account the Leibnitz formulas and Eqs. (5) and (6), one can write the derivative of the product $(GG_0)^{(s)}$ with respect to ξ in the form:

$$(G G_0)^{(s)} = G\, G_0^{(s)} + C_s^{s-1}\, G'\, G_0^{(s-1)} +$$
$$+ C_s^{s-2}\, G''\, G_0^{(s-2)} + \sum_{k=0}^{s-3} C_s^k\, G^{(s-k)}\, G_0^{(k)} =$$
$$= (a_s \gamma^s + a_{s-1} \gamma^{s-1} + a_{s-2} \gamma^{s-2} + \ldots)\, \exp(i\phi)\, G_0,$$

where

$$a_s = P\xi^s;$$
$$a_{s-1} = C_s^{s-1} G'\, \xi^{s-1} + P C_s^2\, \xi^{s-2} = G'(\xi^s) + P(\xi^s)''/2;$$
$$a_{s-2} = C_s^{s-2} G''\, \xi^{s-2}\, C_s^{s-1} C_{s-1}^2 G'\, \xi^{s-3} + P C_s\, \xi^{s-4} =$$
$$= P\, C_s\, \xi^{s-4} + G''(\xi^s)''/2 + C'(\xi^s)'''/2.$$

Similarly written can be the expression for the derivative with respect to η:

$$(GG_0)^{(t)} = (b_t \gamma^t + b_{t-1} \gamma^{t-1} +$$
$$+ b_{t-2} \gamma^{t-2} + \ldots)\, \exp(i\phi)\, G_0.$$

Supposing $\gamma = -iu$, we obtain

$$(G\,G_0)^{(s)}\,(G\,G_0)^{*(t)} = i^{t+s}\,(-1)^s\,(a_s\,u^s + i\,a_{s-1}\,u^{s-1} - a_{s-2}\,u^{s-2} + \ldots)(b_t^*\,u^t - i\,b_{t-1}^*\,u^{t-1} - b_{t-2}^*\,u^{t-2} + \ldots) = i^{t+s}\,(-1)^s\,[a_s\,b_t^*\,u^{s+t} - i\,(a_s\,b_{t-1}^* - a_{s-1}\,b_t^*)u^{s+t-1} - (a_s\,b_{t-2}^* - a_{s-1}\,b_{t-1}^* + a_{s-2}\,b_t^*)\,u^{s+t-2} + \ldots].$$

Since h_{st} is real, the imaginary part in brackets should be equal to zero, and the real parts of the corresponding terms are:

$$\mathrm{Re}\,a_s\,b_t^* = P^2\,\xi^s\,\eta^t\,,$$

$$\mathrm{Re}\,i\,(a_s\,b_{t-1}^* - a_{s-1}\,b_t^*) = P^2\,\phi'_\eta\,\xi^s\,(\eta^t)' + P^2\,\phi'_\xi\,(\xi^s)'\,\eta^t = P^2\,\mathrm{grad}\,\phi\cdot(\xi^s\,\eta^t),$$

$$\mathrm{Re}\,a_s\,b_{t-2}^* = [P\,C_t\,\eta^{t-4} + P'_\eta\,(\eta^t)''' + (P''_\eta - P\,\phi'^2_\eta)\,(\eta^t)'']\,P\,\xi^s/2,$$

$$\mathrm{Re}\,a_{s-2}\,b_t^* = [P\,C_t\,\eta^{t-4} + P'_\xi\,(\xi^s)''' + (P''_\xi - P\,\phi'^2_\xi)\,(\xi^s)'']\,P\,\eta^t/2,$$

$$\mathrm{Re}\,a_{s-1}\,b_{t-1}^* = (P'_\xi\,(\xi^s)' + P\,(\xi^s)''/2)\times(P'_\eta\,(\eta^t)' + P\,(\eta^t)''/2) + P^2\,(\xi^s)'\,(\eta^t)'\,\phi'_\xi\,\phi'_\eta,$$

$$\mathrm{Re}\,(a_s\,b_{t-2}^* - a_{s-1}\,b_{t-1}^*\,a_{s-2}\,b_t^* = F_{st}(P) - (\varphi''_{st}\,\phi',\,\phi')\,P^2/2,$$

where $F_{st}(P)$ is the expression depending on P and independent of ϕ; $\phi' = (\phi'_\xi, \phi'_\eta)$ is the vector of derivatives of the function ϕ; $\varphi'_{st} = ((\varphi_{st})'_\xi, (\varphi_{st})'_\eta)$, is the vector of derivatives of the function φ_{st}; φ''_{st} is the matrix of second derivatives of the function $\varphi_{st} = \xi^s\eta^t$.

Thus, we have obtained the following expression for the product:

$$(G\,G_0)^{(s)}\,(G\,G_0)^{*(t)} = i^{t+s}\,(-1)^s\,[P^2\,u^{s+t} + P^2(\phi',\,\varphi'_{st})\,u^{s+t-1} + (F_{st}(P) - (\varphi''_{st}\,\phi',\,\phi')\,\,P^2/2)\,u^{s+t-2} + \ldots]. \quad (7)$$

Substituting Eq. (7) into Eq. (4), we obtain an explicit from for the dependence of the momenta h_{st} on the pupil function and on spatial coordinate along the optical axis:

$$h_{st}(u) = (-1)^{t+s}/s!\,t!\,[u^{s+t}\iint P^2\,\varphi_{st}\,d\xi\,d\eta - u^{s+t-1}\iint P^2(\phi'\,\varphi'_{st})\,d\xi\,d\eta + u^{s+t-2}/2\iint P^2(\varphi''_{st}\,\phi',\,\phi')\,d\xi\,d\eta - u^{s+t-2}\iint F_{st}(P)\,d\xi\,d\eta + \ldots]. \quad (8)$$

In order to avoid introduction of any additional variables, we assume that P^2 in Eq. (8) and everywhere below to be defined by the expression

$$P^2/\iint P^2\,d\xi\,d\eta,$$

which has a meaning of the nonuniformity coefficient of the attenuation, over aperture, of the intensity of wave that forms the image.

It is seen from Eq. (8) that although the momentum h_{st} is a complex function of the pupil function, its higher orders derivatives with respect to u at the point $u = 0$ have a more simple dependence:

$$\frac{d^{s+t}\,h_{st}(0)}{du^{s+t}} = r\iint P^2\,\varphi_{st}\,d\xi\,d\eta, \quad (9)$$

$$\frac{d^{s+t-1}\,h_{st}(0)}{du^{s+t-1}} = r_1\iint P^2(\phi',\,\varphi'_{st})\,d\xi\,d\eta, \quad (10)$$

$$\frac{d^{s+t-2}\,h_{st}(0)}{du^{s+t-2}} = r_2\iint [F_{st}(P) - P^2(\varphi''_{st}\,\phi',\,\phi')/2\,d\xi\,d\eta, \quad (11)$$

where $r = (-1)^{t+s}\,(s+t)!/s!\,t!$; $r_1 = -r/(s+t)$; $r_2 = -r_1/(s+t-1)$.

The relationship (3) allows one to write explicitly the dependence of m_{st} on u:

$$m_{st}(u) = \sum_{k=0}^{s+t\le 2}\sum_{l=0} h_{s-k,\,t-l}(u)\,n_{kl} + \sum_{k+l>2}\sum h_{s-k,\,t-l}(u)\,n_{kl}.$$

The first sum in this expression is a polynomial with respect to u to a power lower than $s + t - 2$. By definition, $n_{00} = 1$. One can assume the momenta n_{10} and n_{01} to be known, because they determine the direction of the optical axis orientation. Particularly, one can take them to be equal to zero, that corresponds to the axis orientation toward the energy center of the object observed. Taking into account this equality and Eqs. (9)–(11), we have

$$m_{st}(u) = h_{st}(u) + h_{s-2,t}(u)\,n_{20} + h_{s-1,\,t-1}(u)\,n_{11} + h_{s,\,t-2}(u)\,n_{02} + \ldots; \quad (12)$$

$$\frac{d^{s+t}\,m_{st}(0)}{du^{s+t}} = \frac{d^{s+t}\,h_{st}(0)}{du^{s+t}}\,,$$

$$\frac{d^{s+t-1}\,m_{st}(0)}{du^{s+t-1}} = \frac{d^{s+t-1}\,h_{st}(0)}{du^{s+t-1}}\,,$$

$$\frac{d^{s+t-2}\,m_{st}(0)}{du^{s+t-2}} = \frac{d^{s+t-2}\,h_{st}(0)}{du^{s+t-2}} + \ldots,$$

where ellipses mean the terms independent of ϕ,

$$\iint P^2 \varphi_{st}\, d\xi\, d\eta = P_{st}, \quad s + t \geq 0, \tag{13}$$

$$\iint P^2(\phi', \varphi'_{st})\, d\xi\, d\eta = \phi_{st}, \quad s + t \geq 1, \tag{14}$$

$$\frac{1}{2} \iint P^2(\varphi''_{st}\, \phi', \phi')\, d\xi\, d\eta = \phi_{2st} + F_{2st}(P), \quad s + t \geq 2, \tag{15}$$

where $P_{st} = (1/r)\, d^{s+t}\, m_{st}(0)/(du^{s+t})$ at $s + t \geq 1$ and $P_{00} = 1$ by definition;

$\phi_{st} = (1/r_1)\, d^{s+t-1}\, m_{st}(0)/(du^{s+t-1})$;
$\phi_{2st} = (1/r_2)\, d^{s+t-2}\, m_{st}(0)/(du^{s+t-2})$;

$F_{2st}(P)$ is the functional of P.

Relationships (13)–(15) are the momentum equalities sought. Parameters P_{st} and ϕ_{st} are the momenta of the functions $P^2(\xi, \eta)$ and $\phi(\xi, \eta)$ relative to the functions $\varphi_{st}(\xi, \eta)$.

Equality (15) is nonlinear relative to ϕ, and this fact makes it difficult to use it for determining the pupil function. However, if it is possible to introduce the controlled variation of the phase function $\Delta\phi(\xi, \eta)$ in the optical system (for example, in adaptive optics), one can determine the variation of the momentum $\Delta\phi_{2st}$ from two measurements of the phase function ϕ and $\phi + \Delta\phi$. Since only the first term in the right-hand side of Eq. (15) depends on the phase function, then

$$\Delta\phi_{2st} = \frac{1}{2} \iint P^2[(\varphi''_{st}\, \phi' + \Delta\phi', \phi' + \Delta\phi') -$$
$$- (\varphi''_{st}\, \phi', \phi')]\, d\xi\, d\eta ,$$

and the equality

$$\iint P^2(\psi_{st}, \phi')\, d\xi\, d\eta = \phi_{1st} , \tag{16}$$

follows from it. Here

$$\psi_{st} = \varphi''_{st}\, \Delta\phi',$$
$$\phi_{1st} = \Delta\phi_{2st} - \frac{1}{2} \iint P^2(\varphi''_{st}\, \Delta\phi', \Delta\phi')\, d\xi\, d\eta .$$

Parameters ϕ_{1st} are the momenta of the function ϕ relative to the functions ψ_{st}.

Momentum equalities (13), (14), and (16) are obtained supposing that the functions $P^2(\xi, \eta)$ and $\phi(\xi, \eta)$ are quite smooth, but they are also valid for $P^2 \in L_2(\Omega)$ and $\phi \in W_2^1(\Omega)$, because these momentum equalities can be continued by continuity to the noted spaces.

Let us note that the momentum equality (16) can be effectively used in the adaptive optical systems, because it allows one to obtain sufficient number of equalities, linear relative to ϕ, in terms of momenta of the low order $s + t \geq 2$ for different $\Delta\phi$.

THEOREM OF RECONSTRUCTION OF THE WAVE FUNCTION

Equalities (13) are the momenta of the function $P^2(\xi, \eta)$ relative to the full system of functions $\{\varphi_{st}(\xi, \eta)\}_{s+t>0}$ in $L_2(\Omega)$. They make it possible to determine the function $P^2(\xi, \eta)$, if it is unknown. By orthogonalization of the sequence $\{\varphi_{st}\}$ in $L_2(\Omega)$, one can pass to the orthonormalized sequence $\{\bar{\varphi}_{st}\}$ and the corresponding momenta $\bar{P}_{st}$, that are the coefficients of the Fourier function $P^2(\xi, \eta)$, and so

$$P^2(\xi, \eta) = \sum_{s+t\geq 0}\sum \bar{P}_{st}\, \bar{\varphi}_{st}(\xi, \eta). \tag{17}$$

We have supposed in the beginning of the paper that the phase function $\phi(\xi, \eta)$ is an element of the Sobolev space $W_2^1(\Omega)$ with the norm[10]

$$|\phi|_{W(\Omega)} = |\phi|_{L(\Omega)} + |\phi|_{w(\Omega)};$$

where

$$|\phi|^2_{L(\Omega)} = \iint \phi^2\, d\xi\, d\eta ;$$

$$|\phi|^2_{w(\Omega)} = \iint (\phi', \phi')\, d\xi\, d\eta .$$

Let us take into account peculiarities of the problem and introduce the equivalent and a convenient norm. First, let us suppose that the function P^2 satisfies the condition $P^2(\xi, \eta) \geq P^2_{\min} > 0$ in the region Ω almost everywhere. This condition is satisfied in applications. Second, it is sufficient to determine the function ϕ accurate to a constant factor, so $W_2^1(\Omega)$ is considered to mean the subspace of functions satisfying the condition

$$\iint \phi\, d\xi\, d\eta = 0. \tag{18}$$

Then we have the inequality

$$\iint \phi^2\, d\xi\, d\eta \leq$$
$$\leq C\left[\left(\iint \phi\, d\xi\, d\eta\right)^2 + \iint (\phi', \phi')\, d\xi\, d\eta\right] =$$
$$= C\iint (\phi', \phi')\, d\xi\, d\eta \leq C/P^2_{\min} \iint P^2(\phi', \phi')\, d\xi\, d\eta ,$$

from the Poincare inequality,[10] taking into account the condition (18).

It follows that one can take the value

$$|\phi| = \left(\iint P^2(\phi', \phi')\, d\xi\, d\eta \right)^{1/2} \qquad (19)$$

as an equivalent norm in $W_2^1(\Omega)$.

For such a definition of the norm, one can consider the left sides of the momentum equalities (14) as a scalar product of the function ϕ and functions $\varphi_{st}(\xi, \eta)$, and, due to the completeness of the sequence of functions $\{\varphi_{st}\}$, the function ϕ is unambiguously determined in $W_2^1(\Omega)$ by its momenta ϕ_{st}.

In our case since the norm (19) and the momentum equalities (14) are determined by the scalar product, one can write the solution, similarly to Eq. (17), in the form:

$$\phi(\xi, \eta) = \phi_0 + \sum_{s+t>0}\sum \bar{\phi}_{st}\, \bar{\varphi}_{st}(\xi, \eta),$$

where ϕ_0 is an arbitrary constant; $\{\bar{\varphi}_{st}\}$ is the orthogonal sequence of functions $\{\varphi_{st}\}$ with the norm (19); $\{\bar{\phi}_{st}\}$ is the sequence of momenta corresponding to the functions $\bar{\varphi}_{st}$.

Thus the following theorem is proved:

When the conditions $P^2(\xi, \eta) \geq P^2_{\min} > 0$ are satisfied almost everywhere on Ω and $\phi \in W_2^1(\Omega)$, the momentum equalities (13) and (14) unambiguously determine the pupil function $G(\xi, \eta) = \varphi(\xi, \eta)\exp(i\phi(\xi, \eta))$ accurate to a constant factor.

DETERMINATION OF MOMENTA BY THE METHOD OF TIME MODULATION

The technique described below gives a clear view of the dependence of the pupil function momenta P_{st}, ϕ_{st}, and ϕ_{1st} on the spatial distribution of the intensity. Let us suppose that the measurement plane OXY moves along the z axis according to a harmonic law $u = u_0 \sin\omega t$, and the period $T = 2\pi/\omega$ is small in comparison with the "frozen" duration of the pupil function. Since the momentum $m_{st}(u)$ is a polynomial of $(s + t)$ power relative to u, then the temporal function $m_{st}(u_0 \sin\omega t)$ is a trigonometric polynomial of $(s + t)$ power, and its higher-orders Fourier coefficients determine the momenta P_{st}, ϕ_{st}, and ϕ_{1st} of the pupil function.

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