On integral resolution of the turbulent atmosphere and a telescopic system for the Knox–Thompson method

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The integral resolution of the Knox–Thompson method is investigated in its classical formulation, when image spectra are shifted by some fixed value of the spatial scale. In this case, the integral resolution of the method can be determined as an integrated form of its optical transfer function at the current spatial scale. The integral resolution of the system "turbulent atmosphere – telescope" is shown to decrease monotonically for this method, as the fixed shift of the spatial scale increases. It is noted that the integral resolution of the Knox–Thompson method is always lower than that of the Labeyrie method. The classical Knox–Thompson method has a higher integral resolution at shifts of speckle-interferograms within a speckle, while the extended Knox–Thompson method better processes the well advanced speckle-structure of an image for large shifts of speckle-interferograms.

Fried[1,2] has considered the integral resolution of the optical system "turbulent atmosphere–telescope" for the cases of an average image and a series of short-exposure images at their processing by the Labeyrie method. This issue has received further development in my earlier paper,[3] which proposed an original approach to estimation of the integral resolution of different methods for postdetector processing of an image of an incoherently illuminated object observed by a telescopic optical system through the turbulent atmosphere. In Ref. 3 the following image processing methods were treated: the method of average image registration, Labeyrie method, Knox–Thompson method, and method of triple correlation of the image intensity. The Knox–Thompson method was studied in its extended version as a result of bispectral transformation of the correlation function of the image intensity.

In this paper, somewhat different version of determination of the integral resolution for the Knox–Thompson method is formulated. The case is treated, when image spectra are shifted to some fixed value of the spatial scale ($\Delta\mathbf{p} \neq 0$), and the method performance characteristic is related to the optical transfer function (OTF) of the system through the ordinary Fourier transformation. In this situation, the integral resolution of the method can be determined as the integrated form of its optical transfer function at the current spatial scale $\mathbf{p}$.

The optical transfer function of the optical system "turbulent atmosphere–telescope" can be written in the form[4]:

$$M(\mathbf{p}) = \iint_{-\infty}^{\infty} \mathrm{d}\boldsymbol{\rho}\, U(\boldsymbol{\rho}) U^*(\boldsymbol{\rho}+\mathbf{p}) K(\boldsymbol{\rho}) K^*(\boldsymbol{\rho}+\mathbf{p}),$$

where $U(\boldsymbol{\rho})$ is the complex amplitude of the field at the point $\boldsymbol{\rho}$ of the receiving aperture generated by a point incoherent source positioned in the object space; $K(\boldsymbol{\rho})$ is the pupil function of the receiving aperture; $\mathbf{p}$ is the spatial scale.

For the extended Knox–Thompson method studied in Ref. 3, the following is true:

$$\left\langle M(\mathbf{p}_1) M^*(\mathbf{p}_2)\right\rangle = \iint_{-\infty}^{\infty} \mathrm{d}\boldsymbol{\rho}' \iint_{-\infty}^{\infty} \mathrm{d}\boldsymbol{\rho}'' \left\langle I(\boldsymbol{\rho}') I(\boldsymbol{\rho}'')\right\rangle \times$$
$$\times \exp\left[-\frac{ik}{F}(\mathbf{p}_1\boldsymbol{\rho}' - \mathbf{p}_2\boldsymbol{\rho}'')\right],$$

where $I(\boldsymbol{\rho}) = U(\boldsymbol{\rho})\, U^*(\boldsymbol{\rho})$ is the intensity of the optical field at the point $\boldsymbol{\rho}$ of the receiving aperture generated by a point incoherent source positioned in the object space. Thus, bispectral transformation of the correlation function of the image intensity is performed: $\left\langle I(\boldsymbol{\rho}') I(\boldsymbol{\rho}'')\right\rangle$.

If the classical Knox–Thompson method is considered, then

$$\left\langle M(\mathbf{p}) M^*(\mathbf{p}+\Delta\mathbf{p})\right\rangle = \iint_{-\infty}^{\infty} \mathrm{d}\boldsymbol{\rho} \exp\left(-\frac{ik}{F}\mathbf{p}\boldsymbol{\rho}\right) \times$$
$$\times \left[\iint_{-\infty}^{\infty} \mathrm{d}\boldsymbol{\rho}' \left\langle I(\boldsymbol{\rho}') I(\boldsymbol{\rho}'+\boldsymbol{\rho})\right\rangle \exp\left(\frac{ik}{F}\Delta\mathbf{p}\boldsymbol{\rho}'\right)\right]$$

and the performance characteristic equal to

$$\left[\iint_{-\infty}^{\infty} \mathrm{d}\boldsymbol{\rho}' \left\langle I(\boldsymbol{\rho}') I(\boldsymbol{\rho}'+\boldsymbol{\rho})\right\rangle \exp\left(\frac{ik}{F}\Delta\mathbf{p}\boldsymbol{\rho}'\right)\right]$$

is related to the optical transfer function through the ordinary Fourier transformation. Estimate the

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potentialities of these two versions of the method for an object observed through the turbulent atmosphere from the viewpoint of integral resolution.

Calculate the integral resolution $\Re_{KT}(\Delta\mathbf{p})$ for the classical formulation of the Knox–Thompson method defined as follows:

$$\Re_{KT}(\Delta\mathbf{p}) = 2\frac{k^2}{F^2}\int\int_{-\infty}^{\infty} \mathrm{d}\mathbf{p}\tau_{KT}(\mathbf{p},\mathbf{p}+\Delta\mathbf{p}) =$$

$$= 2\frac{k^2}{\langle M(0)\rangle^2 F^2}\int\int_{-\infty}^{\infty} \mathrm{d}\mathbf{p}\langle M(\mathbf{p})M^*(\mathbf{p}+\Delta\mathbf{p})\rangle, \quad (1)$$

where $\tau_{KT}(\mathbf{p},\mathbf{p}+\Delta\mathbf{p})$ is the OTF of the system "turbulent atmosphere–telescope" for the Knox–Thompson method[4]; $\langle M(0)\rangle = \pi K_0^2 R^2$ is the normalization factor[4]; K_0 is the amplitude transmittance of the telescope at the optical axis of the system; R is the radius of the receiving aperture; $k = 2\pi/\lambda$, λ is the wavelength of optical radiation in vacuum; F is the focal length of the receiving lens. Take the transmission function of the optical receiving system as a gaussoid[3,4]:

$$K(\boldsymbol{\rho}) = K_0 \exp\left(-\frac{\rho^2}{2R^2}\right).$$

It follows from the definitions of the Labeyrie methods[1,4] and the classical formulation of Knox–Thompson method (1) that the integral resolutions of these methods are related as follows:

$$\Re_{KT}(\Delta\mathbf{p} = 0) \equiv \Re_{LM}, \quad (2)$$

where $\Re_{LM}$ is the integral resolution of the Labeyrie method.

The equation for the integral resolution of the Knox–Thompson method can be obtained by substituting the OTF of the system "turbulent atmosphere–telescope" $\tau_{KT}(\mathbf{p},\mathbf{p}+\Delta\mathbf{p})$ in the form found in Ref. 4:

$$\Re_{KT}(\Delta\mathbf{p}) = \frac{k^2}{\pi R^2 F^2}\exp\left(-\frac{3\Delta p^2}{8R^2}\right)\times$$

$$\times\int\int_{-\infty}^{\infty} \mathrm{d}\boldsymbol{\rho}\exp\left[-\frac{p^2+\Delta\mathbf{p}\mathbf{p}}{2R^2} - \frac{1}{2}D(\mathbf{p}) - \frac{1}{2}D(\mathbf{p}+\Delta\mathbf{p})\right]\times$$

$$\times\int\int_{-\infty}^{\infty} \mathrm{d}\boldsymbol{\rho}\exp\left[-\frac{\rho^2-\Delta\mathbf{p}\boldsymbol{\rho}}{2R^2} - \frac{1}{2}D(\boldsymbol{\rho}) - \frac{1}{2}D(\boldsymbol{\rho}-\Delta\mathbf{p}) + \right.$$

$$\left. + \frac{1}{2}D(\boldsymbol{\rho}-\mathbf{p}-\Delta\mathbf{p}) + \frac{1}{2}D(\boldsymbol{\rho}+\mathbf{p})\right]. \quad (3)$$

into Eq. (1). Here $D(\boldsymbol{\rho})$ is the spatial structural function of fluctuations of the complex phase of a plane optical wave.

For further considerations, the integral representation for this function will be taken in the form

$$D(\boldsymbol{\rho}) = \pi k^2 x \int\int_{-\infty}^{\infty} \mathrm{d}\boldsymbol{\kappa}\,\Phi_\varepsilon(\boldsymbol{\kappa})\left[1-\exp(i\boldsymbol{\kappa}\boldsymbol{\rho})\right],$$

where

$$\Phi_\varepsilon(\kappa) = 0.033 C_\varepsilon^2\left[1-\exp\left(-\kappa^2/\kappa_0^2\right)\right]\kappa^{-11/3}\exp\left(-\kappa^2/\kappa_m^2\right)$$

is the spectrum of fluctuations of the turbulent atmosphere permittivity; C_ε^2 is the structural parameter of the atmospheric turbulence; $\kappa_0 = 2\pi/L_0$, L_0 is the outer scale of atmospheric turbulence; $\kappa_m = 5.92/l_0$, l_0 is the inner scale of atmospheric turbulence; x is the effective thickness of the optically active layer of atmospheric turbulence.[4] The asymptotic relationships for $D(\boldsymbol{\rho})$ have the following form:

$$D(\boldsymbol{\rho}) \cong \begin{cases} 2(\rho/\rho_m)^2, & \rho < l_0, \\ 2(\rho/\rho_0)^{5/3}, & l_0 < \rho < L_0, \\ 2\sigma_S^2\left[1-\alpha_0(\kappa_0\rho)^{-1/3}\right], & \rho > L_0, \end{cases}$$

where

$$\rho_m = \left[\frac{3}{4}0.033\pi^2\Gamma\left(\frac{7}{6}\right)k^2 x C_\varepsilon^2\kappa_m^{1/3}\right]^{-1/2}$$

is the coherence length of a plane wave under the condition that the coherence length of the plane optical wave ρ_c is less than the inner scale of atmospheric turbulence;

$$\rho_0 = \left[2^{-5/3}\frac{18}{5}0.033\pi^2\Gamma\left(\frac{7}{6}\right)\Big/\Gamma\left(\frac{11}{6}\right)k^2 x C_\varepsilon^2\right]^{-3/5}$$

is the coherence length of a plane wave under the condition that the coherence length of the plane optical wave ρ_c is greater than the inner scale of atmospheric turbulence;

$$\sigma_S^2 = \frac{18}{5}0.033\pi^2\Gamma\left(\frac{7}{6}\right)k^2 x C_\varepsilon^2\kappa_0^{-5/3}$$

is the variance of fluctuations of the complex phase of the plane wave;

$$\alpha_0 = 2^{-5/3}\frac{25}{9}\Gamma\left(\frac{11}{6}\right) = 0.82\,.$$

Let us analyze asymptotically Eq. (3) for the integral resolution of the Knox–Thompson method. As commonly accepted,[3,4] consider two cases: (1) a slightly distorted image ($R < \rho_c$) and (2) well advanced speckle-structure of the image ($R > \rho_c$).

For the case $R < \rho_c$, in integral equation (3) expand the factor containing the structure functions of fluctuations of the complex phase of the optical wave into a series and remain two first terms:

$$\Re_{\mathrm{KT}}(\Delta\mathbf{p}) \cong \frac{k^2}{\pi R^2 F^2}\exp\left(-\frac{3\Delta p^2}{8R^2}\right)\times$$

$$\times\int\!\!\int_{-\infty}^{\infty} d\mathbf{p}\exp\left(-\frac{p^2+\Delta\mathbf{p}\mathbf{p}}{2R^2}\right)\int\!\!\int_{-\infty}^{\infty} d\boldsymbol{\rho}\exp\left(-\frac{\rho^2-\Delta\mathbf{p}\boldsymbol{\rho}}{2R^2}\right)\times$$

$$\times\left[1-\frac{1}{2}D(\mathbf{p})-\frac{1}{2}D(\mathbf{p}+\Delta\mathbf{p})-\frac{1}{2}D(\boldsymbol{\rho})-\frac{1}{2}D(\boldsymbol{\rho}-\Delta\mathbf{p})+\right.$$

$$\left.+\frac{1}{2}D(\boldsymbol{\rho}-\mathbf{p}-\Delta\mathbf{p})+\frac{1}{2}D(\boldsymbol{\rho}+\mathbf{p})\right].$$

Further calculations of the asymptotic equations require a concrete definition of the relationships between R, Δp, and the scales of atmospheric turbulence (l_0, L_0). Consider the case of small receiving apertures ($R < l_0$), that is, when the radius of the receiving aperture is less than the inner scale of atmospheric turbulence:

$$\Re_{\mathrm{KT}}(\Delta\mathbf{p}) \cong \Re_{\mathrm{KT}}\begin{pmatrix} R < l_0, \\ \Delta p < l_0 \end{pmatrix} = \Re_0\exp\left(-\frac{\Delta p^2}{8R^2}\right)\times$$

$$\times\left[1-\frac{1}{3}(\kappa_{\mathrm{m}}R)^2\left(\frac{R}{\rho_{\mathrm{m}}}\right)^2-\left(\frac{\Delta p}{\rho_{\mathrm{m}}}\right)^2\right],$$

where $\Re_0 = 4\pi^2 kR^2/F^2$ is the integral resolution of the optical system in vacuum.[3] In this case the range of spatial scales $\Delta p < l_0$ describes almost whole meaningful range of variability of the function $\Re_{\mathrm{KT}}(\Delta\mathbf{p})$. The ranges $l_0 < \Delta p < L_0$ and $\Delta p > L_0$ can be omitted, since they have no practical meaning. As follows from Eq. (2), this asymptotics for the function $\Re_{\mathrm{KT}}(\Delta\mathbf{p})$ at $\Delta p = 0$ coincides with that for $\Re_{\mathrm{LM}}$ at $R < l_0$ obtained earlier in Ref. 3:

$$\Re_{\mathrm{LM}} \cong \Re_0\left[1-\frac{1}{3}(\kappa_{\mathrm{m}}R)^2\left(\frac{R}{\rho_{\mathrm{m}}}\right)^2\right].$$

It follows from the above equations that in this case the ratio of the integral resolution of the Knox–Thompson method to that of the Labeyrie method is always less than unity:

$$\frac{\Re_{\mathrm{KT}}(\Delta\mathbf{p})}{\Re_{\mathrm{LM}}} \cong \exp\left(-\frac{\Delta p^2}{8R^2}\right)\left[1-\left(\frac{\Delta p}{\rho_{\mathrm{m}}}\right)^2\right] \le 1. \tag{4}$$

In the case when the size of the receiving aperture falls between the inner and outer scales of atmospheric turbulence $l_0 < R < L_0$, the range of the spatial scales $\Delta p < R$ covers almost whole meaningful variability range of the function $\Re_{\mathrm{KT}}(\Delta\mathbf{p})$:

$$\Re_{\mathrm{KT}}(\Delta\mathbf{p}) \cong \Re_{\mathrm{KT}}\begin{pmatrix} l_0 < R < L_0, \\ \Delta p < R \end{pmatrix} =$$

$$= \Re_0\exp\left(-\frac{\Delta p^2}{8R^2}\right)\left[1-2^{8/3}\left(2^{1/6}-1\right)\Gamma\left(\frac{11}{6}\right)\left(\frac{R}{\rho_0}\right)^{5/3}-\right.$$

$$\left.-\frac{5}{12}2^{5/6}\,\Gamma\left(\frac{11}{6}\right)\frac{\Delta p^2}{\rho_0^{5/3}\;R^{1/3}}\right].$$

The ranges $R < \Delta p < L_0$ and $\Delta p > L_0$ can be omitted, since in this case they are practically meaningless. This asymptotics for $\Re_{\mathrm{KT}}(\Delta\mathbf{p})$ at $\Delta p = 0$ coincides with the asymptotics for $\Re_{\mathrm{LM}}$ at $l_0 < R < L_0\{\rho_c\}$ obtained in Ref. 3:

$$\Re_{\mathrm{LM}} \cong \Re_0\left[1-2^{8/3}\left(2^{1/6}-1\right)\Gamma\left(\frac{11}{6}\right)\left(\frac{R}{\rho_0}\right)^{5/3}\right].$$

In this case, the ratio of the integral resolutions of the Knox–Thompson and Labeyrie methods has the following form:

$$\frac{\Re_{\mathrm{KT}}(\Delta\mathbf{p})}{\Re_{\mathrm{LM}}} \cong \exp\left(-\frac{\Delta p^2}{8R^2}\right)\left[1-\frac{5}{12}2^{5/6}\Gamma\left(\frac{11}{6}\right)\frac{\Delta p^2}{\rho_0^{5/3}R^{1/3}}\right] \le 1. \tag{5}$$

Let us restrict our consideration to analysis of the integral resolution of the methods for postdetector image processing in only the most-used case of receiving apertures smaller than the outer scale of atmospheric turbulence. The range of huge receiving apertures ($R > L_0$) is beyond the scope of this consideration.

It turns out that as the shift of the spatial scales Δp increases, the integral resolution of the Knox–Thompson method $\Re_{\mathrm{KT}}(\Delta\mathbf{p})$ decreases. In addition, as it follows from Eqs. (4) and (5), the integral resolution of the Knox–Thompson method is always lower than that of the Labeyrie method.

At strong image distortion (advanced speckle-structure) $R > \rho_c$ there are two integration domains, which contribute considerably to the integrand of Eq. (2). In these domains, expand the factor including the structural functions of fluctuations of the complex phase of the optical wave into a series and remain the first two (zero-order and first-order) terms:

$$\Re_{\mathrm{KT}}(\Delta\mathbf{p}) \cong \frac{k^2}{\pi R^2 F^2}\exp\left(-\frac{3\Delta p^2}{8R^2}\right)\times$$

$$\times\int\!\!\int_{-\infty}^{\infty} d\mathbf{p}\exp\left[-\frac{p^2+\Delta\mathbf{p}\mathbf{p}}{2R^2}-\frac{1}{2}D(\mathbf{p})-\frac{1}{2}D(\mathbf{p}+\Delta\mathbf{p})\right]\times$$

$$\times \iint\limits_{-\infty}^{\infty} d\boldsymbol{\rho} \exp\left[-\frac{\rho^2 - \Delta\mathbf{p}\boldsymbol{\rho}}{2R^2}\right]\left[1 - \frac{1}{2}D(\boldsymbol{\rho}) - \frac{1}{2}D(\boldsymbol{\rho} - \Delta\mathbf{p}) + \right.$$

$$\left. + \frac{1}{2}D(\boldsymbol{\rho} - \mathbf{p} - \Delta\mathbf{p}) + \frac{1}{2}D(\boldsymbol{\rho} + \mathbf{p})\right] + \frac{k^2}{\pi R^2 F^2} \times$$

$$\times \exp\left(-\frac{3\Delta p^2}{8R^2}\right) \iint\limits_{-\infty}^{\infty} d\mathbf{p} \exp\left[-\frac{p^2 + \Delta\mathbf{p}\mathbf{p}}{2R^2}\right] \times$$

$$\times \iint\limits_{-\infty}^{\infty} d\boldsymbol{\rho} \exp\left[-\frac{\rho^2 - \Delta\mathbf{p}\boldsymbol{\rho}}{2R^2} - \frac{1}{2}D(\boldsymbol{\rho}) - \frac{1}{2}D(\boldsymbol{\rho} - \Delta\mathbf{p})\right] \times$$

$$\times \left[1 - \frac{1}{2}D(\mathbf{p}) - \frac{1}{2}D(\mathbf{p} + \Delta\mathbf{p}) + \right.$$

$$\left. + \frac{1}{2}D(\boldsymbol{\rho} - \mathbf{p} - \Delta\mathbf{p}) + \frac{1}{2}D(\boldsymbol{\rho} + \mathbf{p})\right].$$

For small receiving apertures $R < l_0$, then $\rho_c < l_0$ since $R > \rho_c$, and, consequently, $\rho_c = \rho_m$. In this case, for the spatial scales $\Delta p < l_0$

$$\Re_{KT}(\Delta\mathbf{p}) \cong \Re_{KT}\begin{pmatrix} R < l_0, \\ \Delta p < l_0 \end{pmatrix} = \Re_0\left[\left(\frac{\rho_m}{R}\right)^2\right] \times$$

$$\times \exp\left(-\frac{\Delta p^2}{2\rho_m^2}\right)\left[1 - \frac{3}{8}\left(\frac{\rho_m}{R}\right)^2 - \frac{1}{4}\left(\frac{\Delta p}{\rho_m}\right)^2\right].$$

The integral resolution of the Labeyrie method for this case has the form (this asymptotics was not obtained in Ref. 3, because the case $R < l_0$ was not considered):

$$\Re_{LM} \cong \Re_0\left(\frac{\rho_m}{R}\right)^2\left[1 - \frac{3}{8}\left(\frac{\rho_m}{R}\right)^2\right].$$

For the intermediate size of the receiving aperture $l_0 < R < L_0$ and $\Delta p < R$ this range covers almost whole practically meaningful variability range of the function. Assume, for certainty, that $\rho_c > l_0$, that is, $\rho_c = \rho_0$. Then the following equation can be obtained:

$$\Re_{KT}(\Delta\mathbf{p}) \cong \Re_{KT}\begin{pmatrix} l_0 < R < L_0, \\ \Delta p < R \end{pmatrix} =$$

$$= \Re_0\left[0.5\left(\frac{\rho_0}{R}\right)^2\right]\exp\left(-\frac{\Delta p^2}{2\rho_0^2}\right) \times$$

$$\times \left[1 + \frac{5}{6}2^{-1/6}\,\Gamma\left(\frac{11}{6}\right)\left(\frac{\rho_0}{R}\right)^{1/3} - \frac{5}{24}2^{5/6}\,\Gamma\left(\frac{11}{6}\right)\frac{\Delta p^2}{\rho_0^{5/3}R^{1/3}}\right].$$

The integral resolution of the Labeyrie method for this case has the form

$$\Re_{LM} = \Re_0\left[0.5\left(\frac{\rho_0}{R}\right)^2\right]\left[1 + \frac{5}{6}2^{-1/6}\Gamma\left(\frac{11}{6}\right)\left(\frac{\rho_0}{R}\right)^{1/3}\right],$$

which is almost identical to the result obtained in Ref. 3:

$$\Re_{LM} = \Re_0\left[2^{-1/5}0.5\left(\frac{\rho_0}{R}\right)^2\right]\left[1 + 2^{-1/5}\frac{5}{6}2^{-1/6}\Gamma\left(\frac{11}{6}\right)\left(\frac{\rho_0}{R}\right)^{1/3}\right].$$

Insignificant discrepancies in the coefficients are explained by differences in the applied asymptotic equations for approximation of the structural functions of fluctuations of the optical wave complex phase by parabolas.

The ratio of the integral resolutions of the Knox–Thompson and Labeyrie methods for the strongly distorted image can be written as follows:

$$\frac{\Re_{KT}(\Delta\mathbf{p})}{\Re_{LM}} \cong \begin{cases} \exp\left(-\dfrac{\Delta p^2}{2\rho_m^2}\right)\left[1 - \dfrac{1}{4}\left(\dfrac{\Delta p}{\rho_m}\right)^2\right], \\ \qquad R < l_0,\ \Delta p < l_0, \\ \exp\left(-\dfrac{\Delta p^2}{2\rho_0^2}\right)\left[1 - \dfrac{5}{24}2^{5/6}\Gamma\left(\dfrac{11}{6}\right)\dfrac{\Delta p^2}{\rho_0^{5/3}R^{1/3}}\right], \\ \qquad l_0 < R < L_0,\ \Delta p < R,\ \rho_c > l_0. \end{cases}$$

It turns out that at the advanced speckle-structure of the image the ratio of the integral resolution of the Knox–Thompson method to that of the Labeyrie method is also less than unity. Thus, it can be concluded that the integral resolution of the Knox–Thompson method is always lower than that of the Labeyrie method.

In conclusion, we give comparative estimates of the integral resolution of different versions of the Knox–Thompson method at postdetector image processing for an arbitrarily large telescope ($R \to \infty$) at infinite value of the outer scale of atmospheric turbulence ($L_0 \to \infty$). It can be shown that the integral resolution of the Knox–Thompson method is characterized by the following limit values.

a) Classic Knox–Thompson method:

$$\lim_{R\to\infty}\Re_{KT}(\Delta\mathbf{p}) = 2\frac{k^2}{F^2}\lim_{R\to\infty}\iint\limits_{-\infty}^{\infty} d\mathbf{p}\,\tau_{KT}(\mathbf{p}, \mathbf{p} + \Delta\mathbf{p}) \cong$$

$$\cong 4\frac{k^2}{F^2}\iint\limits_{-\infty}^{\infty} d\mathbf{p}\exp\left[-\frac{1}{2}D(\mathbf{p}) - \frac{1}{2}D(\mathbf{p} + \Delta\mathbf{p})\right] \cong$$

$$\cong 4 \cdot 2^{-6/5}\exp\left(-\frac{\Delta p^2}{2\rho_0^2}\right)\lim_{R\to\infty}\Re_{LE} =$$

$$= 1.74\exp\left(-\frac{\Delta p^2}{2\rho_0^2}\right)\lim_{R\to\infty}\Re_{LE}, \tag{6}$$

where

$$\lim_{R\to\infty}\Re_{LE} = \frac{k^2}{F^2}\lim_{R\to\infty}\iint\limits_{-\infty}^{\infty} d\mathbf{p}\,\tau_{LE}(\mathbf{p}) \cong$$

$$\cong \frac{k^2}{F^2}\iint\limits_{-\infty}^{\infty} d\mathbf{p}\exp\left[-\frac{1}{2}D(\mathbf{p})\right] \cong \begin{cases} \dfrac{\pi k^2\rho_m^2}{F^2}, & \rho_c < l_0, \\ \Gamma\left(\dfrac{11}{5}\right)\dfrac{\pi k^2\rho_0^2}{F^2}, & \rho_c > l_0; \end{cases}$$

$\Re_{LE}$ and $\tau_{LE}(\mathbf{p})$ are, respectively, the integral resolution and OTF of the telescopic system in the turbulent atmosphere at observation of the average image[3,4]; $\Gamma(11/5) = 1.10$.

b) Extended Knox–Thompson method (data from Ref. 3):

$$\lim_{R\to\infty} \Re_{KT} = \frac{k^2}{F^2} \lim_{R\to\infty} \sqrt{\int\int_{-\infty}^{\infty} d\mathbf{p}_1 \int\int_{-\infty}^{\infty} d\mathbf{p}_2\, \tau_{KT}(\mathbf{p}_1, \mathbf{p}_2)} \cong$$

$$\cong \sqrt{2} \lim_{R\to\infty} \Re_{LE} = 1.41 \lim_{R\to\infty} \Re_{LE}. \quad (7)$$

Comparison of Eqs. (6) and (7) shows that

$$\frac{\lim\limits_{R\to\infty} \Re_{KT}(\Delta\mathbf{p})}{\lim\limits_{R\to\infty} \Re_{KT}} \cong \frac{4 \cdot 2^{-6/5}}{\sqrt{2}} \exp\left(-\frac{\Delta p^2}{2\rho_0^2}\right) =$$

$$= 1.23 \exp\left(-\frac{\Delta p^2}{2\rho_0^2}\right).$$

This means that the classic Knox–Thompson method based on the ordinary Fourier transformation has a somewhat higher integral resolution at $\Delta p < \rho_0$, that is, at shifts of speckle-interferograms within a speckle. At $\Delta p > \rho_0$ the pattern is quite opposite: the extended Knox–Thompson method (bispectral Fourier transformation) has an obvious advantage over the classic method.

Consequently, it can be concluded that advantages of the bispectral Fourier transformation over the ordinary Fourier transformation show themselves in better processing of the well advanced speckle-structure of an image for large shifts of speckle-interferograms.

References

1. D.L. Fried, J. Opt. Soc. Am. **56**, No. 10, 1372–1379 (1966).
2. D.L. Fried, J. Opt. Soc. Am. **56**, No. 10, 1380–1385 (1966).
3. I.P. Lukin, Atmos. Oceanic Opt. **8**, No. 3, 248–250 (1995).
4. I.P. Lukin, Atmos. Oceanic Opt. **8**, No. 3, 235–240 (1995).