Model of passive impurity plume with accounting for its internal structure

V.P. Kabashnikov and A.P. Chaikovsky

B.I. Stepanov Institute of Physics, National Academy of Sciences of Belarus, Minsk

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A model of passive impurity plume is considered in the homogeneous turbulent atmosphere with accounting for in-plume fluctuations of the impurity concentration. The model is based on the method of random forces of the turbulence theory and allows calculating the space–time correlation functions of the concentration field. It is shown that the in-plume concentration fluctuations are caused by longitudinal pulsations of the wind velocity. The plume axis fluctuations are caused by transversal pulsations of the wind velocity with the scales equal to typical displacement of a liquid particle due to the turbulent diffusion in the average wind direction. The amplitude of the plume axis fluctuations tends to zero at large distances from the impurity source. At small distances from the source, where the main contribution to the impurity concentration pulsations is made by the plume displacements as a whole, the considered model is close to the Gifford plume model. At large distances, where the in-plume concentration fluctuations are considerable, the indicated models differ much. In particular, the time correlation function of concentration pulsations can be negative in distant plume regions. The calculation results satisfactorily agree with the experiment.

Introduction

Solution of a number of practical problems on the impurity propagation in the atmosphere requires the knowledge of not only average concentrations but also statistical characteristics of their fluctuations. A simple and effective method of calculation of concentration moments of the passive impurity ejected by a point stationary source is the Gifford plume model.[1] The model takes into account two independent processes: plume axis vibrations and jet divergence in the coordinate system connected with the plume axis. The main mechanism causing fluctuations is the transverse movements of the plume as a whole. A disadvantage of the model is the ignoring of concentration fluctuations in the coordinate system connected with the plume axis, so-called in-plume fluctuations.

Recently, a number of works have appeared[2–6] essentially advancing the solution of the problem on the in-plume concentration fluctuations. In them, the probability density of the plume axis positions is postulated[3] or determined based on symmetry reasons,[4] or it is found by the method of Lagrangian statistical modeling[2,5,6] accounting for the full mixing condition.[7] It is taken into account in this case that only wind velocity fluctuations with the scales equal to transverse plume dimensions[5,6] are responsible for the plume axis displacements. Further, a certain parameterization of the single-point concentration probability density is set in the coordinate system connected with the plume axis, which finally allows calculating single-point simultaneous concentration moments of different orders.

However, there are a number of problems, which require the estimation of a more general form for moments of the impurity concentration fluctuation. For example, if the measured quantity is proportional to the sum of concentrations in different points of the plume in different moments, then, in order to estimate this quantity variation caused by the atmospheric turbulence, it is necessary to know the space–time correlation functions of the concentration. Such situation takes place when lidar estimating the impurity source power.[8]

In this paper, a random field of the passive impurity concentration formed by the point stationary source is simulated by the method of random forces in terms of the turbulent theory.[9] The wind velocity field is supposed to be statistically uniform and stationary. Various liquid particles leaving the source move subjected to the statistically independent δ-time-correlated accelerations. Nevertheless, velocities (and positions) of various liquid particles are partially correlated, if the difference in times of their escape from the source do not exceed the Eulerian time of the velocity correlation, and the source size do not exceed the Eulerian length of the velocity correlation.

This approach allows generalization of the Gifford plume model and taking into account of main features of the concentration field in the plume. Within the limits of the model, there naturally occur the in-plume concentration fluctuations. Fluctuations of the plume axis position are caused by transverse fluctuations of the wind velocity, with the scales equal or greater than the typical longitudinal displacement of a liquid particle due to the turbulent diffusion. The amplitude of the mass center fluctuations at large distances from the source tends to zero. Main differences between this model and the Gifford one manifest themselves when the role of the in-plume fluctuations is comparable with transverse displacements of the plume as a whole.

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Basic equations

For the statistical description of the gas motion in the turbulent flow, we use the Lagrange approach and, according to Ref. 9, consider that motion of a single liquid particle is submitted to the stochastic Langevin equation

$$\frac{dv}{dt} = -\lambda v + f(t), \tag{1}$$

where v is one of the Cartesian velocity components; λ^{-1} is the Lagrange time of the velocity correlation; t is the time; $f(t)$ is the component of the random statistically stationary δ-time-correlated force per unit of liquid mass

$$\overline{f(t+s)f(s)} = 2\overline{v^2}\lambda\delta(t) \tag{2}$$

($\delta(t)$ is the Dirac delta-function).

Let us introduce the Cartesian coordinate system with the x-axis along the average wind direction and vertical z-axis. Denote the pulsation components of x-, y-, z-velocity components by u, v, w, respectively. The distribution of liquid particles by space and velocities matching Eqs. (1) and (2), is described by the Fokker–Planck equation.[10] Let us deal with rather high sources, in order to consider the atmospheric layer, where the impurity scatters, approximately homogeneous. In this case, the function of liquid particles distribution over space and velocities at a fixed initial velocity is normal within time α after escaping the point source and determined by moments of coordinates and velocities of the first and the second orders.

The average impurity concentration is known to be proportional to the probability density of the liquid particle coordinate.[10,11] Accounting for the above-mentioned distribution normality of the liquid particle coordinates, the integral η of concentration along the sounding path parallel to y at the moment t at the point x, z at the fixed initial velocity, can be written in the following way:

$$\eta(t,x,z) = \int Q(y',z')dy'\,dz' \int_0^\infty \frac{d\alpha}{2\pi\sqrt{\overline{\Delta x^2(\alpha)}\,\overline{\Delta z^2(\alpha)}}} \times$$

$$\times\exp\left\{-\frac{\left[x - U\alpha - u(t-\alpha,y',z')T_u(\alpha)\right]^2}{2\overline{\Delta x^2(\alpha)}}\right\}\times$$

$$\times\exp\left\{-\frac{\left[z - z' - w(t-\alpha,y',z')T_w(\alpha)\right]^2}{2\overline{\Delta z^2(\alpha)}}\right\}. \tag{3}$$

Here $Q(y', z')$ is the impurity flow distribution in the initial plume cross section ($x = 0$), free of the influence of the initial temperature and outflow velocity; U is the average wind velocity; $u(t', y', z')$ and $w(t', y', z')$ are fluctuations of x- and z-components of the wind velocity in the time t' at the point 0, y', z';

$$T_u(\alpha) = \lambda_u^{-1}\left[1 - \exp(-\lambda_u\alpha)\right]; \tag{4}$$

$$\overline{\Delta x^2(\alpha)} = 2\lambda_u^{-2}\overline{u^2}\times$$

$$\times\left[\lambda_u\alpha - 1.5 + 2\exp(-\lambda_u\alpha) - 0.5\exp(-2\lambda_u\alpha)\right]. \tag{5}$$

Parameters T_w and $\overline{\Delta z^2}$ are determined by analogy with Eqs. (4) and (5), where λ_u and $\overline{u^2}$ are substituted by λ_w and $\overline{w^2}$, respectively.

Assuming that the impurity flow distribution in the initial cross section can be approximated by the Gaussian function

$$Q(y,z) = \frac{M}{2\pi R_y R_z}\exp\left[-\frac{y^2}{2R_y^2} - \frac{z^2}{2R_z^2}\right], \tag{6}$$

where M is the impurity flow; R_y and R_z are the root-mean-square dimensions of the initial cross section along axes y and z, being much smaller than Eulerian velocity correlation lengths, the expression (3) can be presented in the following way:

$$\eta(t,x,z) = \int_0^\infty \frac{M d\alpha}{2\pi\sqrt{\overline{\Delta x^2(\alpha)}\left(\overline{\Delta z^2(\alpha)} + R_z^2\right)}}\times$$

$$\times\exp\left\{-\frac{\left[x - U\alpha - u(t-\alpha)T_u(\alpha)\right]^2}{2\overline{\Delta x^2(\alpha)}} - \frac{\left[z - w(t-\alpha)T_w(\alpha)\right]^2}{2\left(\overline{\Delta z^2(\alpha)} + R_z^2\right)}\right\}, \tag{7}$$

($u(t')$ and $w(t')$ are the fluctuation components of the wind velocity at the moment t' in the center of the initial cross section). Further, the integral η of the concentration along the sounding path will be termed the integral concentration.

Let pulsation components of the wind velocity be characterized by the normal distribution. Then the integrated concentration averaged by initial velocities takes the form

$$\bar{\eta}(x,z) = \int_0^\infty \frac{M d\alpha}{2\pi\sqrt{K(\alpha)L(\alpha)}}\exp\left\{-\frac{(x-U\alpha)^2}{2K(\alpha)} - \frac{z^2}{2L(\alpha)}\right\}. \tag{8}$$

Here

$$K(\alpha) = \overline{\Delta x^2}(\alpha) + \overline{u^2}T_u^2(\alpha); \tag{9}$$

$$L(\alpha) = \overline{\Delta z^2}(\alpha) + R_z^2 + L_{12}(t,\alpha,t,\alpha), \tag{10}$$

$$L_{12}(t_1,\alpha_1,t_2,\alpha_2) = T_w(\alpha_1)T_w(\alpha_2)\overline{w(t_1-\alpha_1)w(t_2-\alpha_2)}. \tag{11}$$

Covariance of the integrated concentrations is determined by the expression

$$\overline{\eta(t_1,x_1,z_1)\eta(t_2,x_2,z_2)} = \int_0^\infty\int_0^\infty \frac{M^2\,\mathrm{d}\alpha_1\mathrm{d}\alpha_2}{4\pi^2\sqrt{\left[K(\alpha_1)K(\alpha_2)-K_{12}^2(t_1,\alpha_1,t_2,\alpha_2)\right]\left[L(\alpha_1)L(\alpha_2)-L_{12}^2(t_1,\alpha_1,t_2,\alpha_2)\right]}}\times$$

$$\times\exp\left\{-\frac{(x_1-U\alpha_1)^2K(\alpha_2)+(x_2-U\alpha_2)^2K(\alpha_1)-2(x_1-U\alpha_1)(x_2-U\alpha_2)K_{12}(t_1,\alpha_1,t_2,\alpha_2)}{2\left[K(\alpha_1)K(\alpha_2)-K_{12}^2(t_1,\alpha_1,t_2,\alpha_2)\right]}\right\}\times$$

$$\times\exp\left\{-\frac{z_1^2L(\alpha_2)+z_2^2L(\alpha_1)-2z_1z_2L_{12}(t_1,\alpha_1,t_2,\alpha_2)}{2\left[L(\alpha_1)L(\alpha_2)-L_{12}^2(t_1,\alpha_1,t_2,\alpha_2)\right]}\right\}, \quad (12)$$

where $K_{12}(t_1, \alpha_1, t_2, \alpha_2)$ is determined by analogy with Eq. (11), when replacing w by u.

If to neglect the fluctuations of longitudinal velocity ($\sqrt{\overline{u^2}} = 0$), in the expressions (8) and (12), we shall come to Gifford's meandering plume model[1]:

$$\bar{\eta}(x_1,z) = \frac{M}{U\sqrt{2\pi L(\bar{\alpha}_1)}}\exp\left\{-\frac{z^2}{2L(\bar{\alpha}_1)}\right\}, \quad (13)$$

$$\overline{\eta(t_1,x_1,z_1)\eta(t_2,x_2,z_2)} =$$

$$= \frac{M^2}{U^2 2\pi\sqrt{\left[L(\bar{\alpha}_1)L(\bar{\alpha}_2)-L_{12}^2(t_1,\bar{\alpha}_1,t_2,\bar{\alpha}_2)\right]}}\times$$

$$\times\exp\left\{-\frac{z_1^2L(\bar{\alpha}_2)+z_2^2L(\bar{\alpha}_1)-2z_1z_2L_{12}(t_1,\bar{\alpha}_1,t_2,\bar{\alpha}_2)}{2\left[L(\bar{\alpha}_1)L(\bar{\alpha}_2)-L_{12}^2(t_1,\bar{\alpha}_1,t_2,\bar{\alpha}_2)\right]}\right\}, \quad (14)$$

where $\bar{\alpha}_i = x_i/U$ ($i = 1, 2$).

Equations (13) and (14) are valid at small distances from the source under the condition

$$G \equiv \xi_w\sqrt{\overline{\Delta x^2}(x/U)}\Big/U \ll 1 \quad (15)$$

(ξ_w^{-1} is the Eulerian vertical velocity correlation time), which allows ignoring longitudinal fluctuations of the wind velocity. The condition (15) can be obtained from Eq. (3), taking into account that the integration interval in Eq. (3) over α is limited by the interval $\Delta\alpha \approx \sqrt{\overline{\Delta x^2}(x/U)}\Big/U$. If the given interval is much less than the Eulerian vertical velocity correlation time ξ_w^{-1}, then the change in vertical velocity $w(t-\alpha)$ within the limits of integration can be ignored. It allows one to approximate integration of Eq. (3) over α and after averaging by initial velocities come to Eqs. (13) and (14).

In-plume concentration fluctuations

An instantaneous value of the mass center z_c for the transverse concentration distribution is determined by the following expression:

$$z_c(x,t) \equiv \int z\,\eta(t,x,z)\,\mathrm{d}z\Big/\int\eta(t,x,z)\,\mathrm{d}z =$$

$$= \int_0^\infty \frac{\mathrm{d}\alpha\left[w(t-\alpha)T_w(\alpha)\right]}{\sqrt{\overline{\Delta x^2}(\alpha)}}\exp\left\{-\frac{\left[x-U\alpha-u(t-\alpha)T_u(\alpha)\right]^2}{2\overline{\Delta x^2}(\alpha)}\right\}\Bigg/$$

$$\Bigg/\int_0^\infty \frac{\mathrm{d}\alpha}{\sqrt{\overline{\Delta x^2}(\alpha)}}\exp\left\{-\frac{\left[x-U\alpha-u(t-\alpha)T_u(\alpha)\right]^2}{2\overline{\Delta x^2}(\alpha)}\right\}. \quad (16)$$

After having passed to the coordinate system connected with the mass center $\tilde{z} = z - z_c(x,t)$, and substituting equations $z = \tilde{z} + z_c(x,t)$ and (16) into Eq. (7), we obtain the expression for the instantaneous concentration, being a complex functional of velocity fluctuations, whose strict averaging is hardly possible. For practical estimations, one can use the fact that an instantaneous value of mass center (16) is a certain vertical displacement of a liquid particle averaged over the finite time interval. Approximately, it can be presented as:

$$z_c(x,t) \approx \frac{1}{2\Delta\alpha}\int_{\bar{\alpha}-\Delta\alpha}^{\bar{\alpha}+\Delta\alpha} w(t-\alpha')T_w(\alpha')\,\mathrm{d}\alpha', \quad (17)$$

where

$$\bar{\alpha} = x/U;\quad \Delta\alpha = \sqrt{2\overline{\Delta x^2}(x/U)}\Big/U. \quad (18)$$

As it follows from Eqs. (17) and (18), the transverse fluctuations of the wind velocity are responsible for the mass center displacements with the spatial scales equal to the typical longitudinal displacement of a liquid particle owing to the turbulent diffusion $(\overline{\Delta x^2})^{1/2}$. Substituting Eq. (17) into Eq. (7), we found that the average integral concentration and the covariance of the integral concentrations in the coordinate system connected with the plume mass center, are determined according to the Eqs. (8) and (12) with substitution of z by $\tilde{z}$, $L(\alpha)$ by $\tilde{L}(\alpha)$ and $L_{12}(t_1, \alpha_1, t_2, \alpha_2)$ by $\tilde{L}_{12}(t_1,\alpha_1,t_2,\alpha_2)$, where

$$\tilde{L}(\alpha) = \overline{\Delta z^2}(\alpha) + \tilde{L}_{12}(t,\alpha,t,\alpha) + R_z^2, \quad (19)$$

$$\tilde{L}_{12}(t_1,\alpha_1,t_2,\alpha_2) =$$

$$= \overline{\left(z_c(x,t_1)-w(t_1-\alpha_1)T_w(\alpha_1)\right)\left(z_c(x,t_2)-w(t_2-\alpha_2)T_w(\alpha_2)\right)} =$$

$$= L_{12}(t_1,\alpha_1,t_2,\alpha_2) + \frac{1}{4\Delta\alpha^2}\int_{\bar{\alpha}-\Delta\alpha}^{\bar{\alpha}+\Delta\alpha}\int_{\bar{\alpha}-\Delta\alpha}^{\bar{\alpha}+\Delta\alpha} L_{12}(t_1,\alpha',t_2,\alpha'')\,\mathrm{d}\alpha'\mathrm{d}\alpha'' -$$

$$-\frac{1}{2\Delta\alpha}\int\limits_{\bar{\alpha}-\Delta\alpha}^{\bar{\alpha}+\Delta\alpha} L_{12}(t_1,\alpha',t_2,\alpha_2)\,d\alpha' -$$

$$-\frac{1}{2\Delta\alpha}\int\limits_{\bar{\alpha}-\Delta\alpha}^{\bar{\alpha}+\Delta\alpha} L_{12}(t_1,\alpha_1,t_2,\alpha')\,d\alpha'. \qquad (20)$$

For simplicity, the expression (20) is written only for $x_1 = x_2 = x$.

The average value of the mass center of the transverse concentration distribution is equal to zero and, as follows from Eqs. (17) and (11), the dispersion is determined as

$$\overline{z_c^2(t,x)} \approx \frac{1}{4\Delta\alpha^2}\int\limits_{\bar{\alpha}-\Delta\alpha}^{\bar{\alpha}+\Delta\alpha}\int\limits_{\bar{\alpha}-\Delta\alpha}^{\bar{\alpha}+\Delta\alpha} L_{12}(t,\alpha',t,\alpha'')\,d\alpha'd\alpha''. \qquad (21)$$

As follows from Eqs. (21), (19), (4) and (5), the dispersion of plume axis displacements $\overline{z_c^2}$ at large distances decreases following $x^{-1/2}$. Thus, $\tilde{L}_{12}$ tends to L_{12}, i.e., the resulting concentration fluctuations become completely in-plume. When no longitudinal pulsations of the wind velocity exists (at $\sqrt{\overline{u^2}} = 0$), hence $\tilde{L}_{12}(t_1,\bar{\alpha},t_2,\bar{\alpha}) = 0$. Expressions (13) and (14) being valid in this case, show that the average product of concentrations (14) is equal to the product of the averages (13) and there are no in-plume fluctuations.

Figure 1 presents the root-mean-square amplitudes of relative fluctuations for the integral concentration in the fixed coordinate system as the distance function x from the impurity source for two distances from the average position of jet axis $z = 0$ and $z = \sqrt{L(x/U)}$, calculated in two ways: taking into account longitudinal pulsations of wind velocity (by Eqs. (8) and (10)) and ignoring them (by Eqs. (13) and (14)). The fluctuation amplitudes calculated by the models (7), (10) and (17), (18) approximately coincide at small distances. The fluctuation maximum is found approximately in the cross section, where the ratio of the root-mean-square displacement of the plume axis to its cross sectional dimension is maximal. Figure 1 also presents the root-mean-square fluctuation amplitudes in the coordinate system connected with mass center, at $\tilde{z} = 0$ and $\tilde{z} = \sqrt{L(x/U)}$. It is seen that in-plume fluctuations, small near the source, become dominating at large distances.

The fluctuation dispersion of the concentration integral by z in the ground jet with recording of horizontal meandering and separation of in-plume fluctuations was experimentally investigated.[12] Figure 2 presents the comparison of the full-scale measurement data[12] with calculation results of the integrated concentration fluctuations in the fixed coordinate system on the average jet axis and in the coordinate system connected with the plume mass center, on its instantaneous axis.

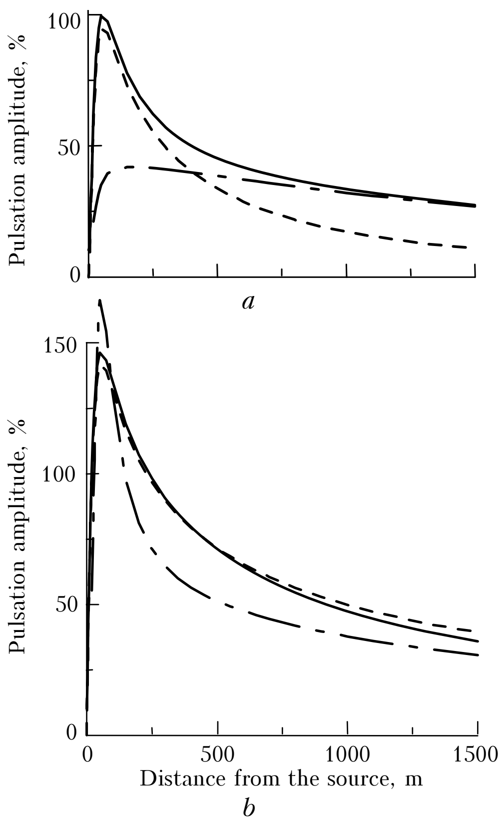

Fig. 1. Root-mean-square pulsation amplitude of the integral concentration attributed to the local average integral concentration depending on the distance from the source in fixed (solid and dashed lines) and moving coordinate systems connected with meandering jet axis (dot-dashed lines). Solid lines denote calculation by Eqs. (8) and (12), dashed lines denote calculation by Eqs. (13) and (14), dot-dashed lines denote calculation by Eqs. (8) and (12) allowing for Eqs. (19) and (20): on the plume axis at $z = 0$ (*a*), at a distance $z = \sqrt{L(x/U)}$ from the axis (*b*). The calculations were made at $U = 4$ m/s, $\sqrt{\overline{u^2}} = 0.4$ m/s, $\sqrt{\overline{w^2}} = 0.3$ m/s, $\lambda_u^{-1} = 240$ s, $\lambda_w^{-1} = 90$ s, $\xi_u^{-1} = 40$ s, $\xi_w^{-1} = 20$ s, $R_z = 1$ m.

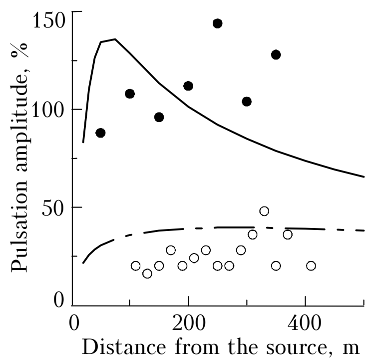

Fig. 2. Root-mean-square pulsation amplitude of the integral concentration on the jet axis attributed to the local average integral concentration depending on the distance from the source in the fixed (solid lines and dark circles) and unfixed coordinate system connected with meandering jet axis (dot-dash lines and light circles). Circles denote the experiment,[12] solid lines denote the calculation by Eqs. (8) and (12), dot-dash lines denote calculation by Eqs. (8) and (12) with regard for Eqs. (19) and (20). The calculations were made at $U = 4$ m/s, $\sqrt{\overline{u^2}} = 0.4$ m/s, $\sqrt{\overline{v^2}} = 0.4$ m/s, $\lambda_u^{-1} = 240$ s, $\lambda_v^{-1} = 240$ s, $\xi_u^{-1} = 40$ s, $\xi_v^{-1} = 40$ s, $R_y = 1$ m.

Figure 3 presents the root-mean-square fluctuations of the integrated concentration averaged by the plume axis from 50 up to 350 m depending on the distance from the average plume axis normalized to the plume-half-width.

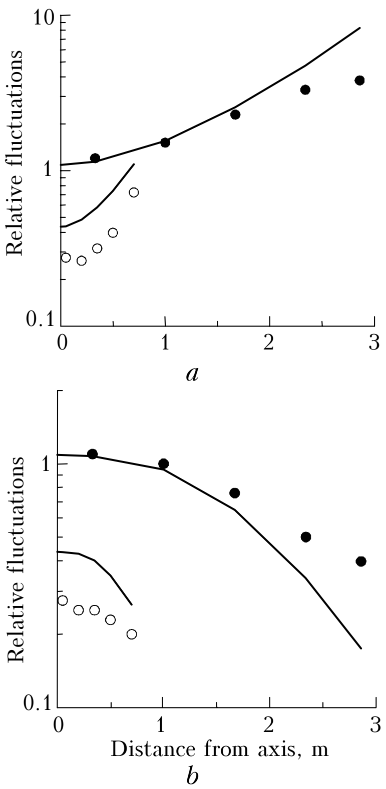

Fig. 3. Root-mean-square pulsation amplitude of the integral concentration as a function of the distance from the jet axis normalized to the average jet half-width in the fixed (upper lines and dark circles) and unfixed coordinate systems connected with meandering jet axis (lower lines and light circles) attributed to the local (*a*) and axial (*b*) average concentrations. The circles denote the experiment,[12] upper lines denote calculation by Eqs. (8) and (12), lower lines denote calculation by Eqs. (8) and (12) allowing for Eqs. (19) and (20). The calculations were made at the same initial data as for Fig. 2.

Figure 3*a* presents the root-mean-square fluctuations attributed to the local average values of the integrated concentration, and Fig. 3*b* – to the axial average values of the integrated concentration. Data on meteorological conditions during the experiment are absent in Ref. 12, therefore, in calculations we used the typical values of input parameters.[13,14] Nevertheless, the calculation results on the whole satisfactorily agree with the experiment. Unlike Refs. 2–6, where the relative in-plume fluctuations are assumed independent of the transverse coordinates, the Ref. 12 and our calculations (Fig. 3*a*) confirm that such dependence exists and it is expressed rather clearly.

Correlation functions

For rather large distances from the initial plume cross section, when the inequality sign in the condition (15) changes to the opposite one and the following relations are fulfilled

$$\mathrm{Max}\left[\xi_u^{-1},\xi_w^{-1}\right] \ll \mathrm{Min}\left[\sqrt{K(x/U)}/U,\sqrt{L(x/U)}/U\right]; \quad (22)$$

$$\begin{aligned} &K_{12}(t,t,x/U,x/U) \ll K(x/U);\\ &L_{12}(t,t,x/U,x/U) \ll L(x/U), \end{aligned} \quad (23)$$

the correlation function can be found analytically. Accounting for Eq. (23), expand Eq. (12) into a series by K_{12}, L_{12} and limiting by the linear approximation, obtain

$$\overline{\eta(t,x,z)\eta(t+\Delta t,x,z)} \approx$$

$$\approx \int_0^\infty \int_0^\infty \mathrm{d}\alpha_1 \mathrm{d}\alpha_2 \left[4\pi^2 \left(K(\alpha_1)K(\alpha_2)L(\alpha_1)L(\alpha_2)\right)\right]^{-1/2} \times$$

$$\times \exp\left\{-0.5\left[(x-U\alpha_1)^2 K^{-1}(\alpha_1) + (x-U\alpha_2)^2 K^{-1}(\alpha_2) + \right.\right.$$

$$\left.\left. + z^2 L^{-1}(\alpha_1) + z^2 L^{-1}(\alpha_2)\right]\right\} \times$$

$$\times \left\{1 + \frac{(x-U\alpha_1)(x-U\alpha_2)K_{12}(t+\Delta t,\alpha_1,t,\alpha_2)}{K(\alpha_1)K(\alpha_2)} + \right.$$

$$\left. + \frac{z^2 L_{12}(t+\Delta t,\alpha_1,t,\alpha_2)}{L(\alpha_1)L(\alpha_2)}\right\}. \quad (24)$$

Functions K_{12}, and L_{12}, proportional to the Eulerian correlation functions, differ from zero only at $|t_1-\alpha_1-t_2+\alpha_2| \le \xi_{u,w}^{-1}$ and, following Eq. (22), act as delta-functions $\delta(t_1-\alpha_1-t_2+\alpha_2)$ relative to other terms of the expansion. Therefore, the exact form of the Eulerian correlation function is not necessary for calculating integrals; it is enough to know the Eulerian correlation times. As a result, we obtain

$$B(\Delta t) \equiv \frac{\overline{\eta(t,x,z)\eta(t+\Delta t,x,z)}}{\overline{\eta(x,z)}\;\overline{\eta(x,z)}} - 1 =$$

$$= \frac{\exp(-\gamma^2)}{2\sqrt{\pi}}\left\{(1-2\gamma^2)\frac{\overline{u^2}\,T_u^2(x/U)\xi_u^{-1}U}{K^{3/2}(x/U)} + \right.$$

$$\left. + \frac{2z^2\overline{w^2}\,T_w^2(x/U)\xi_w^{-1}U}{L^2(x/U)K^{1/2}(x/U)}\right\}, \quad (25)$$

where

$$\gamma = \Delta t\, U / 2K^{1/2}(x/U). \quad (26)$$

As it follows from Eqs. (25) and (26), the time scale of the integral concentration correlation is equal to $K^{1/2}(x/U)/U$. The main contribution to the integrated concentration pulsations is made by the concentration inhomogeneities with longitudinal dimensions about $K^{1/2}(x/U)$, shifting with wind velocity relative to the observation point. At small distances from the plume axis, the integrated concentration pulsations of the impurity are caused only by longitudinal fluctuations of the wind velocity. However, the role of longitudinal velocity fluctuations in generation of the integral concentration pulsations is significant at all distances from the plume axis. The expression (25) also shows that correlation function near the plume axial region can be negative.

In the absence of longitudinal fluctuations of wind velocity, the correlation function can be obtained through expanding Eq. (14) into a series by L_{12} up to the quadratic term inclusive:

$$B(\Delta t) = \frac{z^2 L_{12}(t+\Delta t, x/U, t, x/U)}{L^2(x/U)} + $$

$$+\frac{L_{12}^2(t+\Delta t, x/U, t, x/U)}{2L^2(x/U)}\left[1 - z^2/L(x/U)\right]^2. \quad (27)$$

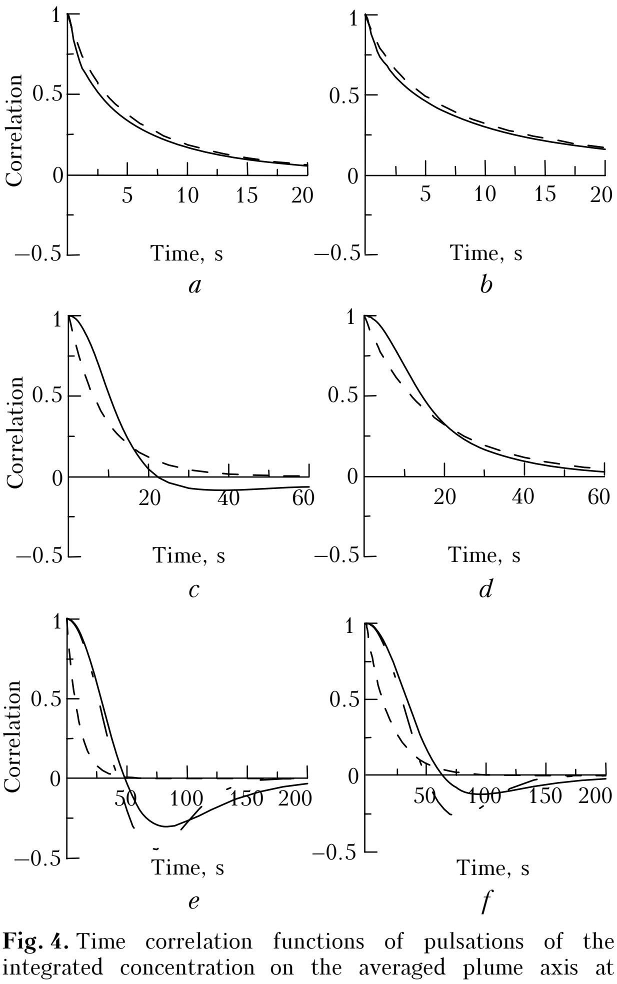

Fig. 4. Time correlation functions of pulsations of the integrated concentration on the averaged plume axis at $z = 0$ (*a*, *c*, *e*) and at a distance $z = \sqrt{L(x/U)}$ from the axis (*b*, *d*, *f*) at different distances from the initial cross section: 50 m (*a*, *b*), 500 m (*c*, *d*), 1500 m (*e*, *f*). Solid lines denote calculation by Eqs. (8) and (12), dot lines denote calculation by Eqs. (13) and (14), dot-dash lines denote calculation by Eq. (25).

The expression (27) leads to the results much differing from those given by the formula (25), obtained with accounting for longitudinal fluctuations of the wind velocity. In particular, time scale of the correlation function (27) in the order of magnitude is equal to the Eulerian time scale ξ_w^{-1} of the wind velocity vertical component correlation, that, in view of Eq. (22), is much less than the time scale of the correlation function (25).

Time correlation functions of the integrated concentration calculated by Eqs. (8), (12)–(14) for three distances from the source, are presented in Fig. 4.

It is seen that at a distance of 50 m, characterized by the criterion $G = 0.03$ under the chosen parameters and being considered small, the taking into account of the longitudinal pulsations weakly affects the results. At an intermediate distance of 500 m ($G = 0.3$) and at a distance of 1500 m, where $G = 0.7$, accounting for longitudinal pulsations essentially affects the form of the correlation function and increases the time correlation scale by several times. The negative section of the correlation function appears at a distance of 500 m.

Conclusion

Random field of the passive impurity concentration formed by the point stationary source in the statistically inhomogeneous medium and stationary atmosphere was simulated by the method of random forces in terms of the turbulent theory. This allowed taking into account of the in-plume concentration fluctuations and generalization of the Gifford plume model. It is shown that the division of concentration fluctuations into in-plume fluctuations and fluctuations caused by vibrations of the plume mass center is possible only when accounting for the longitudinal wind velocities. Fluctuations of the plume mass center position were caused by transverse fluctuations of the wind velocity with the scales equal to the typical displacement of a liquid particle due to the turbulent diffusion in the average wind direction. The wind velocity fluctuations of smaller scales are responsible for in-plume concentration fluctuations, i.e., for the concentration fluctuations in the coordinate system connected with the plume mass center. If to neglect the longitudinal pulsations of the wind velocity, the diffuse displacement of a liquid particle in the average wind direction becomes zero. The in-plume concentration fluctuations disappear and only vibrations of the plume mass center cause the concentration fluctuations in the plume. In this case, the considered model transforms into the Gifford plume model. The main differences of the given model from the Gifford model manifest themselves, when the role of in-plume fluctuations becomes comparable with the effect of transverse displacements of the plume as a whole. In particular, the fluctuation characteristics of the integral from the concentration over the plume transverse cross section do not depend on transverse displacements of the plume as a whole. They are determined only by the in-plume fluctuations and therefore, are not described by the Gifford model. Significant differences in pulsation characteristics of concentrations of the two models always take place at large distances from the source. The time scale of the correlation function of the integral concentration of the impurity at large distances are equal to the ratio of dimensions of longitudinal plume inhomogeneities to the wind velocity in the order of magnitude that can considerably increase the time scale following from the Gifford model. The concentration correlation function itself near the axial plume region can take the negative values.

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