REMOTE TECHNIQUE FOR SIMULTANEOUS MEASUREMENT OF THE PARTICLES' VELOCITY AND SIZE DISTRIBUTION FUNCTION

V.V. Nosov

Institute of Atmospheric Optics,
Siberian Branch of the Russian Academy of Sciences, Tomsk
Received December 20, 1994

The paper describes the optical remote technique for simultaneous measurement of the particles' size distribution function and mean velocity. The technique is based on the spatial frequency filtration of the particles' optical image in a receiving system. The temporal correlation function of the frequency-modulated optical flux received by a photodetector has been constructed based on the theory of optical wave propagation through random media. The particle size distribution function is reconstructed by taking the inverse Laplace transform of the spatial correlation coefficient of modulated flux fluctuations. In the specific case of lognormal distribution the relationships have been obtained for determining the distribution parameters.

1. INTRODUCTION

The paper considers the optical remote technique for simultaneous measurement of the particles' size distribution function and mean velocity. It is assumed that the particles are suspended in rarefied gaseous (for example, in the atmosphere) or liquid flows, and in the case of coherent illumination the nonspecular reflection of optical waves occurs. A source of particle illumination (coherent or incoherent) is outside the particle flow. The source can be on either sides of the flow, realizing the schemes of illumination by reflected or transmitted radiation. Self-glowing particles are also considered. The particle size must exceed the illumination wavelength (or the maximum wavelength of the incoherent source emission spectrum).

The method is based on spatial filtration of the particles' optical image in a receiving system.[1–4] The image in a receiver is split into a series of channels. In every channel the phase modulation at different frequencies is used. This is achieved by the application of, for example, diffraction gratings with different grating periods.[3] A single channel also can be used, when the radiation passes through one diffraction grating with a variable period. For every grating period we measure the temporal correlation function of optical flux fluctuations (reflected from particles for the scheme with reflection) at a receiving telescope.

This method can be used for remote diagnostics of wind velocity in the atmosphere, for remote measurement of the particle size distribution near the months of stacks of industrial plants (with subsequent analysis of the level of environmental pollution), for remote diagnostics of the objects' dimensions on the Earth's surface (in this case the particle size means the transverse size of inhomogeneities on the surface for a given illumination wavelength), and so on.

The applicability limits of the proposed method are mainly determined by the ratio of the longitudinal size of the observed particle flow L_x (in the observation direction) to the optical path length x (the distance between the observation plane in the particle flow producing a sharp image in a photodetector and by photodetector). The smaller the ratio L_x/x, the more precise is the method. The last condition is well fulfilled for most of existing schemes of pulsed laser sounding of the aerosol atmosphere when the longitudinal dimensions of the volume being observed are determined by an optical pulse length (or by a gating pulse length in the photodetector) and make up insignificant part of the entire optical path.[5] When the above condition is violated, the method will overestimate the particle size distribution function being reconstructed in the region of small particle size.

Hereafter we restrict our consideration to the case of short atmospheric paths realized, for example, in remote ground-based measurement of the particles' size and velocity near the months of stacks of industrial plants. For short paths the disturbing effect of the atmospheric turbulence on the light wave propagation may be neglected[1,2,4] in subsequent calculations.

2. TEMPORAL CORRELATION FUNCTION OF FREQUENCY-MODULATED OPTICAL FLUX RECEIVED BY A PHOTODETECTOR

Let us assume that the particle flow transverse to the observation axis is at a distance x from the input aperture of a conventional optical telescope in a homogeneous medium. For the incoherent (thermal) source of illumination we consider the arbitrary

0235-6880/96/01 56-04 $02.00

reflecting characteristics of particles. In the case of a coherent source the particles are considered diffusely reflecting (that is typical of the atmospheric aerosol).

The incoherent radiation wavelength λ characterizes the maximum of emission in a certain given spectral range $\Delta\lambda$. A radius of a receiving (input) lens of the telescope is designated by a_t, and the focal length - by F. The input window of a square-law photodetector (photomultiplier, photodetector, and so on) with the radius a_p is at the distance F_0 from the receiving lens plane. A transparency with the intensity transmission coefficient

$\tau_\xi(y) = (1 + \cos(\xi\, y))/2, \quad \xi[\mathrm{m}^{-1}]$

is placed directly before the input window of the photodetector. A simple model of the transparency τ_ξ is the diffraction grating with the distance between the line centers[3] $d = 2\pi/\xi$ (here, $d/2$ is the linewidth of the diffraction grating, d is its period, and ξ is its frequency).

An electric signal at the photodetector output is proportional to the optical flux at its input:

$$P(t,\xi) = \int \mathrm{d}^2\rho \exp(-\rho^2/a_p^2)\, I(F_0,\rho)\, \tau_\xi(y), \quad \rho = (x, y), \tag{1}$$

where t is the observation time, $I(F_0, \rho)$ is the intensity of incoherent radiation entering the photodetector in the plane F_0.

For conventional photodetectors the time of response (of signal averaging by the detector) exceeds the coherence time of the incoherent source.[2] This enables one to use the relationships[1,2]

$$\langle U(\rho_1)\, U^*(\rho_2)\, U(\rho_3)\, U^*(\rho_4)\rangle = $$
$$= \langle U(\rho_1)\, U^*(\rho_2)\rangle \langle U(\rho_3)\, U^*(\rho_4)\rangle,$$

$$\langle U(\rho_1)\, U^*(\rho_2)\rangle = I_A(\rho_1)\, \delta(\rho_1 - \rho_2),$$

in calculations of statistical moments of flux (1) for the complex amplitude $U(\rho)$ of the incoherent source field where $I_A(\rho_1)$ is the transverse profile of the source intensity. The latter formula describes an averaging of the intensity $I(F_0, \rho)$ in Eq. (1) over the phase fluctuations of complex amplitude of the incoherent source. In this case according to Refs. 1–3

$$I(F_0,\rho) = \frac{k^2 a_t^2}{4\pi^2 x^2 a_0^2} \int \mathrm{d}^2\rho'\, I_A(\rho') \exp\left\{-\left(\rho + \frac{F_0}{x}\rho'\right)^2 / a_0^2\right\}, \tag{2}$$

$$a_0^2 = a_t^2 \left[\left(1 - \frac{F_0}{F} + \frac{F_0}{x}\right)^2 + \Omega_t^{-2}\right], \; \Omega_t = \frac{k\, a_t^2}{F_0}, \; k = \frac{2\pi}{\lambda}.$$

Here a_0 is the radius of the point-source image in the plane F_0 of the photodetector. The value of a_0 in the sharp image plane, in which $1 - F_0/F + F_0/x = 0$, is minimum, $a_0 = F_0/(ka_t)$. In the subsequent discussion we assume that the sharp image plane is the observation plane on the object, being coincident with the plane of the leading edge of particle flow.

The intensity $I_A(\rho')$ in Eq. (2) is specified by the expression:

$$I_A(\rho') = I_{0A}\left[m_1 - m_2 \sum_{i=1}^{N} c_i \exp\left\{-\frac{(\rho' - \rho_i)^2}{a_i^2}\right\}\right], \; I_{0A} = \frac{4\pi^3}{k^2},$$

where $\rho_i = (y_i, z_i)$ are the transverse coordinates of the ith particle center, a_i and c_i are the radius in the observation plane and the reflection coefficient by intensity (for nontransparent particles $c_i = 1$) of the ith particle, and N is the particle number density in the flow. For the scheme of particle illumination by reflected radiation $m_1 = 0$, $m_2 = -1$. For their illumination by transmitted radiation $m_1 = m_2 = 1$. A simple analysis indicates that the radius of particles beyond the observation plane is given by the expression $a_i(1 + L_x/x)^{-1}$ and for the assumed conditions of applicability of the method ($L_x/x \ll 1$) this radius differs little from a_i.

Calculating Eq.(2) and substituting Eq.(2) into Eq.(1) we obtain

$$P(t,\xi) = \frac{1}{2} R(t, 0) + \frac{1}{4} R(t,\xi) + \frac{1}{4} R(t, -\xi), \tag{3}$$

$$R(t,\xi) = k_0\, [m_1 \exp(-L^2/d_x^2) - m_2\, Y(t,\xi)\, L^{-2}],$$

$$k_0 = \frac{\pi^3\, a_t^2\, a_p^2}{F_0^2},$$

$$Y(t,\xi) = \sum_{i=1}^{N} \frac{c_i\, a_i^2\, L^2}{a_{ie}^2} \times$$
$$\times \exp\left\{-\frac{L^2}{\varepsilon^2} \frac{(a_i^2 + a_0^2\, x^2/F_0^2)}{a_{ie}^2} - \frac{\rho_i^2}{a_{ie}^2} - i\, \frac{2\, y_i\, L^2}{d_x\, a_{ie}^2}\right\},$$

$$a_{ie}^2 = a_i^2 + L^2 + a_0^2 \left(\frac{x}{F_0}\right)^2, \quad L = \frac{a_p\, x}{F_0},$$

$$d_x = d_x(\xi) = \frac{2x}{\xi F_0} = \frac{x\, d}{\pi F_0}.$$

Here a_{ie} is the efficient transverse radius of the observation zone in the observation plane (the leading edge of the particle flow), L is the transverse radius of the observation zone on the observation object cut out by a diaphragm of the photodetector receiving window (with the radius a_p), and d_x is the effective linewidth of the diffraction grating in the observation plane.

In what follows we consider that the observation zone radius L in the flow exceeds significantly the particle size a_i and the point-object image size (in the plane of the sharp image) a_0x/F, recalculated by the rules of geometric optics in the plane of the leading edge of particle flow, is much larger than L, that is,

$L \gg a_i, \; L \gg a_0x/F$.

These conditions are usually fulfilled in practice, with $a_{ie} = L$.

It is also assumed that the observation zone is inside the particle flow and does not intersect its transverse boundaries. If the transverse flow dimensions are denoted by L_y and L_z, then the latter condition is equivalent to the fulfilment of the conditions $L_y \gg L$ and $L_z \gg L$.

The particle motion in the flow is described by the expression:

$$\boldsymbol{\rho}_i = \boldsymbol{\rho}_i(t) = \boldsymbol{\rho}_i(t_0) + \mathbf{v}(t - t_0),$$

where t_0 is the starting time, $\mathbf{v} = (v_y, v_z)$ is the mean particles' velocity in the flow (flow velocity), and $\boldsymbol{\rho}_i(t_0)$ are the initial transverse coordinates of the ith particle center.

To obtain the statistical moments of the received optical flux P we further use the generally accepted assumptions, namely: the initial coordinates of particles' centers are considered distributed uniformly and the number density of particles N is considered distributed by the Poisson law.[1,2,4,5] The Poisson statistics of the particle number density is well fulfilled for the artificial particle flows considered in the paper (for example, industrial emissions). In the real atmosphere because of the dynamic turbulence one can observe deviations from the above-mentioned distribution law. However, even in this case the smaller the transverse dimensions of the observation zone (provided by the conditions $L \ll L_z,\ L_y$), the closer is the statistics of the particle number density to the Poisson one. After independent averaging[1] for the first two statistical moments of the optical flux we find the expressions:

$$<P(t,\xi)> = \frac{k_0}{2}\left\{ m_1\left(\exp\left(-L^2/d_x^2\right)+1\right) - m_2\,\pi <c> \times \right.$$
$$\left. \times <n>\left[S_2(0) + S_2\left(\frac{1}{d_x^2}\right)\exp\left(-\frac{L^2}{d_x^2} - l^2(\xi)\right)\right]\right\}, \tag{4}$$

$$B_p(\tau, \xi_1, \xi_2) = <P(t_1,\xi_1)\,P(t_2,\xi_2)> -$$
$$- <P(t_1,\xi_1)><P(t_2,\xi_2)> = \frac{k_0^2\,\pi}{8L^2}<n><c^2>\exp\left(\frac{v^2\tau^2}{2L^2}\right)\times$$
$$\times\left\{S_4(0) + S_4\left(\frac{1}{d_{x1}^2}\right)\exp\left[-\frac{L^2}{2\,d_{x1}^2} - l_1^2\right]\times\right.$$
$$\times\cos\left(\frac{v_y\tau}{d_{x1}}\right) + S_4\left(\frac{1}{d_{x2}^2}\right)\exp\left[-\frac{L^2}{2\,d_{x2}^2} - l_2^2\right]\cos\left(\frac{v_y\tau}{d_{x2}}\right) +$$
$$+\frac{1}{2}S_4\left(\frac{1}{d_{x1}^2}+\frac{1}{d_{x2}^2}\right)\exp(-l_1^2-l_2^2)\left[\exp\left(-\frac{L^2}{2}\left(\frac{1}{d_{x1}}+\frac{1}{d_{x2}}\right)^2\right)\times\right.$$
$$\times\cos\left(v_y\tau\left[\frac{1}{d_{x1}}-\frac{1}{d_{x2}}\right]\right) + \exp\left(-\frac{L^2}{2}\left(\frac{1}{d_{x1}}-\frac{1}{d_{x2}}\right)^2\right)\times$$
$$\left.\left.\times\cos\left(v_y\tau\left[\frac{1}{d_{x1}}+\frac{1}{d_{x2}}\right]\right)\right]\right\}. \tag{5}$$

Here $\tau = (t_2 - t_1)$ is the time delay between the two moments of observation, $v = \sqrt{v_y^2 + v_z^2}$, $d_{x1} = d_x(\xi_1)$, $d_{x2} = d_x(\xi_2)$, $l(\xi) = a_0\xi/2$, $l_1 = l(\xi_1)$, $l_2 = l(\xi_2)$, $<c>$ and $<c^2>$ are the first two statistical moments of the coefficient of light reflection by particles, $<n>$ is the mean value of the random particle number density in the flow $n = N/(L_zL_y)$,

$$S_k(X) = <a^k\exp(-a^2X)> = \int\limits_0^\infty \mathrm{d}r\,P_a(r)\,r^k\exp(-Xr^2),$$
$$k = 1, 2, \ldots, \tag{6}$$

and the function $P_a(r)$ is the probability density (function) of particle size distribution.

3. MEAN PARTICLES' VELOCITY IN A FLOW

From Eq. (5), assuming $\xi_1 = \xi_2 = 0$ we obtain

$$b_p(\tau, 0, 0) = \frac{B_p(\tau, 0, 0)}{B_p(0, 0, 0)} = \exp\left(-\frac{v^2\,\tau^2}{2\,L^2}\right).$$

By measuring the correlation coefficient $b_p(\tau, 0, 0)$ of fluctuations of the total light flux ($\xi_1 = \xi_2 = 0$) we find the mean particles' velocity in a flow. Really, if we know the value of the time correlation scale τ_k of fluctuations of the total light flux, for which $b_p(\tau_k, 0, 0) = \exp(-1)$, we have

$$v = \sqrt{2}\,(L/\tau_k).$$

4. PARTICLE SIZE DISTRIBUTION FUNCTION

Using the spatial correlation coefficient of light flux fluctuations $b_p(0, \xi, \xi) = B_p(0, \xi, \xi)/B_p(0, 0, 0)$, from Eq. (5) we can obtain the following asymptotic expressions:

$$S_4(X)/[S_4(0)] = \varphi\,[(x/F_0)\sqrt{2\,X}],\quad X = 2\,d_x^{-2}, \tag{7}$$
$$\varphi(\xi) = \varphi_1(\xi) = 2\,[4\,b_p(0,\xi,\xi) - 1]\exp[2\,l^2(\xi)],\ \xi > 2/a_p,$$
$$\varphi(\xi) = \varphi_2(\xi) = 2\,b_p(0, \xi, \xi) - 1,\ \xi < 2/a_p.$$

The condition $\xi > 2/a_p$ ($d < \pi a_p$ or $d_x < L$) in Eq. (7) corresponds to the use of standard diffraction gratings with a sufficiently small period, when more than one grating line falls on the input window of the photodetector.

The relationships (7) can be used to reconstruct the particle size distribution function $P_a(r)$ from the measured values of the correlation coefficient $b_p(0, \xi, \xi)$. Really, according to Eq. (6) the quantity $S_4(x)$ in Eq. (7) is the Laplace transform of the function $r^{3/2}\,P_a(\sqrt{r})/2$. Therefore, by measuring $b_p(0, \xi, \xi)$ from Eq. (7) we find

$$P_a(r)/[S_4(0)] = 2\,r^{-3}\,G^{-1}\{\varphi[(x/F_0)\sqrt{2\,y}]\}(r^2),\ r \ge 0, \tag{8}$$

accurate to within the unknown numerical coefficient $S_4^{-1}(0)$. Here $G^{-1}\{f(y)\}(x) = \frac{1}{2\pi i}\int_{x_0-i\infty}^{x_0+i\infty} \exp(px)f(p)\mathrm{d}p$ is the inverse Laplace transform.

To determine the coefficient $S_4^{-1}(0)$, we integrate Eq. (8) and apply the normalization condition

$$\int_0^\infty P_a(r)\,\mathrm{d}r = 1,$$

$$S_4^{-1}(0) = \langle a^4\rangle^{-1} = 2\int_0^\infty r^{-3} G^{-1}\left\{\varphi\left(\frac{x}{F_0}\sqrt{2\,y}\right)\right\}(r^2)\,\mathrm{d}r.$$

5. RECONSTRUCTION OF THE PARAMETERS OF LOGNORMAL DISTRIBUTION

It is known[5] that for aerosols of industrial origin the particle size distribution function $P_a(r)$ is well described by the lognormal law $P_{\mathrm{LN}}(r)$, where

$$P_{\mathrm{LN}}(r) = 1/(r\sqrt{2\,\pi}\,\sigma_{\ln a}) \times$$
$$\times \exp\{-[\ln r - \langle\ln a\rangle]^2/(2\,\sigma^2_{\ln a})\},\quad 0 < r < \infty, \qquad (9)$$

$$\sigma^2_{\ln a} = \ln(1 + m_a),\quad \langle\ln a\rangle = \ln[\langle a\rangle/(\sqrt{1 + m_a})],$$

$$m_a = \sigma_a^2/\langle a\rangle^2,\quad \sigma_a^2 = \langle a^2\rangle - \langle a\rangle^2.$$

If in Eq. (6) we set $P_a(r) = P_{\mathrm{LN}}(r)$ and use Eq.(7), then the problem of determining the particle size distribution function reduces to reconstruction of the parameters of the lognormal distribution σ_a and $\langle a\rangle$.

To determine the basic calculation relations, we find an asymptotic expression for $S_4(X)$, where $X = 2d_x^{-2}$, at $P_a(r) = P_{\mathrm{LN}}(r)$ in the region of large values of the argument X. Using the Laplace method for estimating this integral in Eq.(6), we obtain

$$S_4(2\,d_x^{-2}) = \frac{Y_*^3\,\mu^{-(1+1/2\,\sigma^2_{\ln a})}}{\sqrt{2}} \times$$
$$\times \exp\left[4\,\langle\ln a\rangle - \frac{\ln^2 Y_*}{2\,\sigma^2_{\ln a}}\right][1 + \mathrm{o}(Y_*)], \qquad (10)$$

$$\mu = \frac{2\,\langle a\rangle}{d_x}\,\frac{\sigma_{\ln a}}{\sqrt{1 + m_a}},\quad \mu \gg 1.$$

Here $Y_* = Y_*(\mu)$ is the asymptote for $\mu \gg 1$ solution of the transcendental equation $\mu^2 Y_*^2 + \ln Y_* = 0$. The value of $Y_*(\mu)$ is positive and decreases with the increase of μ. For $\mu \geq 2.5$ it is described by the expression

$$Y_* = \ln^{1/2}[\mu/\ln^{1/2}[\mu/\ln^{1/2}\mu]]/\mu$$

with an accuracy of 10% or better.

The parameter μ in Eq. (10) has a meaning of the ratio of the lognormal distribution width (being equal to $2\sigma_a$ for $\mu_a \ll 1$ or $2\langle a\rangle$ for $\mu_a \geq 1$) to the diffraction grating linewidth in the observation plane d_x.

By measuring the correlation coefficient $b_p(0, \xi, \xi)$ with the use of three diffraction gratings with different periods d_1, d_2, and d_3, such as $d_2 = \nu_2 d_1$ and $d_3 = \nu_3 d_1$ (where ν_2 and ν_3 are the numerical coefficients, for example, from the interval $\nu_i \in (1/2, 2)$, $\nu_i \neq 1$, $i = 2$, 3) in the applicability range of Eq. (10), taking into account Eq. (7) with $\varphi = \varphi_1$, we obtain:

$$2\,\sigma^2_{\ln a} = \frac{\ln\nu_2\,\ln\nu_3\,\ln(\nu_2/\nu_3)}{\ln\nu_2\,\ln s_{13} - \ln\nu_3\,\ln s_{12}},$$

$$2\,\langle\ln a\rangle = \ln\left[\frac{d_{x1}^2}{4\ln\nu_2}\,\alpha^{2\,\sigma^2_{\ln a}/\ln\nu_2}\,\ln\alpha\right],$$

$$\alpha = s_{12}\,\nu_2^{-(4+1/2\,\sigma^2_{\ln a})}\exp\left[\frac{\ln^2\nu_2}{2\,\sigma^2_{\ln a}}\right],$$

$$s_{1i} = \frac{S_4(2\,d_{xi}^{-2})}{S_4(2\,d_{x1}^{-2})},\ i = 2, 3.$$

Therefore, the values of the sought-after parameters σ_a and $\langle a\rangle$ are readily calculated by Eq. (9) from the known values of the parameters $\sigma_{\ln a}$ and $\langle\ln a\rangle$.

REFERENCES

1. S.M. Rytov, Yu.A. Kravtsov, and V.I. Tatarskii, *Introduction to Statistical Radiophysics.* Part. 2, *Random Fields* (Nauka, Moscow, 1978), 463 pp.
2. V.E. Zuev, V.A. Banakh, and V.V. Pokasov, *Optics of the Turbulent Atmosphere* (Gidrometeoizdat, Leningrad, 1988), 270 pp.
3. V.V. Nosov, Atm. Opt. **1**, No. 1, 122–125 (1988).
4. V.L. Mironov, *Laser Beam Propagation through the Turbulent Atmosphere* (Nauka, Novosibirsk, 1982), 248 pp.
5. V.E. Zuev and M.V. Kabanov, *Optics of the Atmospheric Aerosol* (Gidrometeoizdat, Leningrad, 1987), 254 pp.