Peculiarities in propagation of high-power ultra-short laser pulses in the atmosphere

S.A. Bakhramov, A.K. Kasimov, and Sh.D. Paiziev

Scientific Production Association "Akadempribor", Tashkent, Uzbekistan

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A new technique for determination of distance before the beginning of filamentation of a high-power ultra-short laser radiation in a turbulent atmosphere is proposed. Methods of geometrical optics allows a considerable reducing of computation time consumption. The results satisfactorily agree with those obtained by use of the method of slowly varying amplitudes.

Filamentation of ultrashort laser pulses in the atmosphere can be widely applied in wide-band sensing of the atmosphere, improvement of laser location devices, etc.[1] This has stimulated detailed study of filaments formation and propagation in the atmosphere. Computer simulation plays a leading part in studying the process of the filament formation and propagation due to difficulties in analytical description and real experiments.

To completely describe the process of pulse propagation, one should take into account joint manifestation of radiation diffraction and dispersion, self-modulation connected with Kerr non-linearity, laser plasma non-linearity, and, finally, fluctuations of radiation phase arising under turbulence conditions.[2] In this case, numerical describing of slowly varying amplitude of the electric field needs high consumption of computer time. At the same time, quick determination of the distance before the filamentation beginning is a key demand in some applied problems, such as spectroscopic determination of atmospheric components or finding the desirable intensity in laser location of remote objects.[3] Therefore, it is necessary to find new approximation methods simulating the process of the filament formation and propagation.

In this paper, we propose to determine the distance before the beginning of filamentation by the use of the geometrical optics approximation with a correction taking into account the diffraction broadening of laser beam. Suppose that the spatial distribution of intensity at the laser output is described by the Gauss distribution. The inertial-free nonlinear additive $n_2(I)$ to the refractive index of the atmosphere n_0, which is induced by the Kerr effect, is proportional to the intensity in a particular point. Then the medium can be treated as axially symmetric at the distance Δz, where variations of the beam radius due to self-focusing are insignificant. This assumption permits one to reduce the problem of determination of the nonlinear focus to propagation of laser radiation in some cylindrically stratified media along the z axis under the condition that $dz < \Delta z$ at each step of integration. By analogy with the equation of a ray trajectory in a spherically stratified medium,[4] we can write for the axially stratified medium

$$\frac{dr}{dz} = \sqrt{1 + \frac{2n_2}{n_0}\frac{dI}{dr}dr - \cos^2\theta} \Big/ \cos\theta, \qquad (1)$$

where r is the transverse coordinate; n_0 is the refractive index of the medium; dI/dr is the derivative of the intensity with respect to the transverse coordinate; θ is the angle between the ray and the z axis.

After some manipulations, the increment of the transverse coordinate dr at each step dz can be defined as

$$dr = \left[\frac{n_2}{n_0}\frac{dI}{dr} + \sqrt{\left(\frac{n_2}{n_0}\frac{dI}{dr}\right)^2 + \frac{\cos^2\theta(1-\cos^2\theta)}{dz^2}}\right]\frac{dz^2}{\cos^2\theta}. \qquad (2)$$

Based on this equation, a computer model was developed, in which the radiation cross section area was divided by a 100×100 square grid with a step $\Delta x = \Delta y = 0.12$ mm. Within a cell, the intensity and, consequently, the nonlinear additional term $n_2(I)$ for the refractive index were supposed to be constant. The calculations deal with a trajectory of rays emanating from the center of each cell with a weight corresponding to the value of the initial intensity distribution for a given coordinate of the cell center. In the initial plane, the phase front was supposed to be planar; thus, the rays were supposed to be parallel to the z axis. At each step dz for each ray, dr and corresponding new spatial coordinates of rays are determined in accordance with Eq. 2. New direction cosines are defined, corresponding to initial and new spatial coordinates for each ray. New arrangement of the rays defines the intensity distribution by transverse coordinates in the corresponding z axis. To improve the accuracy of results in obtaining the intensity distribution, each cell is divided into 10×10

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subcells, for each ray can be considered as a set of parallel rays. The derivative of intensity by the radius at each step dz was determined by approximation of the intensity distribution by the Gaussian curve. At the next step, the obtained coordinates of the ray, its direction cosines, and intensity distribution are used as initial data to determine the next corresponding values. At each step, maximal intensity was determined by z and compared with the value of the threshold ionization of air molecules ($I_i = 5\cdot 10^{13}$ W/cm^2), which served as a criterion for termination of the experiment. The value of dz was chosen, depending on the contribution of nonlinear focusing to the process of propagation, and varied from 1.0 to 0.01 m near the nonlinear focus.

According to preliminary calculations, the process of filament formation is strongly affected by the beam diffraction broadening at a pulse power of $P = (2\div 5)P_{cr}$ and beam radius $a = 0.35$ cm (here and below we use the critical self-focusing power, which was determined experimentally,[5] $P_{cr} \approx 6\cdot 10^9$ W). Therefore, the diffraction was taken into account by the correction to calculations: the value of beam radius was determined at every step from the Gaussian approximation curve of the intensity by z. Then, assuming that the intensity distribution in the lateral plane remains Gaussian during the process of propagation, we determine variation of the lateral beam radius under diffraction at the distance dz as[6]

$$a(dz) = a_0\left[1+\left(\frac{\lambda dz}{\pi a_0^2}\right)^2\right]^{1/2}. \quad (3)$$

The ratio $a(dz)/a_0$ multiplied by corresponding transverse coordinates of the beam defines the shift of each beam along transverse coordinates caused by diffraction. At the distance Δz, self-focusing and diffraction are supposed to be independent. Therefore, the final transverse coordinate of a beam can be defined as the difference of beam replacements caused by each of the above-mentioned processes. Note that the radius of beam section in the process of self-focusing varies from 0.35 cm in the initial plane to 0.01 cm in the point of beginning of filamentation.

Figure 1 presents the length of the way passed by the beam to the beginning of filamentation as a function of radiation power with allowance of diffraction (curve *1*) and without diffraction (curve *2*).

It is seen that the influence of diffraction becomes weaker with increasing pulse power. This is caused by the fact that radiation before the beginning of filamentation passes a short distance in comparison with the diffraction length. Relative elongation of the distance from the beginning of filamentation was 8% at $P = 8P_{cr}$ with allowance of diffraction. Diffraction influence increases with decrease of pulse power and reaches 55% for $P = 2P_{cr}$. Under the conditions that are realized in our calculations, the influence of diffraction is insignificant for $P \geq 2P_{cr}$. For less values of power, it is necessary to take into account diffraction effects on the length of formation of the nonlinear focus.

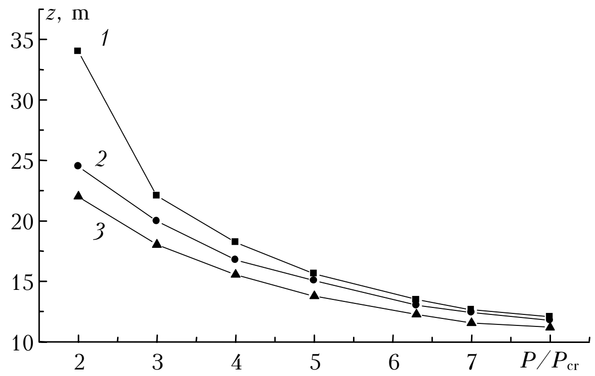

Fig. 1. The length of formation of a nonlinear focus as a function of laser pulse power in a regular medium: with allowance of diffraction (*1*); ignoring diffraction (*2*); with allowance of diffraction in a turbulent atmosphere (*3*).

To take into consideration the influence of atmospheric turbulence, we used a simplified model, according to which the atmosphere is presented as consisting of vortices or spheres with stochastic refractive indices $n = n_0 + n_1$, where $\langle n_1\rangle = 0$. Dimensions of the spheres also change stochastically from the internal scale of turbulence l_0 to the external scale L_0.[7] Small heterogeneities yield non-correlated contributions to the process of beam propagation, so the sum of these contributions is close to zero. Heterogeneities with dimensions that considerably exceed the beam radius can be represented as spheres, whose influence is reduced to stochastic focusing or defocusing of a beam, depending on the sign of n_1. Influence of spheres with different refractive indices along the propagation path can be represented as influence of stochastic lenses. Thus, all path can be represented as a sequence of lenses with stochastic focal distances.

By analogy with the method of phase screens, we suppose that a change of directions of rays consisting the beam, proceeds at the moment of intersection with lenses. Between the lenses, the rays move like in a regular medium. The fluctuating part of the refractive index of the atmosphere n_1 was supposed to be uniformly distributed within the limits $|n_1| \leq \alpha$. To obtain different states of turbulence, the value of α varied within the limits $\alpha = 5\cdot 10^{-7}\div 5\cdot 10^{-6}$, the dispersion of fluctuation of the refractive index was $\sigma_n^2 = \langle n_1^2\rangle = \alpha^2/3$.

To estimate the adequacy of the described model to the real atmosphere, we use the approximate expression from Ref. 7. According to Ref. 7, the value of the structural characteristic of the refractive index, realized in our calculations, corresponds to the atmospheric turbulence with the structural characteristic of the refractive index

$$C_n^2 \approx 1.9\sigma_n^2 \left(\frac{2\pi}{L_0}\right)^{2/3} \approx 2\cdot 10^{-14} \div 2\cdot 10^{-12}\,\text{cm}^{-2/3},$$

where $L_0 = 1$ m. Since in Eq. 2 $\frac{n_2}{n_0}\frac{dI}{dr}$ is used, which, in fact, is the derivative of the refractive index in the Kerr medium, then to take the turbulence into consideration, we add the value of the refractive index derivative, caused by atmospheric turbulence. The latter can be approximately written as n_1/d, where d is the diameter of a stochastic lens or, equivalently, the dimension of spherical heterogeneity.

Figure 2 presents the distribution of intensity in the cross section for the beam of the power $P = 3.8\cdot 10^9$ W, radius $a = 0.35$ cm at different distances from the initial plane for $d \sim L_0$.

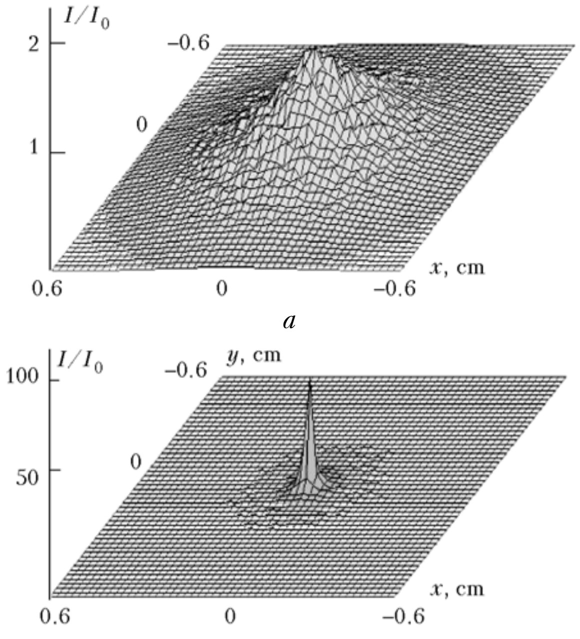

Fig. 2. Distribution of intensity on the plane: $z = 6.0$ m (*a*), $z = 11.5$ m (*b*).

The distribution in Fig. 2*a* corresponds to distribution of intensity in the plane z, where the process of avalanche-like growth of intensity in the center of a beam only begins. Figure 2*b* presents the intensity distribution near the nonlinear focus. Fluctuations of intensity (see Fig. 2*a*) are caused by discreteness of the applied method. With increase of the number of cells these fluctuations decrease, but the computation time increases significantly. Therefore, to exclude effects of fluctuations, caused by the discreteness of the method, the derivative of the intensity was calculated by the use of the Gaussian approximation of the intensity distribution under the assumption that the distribution remains Gaussian for the central part, where the basic part of the beam energy is concentrated. This approach leads to smoothing of small-scale fluctuations, what (as was shown by numerical experiments) is not significant in determining the length of formation of a single filament. Besides, it considerably saves the computation time.

Comparison of the obtained results with those from Ref. 2, where the slowly varying amplitude of the electric field of a pulse was calculated, demonstrates a satisfactory agreement. A similarity of the results is seen both in the character of self-focusing and in determining the mean value of the distance before filamentation. In contrast to Ref. 2, the nonlinear focus is always formed in the center of the laser beam.

It is well known that peak intensity in self-focusing grows nonlinearly. A slow increase of intensity in the beginning of the path gradually changes by rapid growth as it approaches the nonlinear focus. Figure 3 presents peak intensity in the central temporal layer as a function of the passed distance.

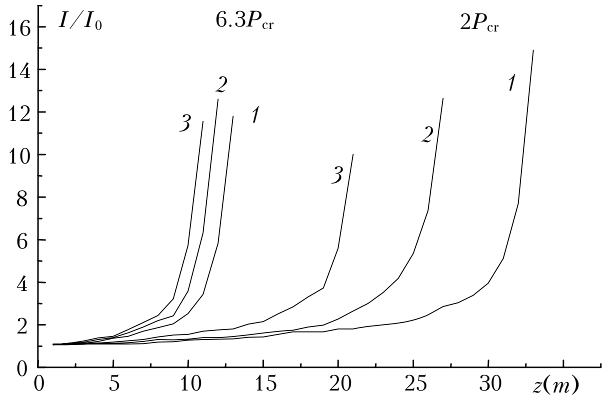

Fig. 3. Relative change of the intensity maximum in the central section of a pulse with power $P = 6.3P_{\rm cr}$ and $2P_{\rm cr}$ in a regular medium (*1*) and in the turbulent atmosphere for two values C_n^2 (C_n^2 (*3*) > C_n^2 (*2*)).

As is seen from the presented results, the process of intensity growth in calculations by our technique completely corresponds to the solution obtained in Ref. 2. Under turbulence, peak intensity grows with acceleration.

As it was shown earlier, we used the atmospheric model, which smoothed small-scale fluctuations. In different realizations of the turbulence with similar C_n^2 the range of lengths in formation of the nonlinear focus was not large, less than 10% of the mean length in formation of the nonlinear focus. Figure 3 presents intensities as functions of the distance for two values of power at two different values of C_n^2. As it should be expected, the turbulence influence on filament formation manifested itself stronger than at $P = 6.3\mathrm{P}_{\rm cr}$. The increase of turbulence in each case resulted in the decrease of the length of formation of the nonlinear focus.

Curve *3* in Fig. 1 presents the mean length of filament formation as a function of the pulse power in a turbulent atmosphere at $C_n^2 = 2 \cdot 10^{-13}$ cm$^{-2/3}$. Turbulence effects, due to instability of positive intensity fluctuations, accelerate the process of self-focusing. This can be easily seen by comparing curves *1* and *3*. Turbulence effects lead to shortening of the mean length of filament formation and fluctuations of the length of the nonlinear focus formation, which are somewhat less than in Ref. 2 due to the chosen model. The value of the mean length, at which the nonlinear focus is formed, agrees satisfactorily with Ref. 2.

Conclusions

Application of methods of geometrical optics in calculating the length of filament formation in a turbulent atmosphere yields results that satisfactorily agree with results obtained in the approximation of the method of slowly varying amplitude.

Time consumption for determining the nonlinear focus position considerably decreases as compared with the method of slowly varying amplitudes. The simplified model of the turbulent atmosphere reduces fluctuations in determining the mean length of the filament formation. These peculiarities of the method for calculating the length of the nonlinear focus formation in the approximation of geometrical optics can give an advantage in fast determination of the distance at which the filamentation of the ultrashort laser radiation begins.

Acknowledgements

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