INCREASED SPEED OF WAVEFRONT ADAPTIVE CONTROL OF LIGHT BEAMS BY THE MULTIDITHER ALGORITHM. PART I. ALGORITHM CONSTRUCTION

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Ways for developing faster adaptive control of the wavefront of a light beam are considered. Optimal laws are obtained, which describe the variation of the control constants as functions of the distance to the receiver, the initial beam power, and the detuning of the optimizing parameters from their extreme values.

In recent years the problem of compensating light beam amplitude-phase distortions[1–13] has received close study. In the publications which deal with this problem in particular, the influence of different factors on the compensation quality is considered, such as, for example, nonlinearity of the light beam propagation process, medium turbulence, and the receiver parameters. One of the main characteristics of the adaptive system which determines the compensation quality is its speed. Obviously, if the time it takes to reach the optimum light beam parameters exceeds the typical time of variation in the state of the medium and the receiver, the adaptive control will be inefficient. Therefore, it is necessary to organize the control by algorithms possessing the highest possible speed. Part I of this article (the present paper) is devoted mainly to the construction of algorithms with the choice of optimum control constants while tuning the adaptive system according to the gradient method. Part II (the present Journal) deals with problems of the practical realization of the proposed principles of varying the constants, some other algorithms different from those based on gradient methods, as well as questions of increasing the speed of multichannel systems.

As is well known, the speed of adaptive control can be increased in several ways: first, by decreasing the number of parameters being optimized; second, by selecting a reasonable starting approximation of these parameters close to their optimum values; third, by constructing "fast" algorithms for control, i.e., possessing maximum speed. Obviously the last way is more universal. It includes the improvement of a widely used algorithm based on the gradient method and the development of new algorithms different from but more effective than those based on the gradient method.

It should be noted that the increments of the parameters with a regular optimization step are determined by the distance from the receiver and the parameters of the receiver (size, reflection coefficient), the characteristics of the light beam (power, radius, etc.), as well as by the geometry of the adaptive mirror (the location of the control elements, the drives). Thus, the construction of a "fast" algorithm should be realized in two stages. The development of new algorithms or the improvement of the gradient method should be aimed eliminating the dependence of the convergence and the speed of the iteration process for generating the optical parameters on the initial power of the optical radiation, the distance from the receiver, and its size and reflection coefficient, i.e., on the parameters that are independent of the adaptive mirror (the active element of a system). This constitutes the first stage. At the second stage one should take into account the dependence of the rate of response of the system on the geometry of the controlling elements (the control channels). Thus, in the proposed approach the control constants are represented as the products of two functions, one of which depends on the light beam parameters (power, etc.), the medium, and the receiver, while the second depends on the location of the driving gears (the geometry of the mirror). In this part of the paper we shall dwell on the creation of the optimal (from the viewpoint of speed control) algorithms without taking into account the geometry of the active element (the mirror).

In general[11] the control of the vector of the light beam parameters being optimized in the multidither algorithm is realized according to the law

$$D\vec{\theta}(t) = -\hat{\gamma}\frac{\partial\Phi(J(\vec{\theta}(t-\tau_{dc}),\ L(t-\tau_{dm}),\ \alpha(t-\tau_{dm})))}{\partial\vec{\theta}(t-\tau_{dc})}, \tag{1}$$

where D is an operator determined by the adaptive system response function; t is dimensionless time; $\hat{\gamma}$ is the matrix of control constants; J is a functional chosen to estimate the quality of the compensations of the distortions, e.g., a functional of the power received at the target; Φ is a function of the functional J, the choice of which determines the control algorithm, in particular when applying the gradient

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method $\Phi = J$; L is the distance to the moving receiver measured In units of the diffraction length $l_d = ka^2/2$, here k is the wave number; a is the initial radius of the beam; a characterizes the degree to which the initial beam power exceeds the beam self-focusing power. Note that the majority of the papers consider a two-point algorithm for which $D\vec{\theta}(t) = \vec{\theta}(t+\tau) - \vec{\theta}(t) = \vec{\theta}_{N+1} - \vec{\theta}_N$, where τ is the time of generation of the control signal. Equation (1) takes account of the time delays τ_{dm} and τ_{dc} in obtaining the information about the receiver and the medium and about the variation of the quality criterion, respectively. It should be emphasized that some problems of beam focusing control in regular nonlinear media are considered in Refs. 12 and 14 in the presence of delay and inertia, and the compensation of phase fluctuations — in Ref. 8, where the extrapolation (prediction) of the state of the medium is proposed to improve the compensation quality. However, it is well known[15] that extrapolation errors rapidly accumulate with growth of the time interval of prediction. Therefore, an increase of the speed of the adaptive system (a decrease of the prediction time interval) is necessary to achieve efficient correction of the light beam distortions. In Ref. 16 on the basis of simplified models for the case of a stationary medium the principles of varying the control constant of focusing of a light beam wavefront are proposed. These principles allowed us to achieve the maximum speed in the numerical experiments of optimization of the wavefront using the gradient method. A method is given below for selecting the control constants for cases different from those considered in Ref. 16 for cases of compensation of nonlinear distortions as well as a mathematical basis for the principles of varying the control constants obtained in Ref. 16. The treatment relies on the theory of Iterative methods.[17]

For the sake of an example, let us first consider the process of light beam focusing (8) onto a stationary receiver ($L(t) = L = \text{const}$) placed in a Kerr nonlinear medium with an adaptive system without delay ($\tau_{dm} = \tau_{dc} = 0$) tuned according to the criterion ($J = J_a$) of minimum width of the light beam at the receiver. Using the aberrationless description of the propagation of the light beam it is not difficult to calculate its width In the plane of the receiver and to obtain from Eq. (1) the following principle of a step-by-step variation of the focusing[18] while optimizing θ according to the gradient method ($\Phi = J$).

$$\theta_{N+1} = \theta_N + 2\gamma_{\gamma+1} L(1 - L\theta_N),\ N = 0,\ 1,\ 2, \ldots \tag{2}$$

or

$$\frac{\theta_{N+1} - \theta_N}{\gamma_{N+1}} + A\theta_N = 2L,\ A = 2L^2,\ N = 0,\ 1, \ldots \tag{2'}$$

where γ_{N+1} is the control constant (for the case in which the light beam passes through a layer of a stationary medium with the thermal nonlinearity mechanism, Θ in Eq. (2) should be replaced by $\Theta - \Theta_{n1}$ here Θ_{n1} is the additional light beam divergence introduced by the layer). It should be noted that Eq. (2′) corresponds to the canonical form of recording in the two-layer iteration schemes,[17] for which the methods of selection of the optimum parameters γ_{N+1} were developed, that realize the maximum rate of convergence of the iteration methods (in our case the attainment of optimum focusing $\Theta_{opt} = 1/L$). Having introduced the detuning of the beam focusing from its optimum value $\xi_N = \Theta_N - 1/L$, we obtain from Eq. (2′)

$$(\xi_{N+1} - \xi_N)/\gamma_{N+1} + A\xi_N = 0. \tag{3}$$

Choosing the parameter γ_{N+1} from the condition that ξ^2_{N+1} be minimum, we can determine its value that gives the maximum speed of the algorithm

$$(\gamma_{N+1})_{opt} = \frac{1}{2} L^2. \tag{4}$$

The method which determines γ_{N+1} in this way is called the minimum discrepancy method. It is important to emphasize that the optimum value of the control constant determined directly from the solution of the difference equation (2) coincides with the value of γ_{opt} in Eq. (4).

The optimum value of the control constant can also be determined from the condition of orthogonality of ξ_{N+1} to ξ_N (the method of steepest descent). It is essential that Eq. (4) is valid for moving receivers. In this case L becomes a function of the iteration $L \to L_N$, and at the optimum value of γ_{N+1} quasistatic control of focusing ($\Theta_{N+1} = 1/L_N$) is realized (see also Ref. 16). Consequently, the current value of the focusing is determined by the previous receiver location. If the moving receiver speed or the speed of the adaptive system is such that the receiver succeeds in getting out of the region of longitudinal focus, then the quality of focusing will be low. In this case after achieving the maximum speed of response it is necessary to extrapolate[16] the receiver location (i.e., to "predict" in terms of Ref. 8).

In practice control based on the criterion of peak intensity $J = J_m$ at the receiver turns out to be more important. Then, instead of Eq. (2) we obtain from Eq. (1)[18]

$$\theta_{N+1} = \theta_N + \frac{2\gamma L(1 - L\theta_N)}{[(1 - L\theta_N)^2 + L^2(1 + \alpha)]^2}, \tag{5'}$$

or, rewriting it in canonical form.

$$B_N \frac{\theta_{N+1} - \theta_N}{\gamma_{N+1}} + \theta_N = 1/L,$$

$$B_N = [(1 - L\theta_N)^2 + L^2(1 + \alpha)]^2/2L. \tag{5''}$$

Note that according to the classification used in the theory of iterative methods,[17] Eq. (5″) represents a non-stationary implicit one-step iterative process, which makes the construction of optimal algorithms more difficult. It is important that both A and B_N are strictly greater than 0, for which reason the representations of the algorithm in the forms (5′) and (2′) are equivalent. According to the correction method[17] the optimal principle for varying the control constant follows from Eq. (5″) by varying the receiver location and the power of the light beam and its focusing

$$\gamma_{N+1} = \gamma_0 \left[(1 - L\Theta_N)^2 + L^2(1 + \alpha)\right]^2/2L^2 \tag{6}$$

(where γ_0 is the Initial value of the control constant), which agrees completely with the expression obtained in Ref. 16.

Thus, the principles of varying the control constant by varying the focusing obtained in Ref. 16 are optimal and cannot be further improved if the focusing algorithm based on the gradient method is used. They enable one in this case to obtain the maximum speed. It should also be emphasized that Eq. (6) is valid in the case of non-stationary thermal self-focusing of the optical radiation in a stationary medium, with the one change that instead of α one should use $\int_0^{t_N} \alpha(t)dt$.

The above method for selecting the optimum values of the control constants gives a scheme for constructing γ_{opt} under other conditions along the propagation path. As an example let us consider the case of compensation of the lateral shift of the center of gravity of a light beam which has passed through a layer of a moving medium with a thermal nonlinearity mechanism. An analysis of control of focusing and the slope of the wavefront for moving and stationary receivers is given in Refs. 16 and 19, respectively. Below the optimal principles of variation of $\gamma_{N+1}^{(x)}$ are written down.

According to Ref. 19, control of the slope $\Theta_N^{(x)}$ of the light beam wavefront (for the correction of its lateral shift during tuning of the adaptive system with respect to the position of the center of gravity of the beam relative to the center of the receiving aperture J_a, the light beam intensity in the center of the target J_t, and that fraction of the power J_p received in the aperture with radius R) is realized according to the principles

$$\xi_{N+1} = \xi_N - 2\gamma_{N+1}^{(x)}L^2 \begin{cases} \xi_N, & J_a; \\ \dfrac{\xi_N}{f_N^3} \exp\left[-\dfrac{L\xi_N}{f_N}\right]^2, & J_t; \\ \exp\left[-\dfrac{R^2 + L^2\xi_N^2}{f_N^2}\right] \times \mathrm{sh}\left\{\dfrac{2RL\xi_N}{f_N^2}\right\}\dfrac{1}{Lf_N}, & I_p, \end{cases} \tag{7}$$

where $f_N^2 = L^2 + (1 - L(\Theta_N - \Theta_{NL}))^2$ is the square of the dimensionless beam width in the plane of the receiver located a distance L behind the layer of nonlinear medium; Θ_α is the additional wavefront slope acquired by the beam in this layer; and $\xi_N = \Theta_N^{(x)} - \Theta_\alpha$. Note that the radius R of the receiving aperture is normalized by α, and in writing down Eq. (7) it is supposed that the direction of motion of the medium coincides with the X axis. Rewriting Eq. (7) in the form of Eq. (5″) and making use of the correction method, we get the following principles of variation of the control constants

$$\gamma_{N+1}^{(x)} = \frac{\gamma_0^{(x)} f_N^3}{2L^2} \exp\left\{\left[\frac{L\left(\Theta_N^{(x)} - \Theta_\alpha\right)}{f_N}\right]^2\right\} \tag{8′}$$

for estimating the quality of beam center position correction according to the criterion J_t;

$$\gamma_{N+1}^{(x)} = \frac{\gamma_0^{(x)} f_N^3}{4RL^2} \exp\left\{\frac{R^2 + L^2\xi_N^2}{f_N^2}\right\} \mathrm{sh}\,\frac{\eta}{\eta},\ \eta = \frac{2RL\xi_N}{f_N^2} \tag{8″}$$

for estimating the quality of correction according to the criterion J_p; $\gamma_0^{(x)}$ is the starting value of the control constant.

As can be seen from Eqs. (8′) and (8″), the optimal variation of the control constant depends exponentially on the beam width if the additional wavefront slope is compensated. At the same time, if the optimization of focusing is carried out with the purpose of achieving the maximum intensity at the receiver (without specifying the point it will be attained at), the control algorithm for Θ will naturally be independent of the value of the slope of the wavefront (the beam center shift), of course, if the beam is located within the receiver. Therefore, it is reasonable to first optimize $\Theta^{(x)}$. When estimating the focusing optimization from the on-axis beam intensity, the adaptation processes Θ and $\Theta^{(x)}$ obey the same exponential dependence.

Note that the characteristics of the beam and the medium enter into Eqs. (8′) and (8″). In a thin nonlinear layer the additional wavefront slope can be evaluated exactly. When the receiver is located within the bulk of the nonlinear medium such estimation will be too rough. Therefore, in practice. If compensating the beam center shift in a thin nonlinear layer, Eqs. (8′) and (8″) should be modified by omitting the Θ_α term in the exponent. The initial value of the control constant should be significantly increased (by a factor of approximately $\exp(L^2\Theta_\alpha^2)$). Note that $L\Theta_\alpha$ is the value of the beam center shift along the X axis due to the influence of the nonlinear layer. Therefore, when the receiver is located within the bulk of the nonlinear medium, $\gamma_0^{(x)}$ should be increased by a factor of $\exp(x_c^2)$, where x_c is the coor dinate of the center of the initially collimated beam in the plane of the receiver. Since in general it is unknown beforehand, it is impossible, generally speaking, to choose the value of $\gamma^{(x)}$ required for the maximum convergence of the iteration process. However, the minimum beam center shift is easy to estimate if one uses the geometric optics approximation for the propagation of a rectangularly shaped beam,[20] because within the class of beams with the same power without a dip in the initial intensity distribution on the axis, the hypo-Gaussian beam whose profile is close to uniform[21] experiences a minimum beam center shift in a moving medium with a thermal nonlinearity mechanism. Hypertubular beams have a similar property.[22] Taking into account the above-mentioned fact, in place of Eq. (8′) we obtain the following expression:

$$\gamma_{N+1}^{(x)} = \frac{\gamma_0^{(x)} f_N^3}{2L^2} \exp\left[- \left(L\theta_N^{(x)}/f_N\right)^2 \right],$$

which is valid for moving receivers as well. For Eq. (8″) one can write down a similar expression.

Recall that the case of the on-axis beam focusing was considered above. But in the case of focusing control along the Y axis a pre-exponential factor – the dimensionless width of the beam along the Y axis – will appear in Eqs. (8′) and (8″). Consequently, the constant in the wavefront slope control channel will depend on the current value of the beam focusing along the Y axis. The other considerations and conclusions remain in force.

Thus, optimal principles of varying the control constants have been constructed for the purpose of achieving maximum speed of control of the adaptive system by the lowest wavefront modes. The proposed approach is applicable to the optimization of any number of parameters, in particular, for flexible mirrors with many degrees of freedom.

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