

## NUMERICAL SIMULATION OF THE SPECTRAL AEROSOL LIGHT SCATTERING PHASE FUNCTION

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*A spectral model of the aerosol scattering phase function has been proposed based on the approximation of the results of calculating the characteristics of light scattering by an ensemble of homogeneous polydisperse spherical particles. The parameters of the model for a specified wavelength and type of aerosol particles (determined by the complex refractive index of the particulate matter) are the aerosol volume scattering coefficient and the aerosol number density. The use of these parameters makes it possible to simulate the dependence of the scattering phase function on the meteorological parameters, particularly, on the meteorological visibility range and humidity of air. Good agreement between the characteristics of the classes of model and experimental scattering phase functions is shown for the scattering phase function classification based on measurement data.*

When calculating the radiation fields in the Earth's atmosphere and particularly, when solving the inverse problems of the remote sensing of the atmosphere, it is necessary to parameterize the aerosol model of the atmosphere. The most difficult problem is the parameterization of the scattering phase function because the approximation of the light scattering phase functions, calculated for an ensemble of aerosol particles, by artificially selected analytical functions (usually they are some modifications of the classical Henyey-Greenstein function) is very rough. Setting of the scattering phase function in tabular form as function of the scattering angles or of the coefficients of expansion in the Legendre polynomials for the specified model parameters (the wavelength, altitude, type of aerosol, humidity, etc.) makes the model to be too cumbersome for practical applications.

In Ref. 1 we proposed an approach to the simulation and parameterization of the phase function of light scattering by an ensemble of homogeneous spherical particles, based on both microphysical (the complex refractive index of particles) and optical characteristics (average scattering cross section of particles of the ensemble) as the parameters. Such an approach makes it possible to relate easily the scattering phase function with such commonly used parameters of aerosol models of the atmosphere as meteorological visibility range, type of aerosol matter, and humidity of air.

As shown in Ref. 1, the relation between the scattering cross section in the given direction  $S(\lambda, n, \kappa, \gamma)$  and the scattering cross section  $s(\lambda, n, \kappa)$  for the ensemble-averaged aerosol particles is well approximated by the formula

$$\ln S(\lambda, n, \kappa, \gamma) = a(\lambda, n, \kappa, \gamma) + b(\lambda, n, \kappa, \gamma) \cdot \ln s(\lambda, n, \kappa), \quad (1)$$

where  $\lambda$  is the wavelength;  $n$  and  $\kappa$  are the real and imaginary parts of the refractive index of aerosol matter, respectively;  $\gamma$  is the scattering angle.

It is easy to calculate the average scattering cross section  $s(\lambda, n, \kappa)$  given that the volume aerosol scattering coefficient  $\sigma$  and number density of particles  $N$  are known

$$s = 10^{-5} \sigma / N, \quad (2)$$

where  $\sigma$  is measured in  $\text{km}^{-1}$ ,  $N$  in  $\text{cm}^{-3}$ , and  $s$  in  $\text{cm}^2$ . The scattering phase function  $x(\gamma)$  is the normalized scattering coefficient in the given direction

$$x(\gamma) = S(\gamma) / \left[ \frac{1}{2} \int_0^\pi S(\gamma) \sin \gamma \, d\gamma \right]. \quad (3)$$

The problem of constructing the model of the scattering phase function reduces to calculation of the coefficients  $a$  and  $b$  in Eq. (1). The technique for calculating these coefficients is described in Ref. 1.

Let us note that the use of the terms "effective radius" and "effective width" of the aerosol particle size distribution function makes it possible to eliminate the dependence on the specific shape of the distribution function from the consideration. Thus, we used four types of distributions for calculations. They were lognormal, Junge, gamma, and inverse gamma distributions, a total of 128 combinations of parameters (see the details in Ref. 1).

Hence, the proposed model of the scattering phase function has five parameters, namely: the wavelength  $\lambda$ , real part of the refractive index  $n$ , imaginary part of the refractive index  $\kappa$ , volume aerosol scattering coefficient  $\sigma$ , and number density of particles  $N$ .

Two last parameters are dependent, because they are related by Eq. (2). The presence of the number density of particles  $N$  as a parameter of the model is its disadvantage, but it is unlikely to avoid the aerosol microphysical characteristics keeping the sufficient accuracy of the approximation of the real scattering phase functions. Humidity of air is not the explicit parameter of the model, because only quantities  $S$  and  $s$  in Eq. (1) vary with the transformation of the particle size spectrum as the humidity changes rather than the coefficients  $a$  and  $b$  that do not depend on the specific shape of the aerosol particle size distribution function and its parameters in the model proposed here. At the same time, humidity affects the parameters  $n$ ,  $\kappa$ , and  $\sigma$ , i.e., it is considered in the model in the implicit form. The simplest example of such consideration is given below.

The coefficients  $a$  and  $b$  were calculated for 37 scattering angles from 0 to 180° at 5° intervals, for 37 wavelengths in the range from 0.2 to 15  $\mu\text{m}$ , for 8 values of the real part of the complex refractive index from 1.33 to 2.20, and for 8 values of the imaginary part of the refractive index from 0 to 0.1. As shown in Ref. 1, the dependences of  $a$  and  $b$  on  $n$  and  $\kappa$  are complex and it is meaningless to approximate them analytically in a wide range of the parameter variation. So the values of the parameters  $a$  and  $b$  calculated for the practical computer implementation of the model were filed. The file was created in a special compressed format, its size was slightly larger than 1 Mbyte, so its storage and use should present no problems for modern computers. The file structure was such that the data could be sampled in the direct access mode, resulting in insignificantly short time of calculation of the scattering phase function. The practical implementation of the model on a computer consisted of the data file and the computer program for calculation of the aerosol scattering phase function by Eqs. (1) and (3) for the specified values of five aforementioned parameters. Linear interpolation between the scattering phase functions was used for intermediate values of the parameters  $\lambda$ ,  $n$ , and  $\kappa$ .

Let us compare the model scattering phase functions with the classification of the experimentally measured scattering phase functions<sup>2,3</sup> as an example of implementation and simultaneous validation of the model proposed. The elongation (asymmetry)  $G$ , i.e., the ratio of scattering into the forward hemisphere and the scattering into the backward hemisphere and the sharpness  $P$ , i.e., the ratio of the value of the phase function of scattering at an angle of 140° to its value at an angle of 105°, determined in Refs. 2–4, were used as parameters of classification. The limiting values of the parameters of the specific class were selected based on

Refs. 2–4. The classes were marked by two numbers divided by the point: the number before the point (class number) indicated the classification on the parameter  $G$ , and the number after the point (subclass number) – on the parameter  $P$ .

The limiting values of the subclass parameters were selected individually for each class (there is a weak dependence of  $P$  on  $G$ ).

When comparing the model and experimental scattering phase functions, it is worthwhile to compare not the scattering phase functions themselves, but their class characteristics. According to Refs. 2–4, they are the meteorological visibility range and the humidity of air. The necessity of comparison of the class characteristics rather than the scattering phase functions follows from the fact that the experimental scattering phase functions are integral over the spectrum; hence, we can only approximately relate them to a certain wavelength (0.55  $\mu\text{m}$ ). In addition, our model has (for the fixed wavelength) four more parameters, so they always can be adjusted to any scattering phase function.

To compare the classifications we simulated the total (aerosol plus molecular) scattering phase function for a wavelength of 0.55  $\mu\text{m}$  and the following ranges of variations of the parameters: real and imaginary parts of the complex refractive index of the dry aerosol matter were the same as in the model; humidity of air varied from 0 to 100% at 20% interval; meteorological visibility range was 1, 3, 7.5, 15, 30, 75, and 150 km; particle number density was  $10^5$ ,  $5 \cdot 10^4$ ,  $10^4$ , 5000, 1000, 500, and  $100 \text{ cm}^{-3}$  for meteorological visibility ranges of 1 and 3 km,  $5 \cdot 10^4$ , 5000, 1000, 500, 100, and  $50 \text{ cm}^{-3}$  for meteorological visibility ranges of 7.5 and 15 km,  $10^4$ , 5000, 1000, 500, 100, 50, and  $10 \text{ cm}^{-3}$  for meteorological visibility ranges of 30 and 75 km, and 5000, 1000, 500, 100, 50, 10, and  $5 \text{ cm}^{-3}$  for a meteorological visibility range of 150 km. Thus, a total of 18816 scattering phase functions was simulated for the classification.

The simplest model was used for simulation of the dependence on the humidity of air. The complex refractive index of the moist aerosol matter  $m_m$  was related to the complex refractive index of the dry aerosol matter  $m_d$  by the formula:

$$m_m = (1 - 0.01 f)m_d + 0.01 f(1.33 - i0), \quad (4)$$

where  $f$  is the relative humidity of air, in %.

The volume aerosol scattering coefficient is expressed in terms of the meteorological visibility range by the formula

$$\sigma_a = 3.9/V - \sigma_{\text{mol}}, \quad (5)$$

where  $V$  is the meteorological visibility range, in km;  $\sigma_{\text{mol}}$  is the volume molecular scattering coefficient equal to  $0.012 \text{ km}^{-1}$  at a wavelength of 0.55  $\mu\text{m}$  in the ground atmospheric layer.

Results of classification of the model scattering phase functions are given in Table I, where the limiting values of the class and subclass parameters are given as well as the model class characteristics, namely: the number of the scattering phase functions of each class  $N$  and the ranges of variations of meteorological visibility values  $V$  and humidity  $f$  for

each class together with the characteristics of the same classes for experimental scattering phase functions,<sup>2,3</sup> namely, the range of variations of meteorological visibility values  $V$  for each class (the data on the meteorological visibility range are lacking in Refs. 2 and 3 for class 3.1 and all the subclasses whose serial numbers are 2 and greater).

TABLE I. Result of classification of the model scattering phase functions and comparison of the model class characteristics with the experimental data.<sup>3,4</sup>

Class	Limiting values of the parameters of the class		Class characteristics			
			Model			Experiment
	$G$	$p$	$N$	$f, \%$	$V, \text{ km}$	$V, \text{ km}$
2.0	<1.9		50	0-40	150	75-150
3.0	1.9-2.5	<1.45	1392	0-100	75-150	30-150
3.1	1.9-2.5	>1.45	82	0-100	150	
4.0	2.5-4.0	<1.35	1736	0-100	1-150	15-150
4.1	2.5-4.0	>1.35	1257	0-100	75-150	30-75
5.0	4.0-6.0	<1.35	3063	0-100	1-75	7.5-75
5.1	4.0-6.0	>1.35	744	0-100	30-75	30-75
6.0	6.0-8.0	<1.30	2074	0-80	1-30	3-30
6.1	6.0-8.0	1.30-2.00	1079	0-100	1-75	15-30
6.2	6.0-8.0	>2.00	0			
7.0	8.0-11.5	<1.10	1194	0-80	1-30	1-15
7.1	8.0-11.5	1.10-2.00	1290	0-100	1-30	7.5-15
7.2	8.0-11.5	2.00-6.00	547	0-100	1-30	
7.3	8.0-11.5	>6.00	0			
8.0	11.5-17.0	<1.10	795	0-80	1-15	1-7.5
8.1	11.5-17.0	1.10-2.00	678	0-80	1-30	1-7.5
8.2	11.5-17.0	2.00-5.00	695	0-100	1-30	
8.3	11.5-17.0	5.00-9.00	0			
8.4	11.5-17.0	>9.00	0			
9.0	17.0-25.0	<1.10	425	0-80	1-15	1-7.5
9.1	17.0-25.0	1.10-2.00	330	0-80	1-30	1-3
9.2	17.0-25.0	2.00-3.50	251	0-100	1-7.5	
9.3	17.0-25.0	>3.50	143	0-100	1-3	
10.0	>25.0	<1.10	566	0-80	1-7.5	1-3
10.1	>25.0	1.10-2.00	277	0-80	1-15	1
10.2	>25.0	>2.00	148	0-100	1-7.5	

We have succeeded in simulation of almost all classes of the scattering phase functions observed in the experiments except classes 6.2, 7.3, 8.3, and 8.4. The lack of these classes is explainable, because such scattering phase functions (sharp with maximum, according to Refs. 2 and 3) were observed under the haze and fog conditions, i.e., in the presence of water droplets in air producing a strong rainbow maximum. The scattering phase functions of droplets and other giant particles cannot be simulated within the scope of our model.

Analysis of the dependences of the model class characteristics on the meteorological visibility range and humidity shows their good agreement with measurement data. The dependence on the humidity of air is weak, the dependence on the meteorological visibility range is statistical rather than deterministic,

i.e., each class of the scattering phase functions encompasses a wide range of variation of the meteorological visibility values. According to Refs. 2 and 3, the scattering phase functions of subclass 0 (smooth) are characteristic of continental air masses, whereas subclass 1 (sharp)—of the marine ones. The model classification reflects this dependence on the type of air mass: the range of the air humidity variations for subclass 0 is narrower than for subclass 1.

The model and experimental ranges of variations of the meteorological visibility values (see Table I) are consistent for the majority of classes. Taking into account a coarse discrete grid of meteorological visibility ranges used for the simulation, we can consider this agreement as good. For a finer grid of meteorological visibility ranges, more detailed dependence of the number density of particles on the meteorological visibility range,

more complicated model of the dependence of the parameters on the air humidity, etc. we can obtain better agreement between the characteristics of the model and experimental classifications. However, by the aforementioned reasons, such an adjustment was not made. The main proof of the adequacy of the model to the real atmospheric situations is the fact that we have obtained almost all classes of the scattering phase functions and that the physical dependences of the class characteristics on the model parameters are in agreement with experimentally observed.

Thus, the model proposed here can be used for obtaining the aerosol scattering phase function at the specified wavelength from *a priori* measured, calculated, or reconstructed parameters of the aerosol model of the atmosphere, i.e., from the known values of the

complex refractive index of aerosol matter, number density of aerosol particles, and volume aerosol scattering coefficient.

#### REFERENCES

1. A.V. Vasil'ev and L.S. Ivlev, *Atmos. Oceanic Opt.* **8**, No. 6, 479–483 (1995).
2. O.D. Barteneva, E.N. Dovgyallo, and E.A. Polyakova, *Trudy GGO*, No. 220 (1967), 244 pp.
3. O.D. Barteneva, A.G. Laktionov, V.N. Adnashkin, and L.I. Veselova, in: *Problems of Atmospheric Physics*, No. 15 (Leningrad State University Publishing House, Leningrad, 1988), pp. 27–43.
4. O.B. Vasil'ev and A.V. Vasil'ev, *Atmos. Oceanic Opt.* **7**, No. 1, 40–49 (1994).