RECONSTRUCTION OF THE TRUE HORIZON BRIGHTNESS FROM SMOOTHED REMOTE SENSING DATA

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We propose a parametrically optimal filtering method for the reconstruction of the true intensity of the horizon from smoothed remote sensing data. The results of the reconstruction modeling, and of reconstruction of the true horizon brightness from experimental data are presented.

The finite spatial resolution of a telescopic radiometer causes smoothing of the horizon brightness u(z) when it is scanned from space (z is the height of the sight line above sea level), and as a result, the device records a smoothed brightness

$$v(z)=\int_{-\infty}^{\infty} k(z-z')u(z')dz'. \tag{1}$$

Here $k(z)$ is the radiometer sesitivity function over the field of view. Because the distance (≈ 2000 km) from a spaceborne observer to the horizon is so large, smoothing can be so strong that actual brightness u must be reconstructed from the smoothed signal v by solving equation (1). This problem is ill-posed, and can be solved only approximately using regularization techniques. The literature contains regularization methods by Tikhonov[1–3] and by Turchin[4–6].

In this paper, a parametrically optimal filtering method[7] is proposed for solving the problem (1). This method can be considered a variant of the Wiener optimal linear filter[8].

Denote the Fourier transforms of the functions $u(z)$, $v(z)$ and $k(z)$ by $\tilde{u}(\omega)$, $\tilde{v}(\omega)$ and $\tilde{k}(\omega)$. In the parametrically optimal filtering method, we seek a regularized solution whose spectral density $\tilde{u}_\tau(\omega)$ takes the form

$$\tilde{u}_\tau(\omega)=r(\omega;\tau)v(\omega), \tag{2}$$

where $r(\omega, \tau)$ is a known function of the wavenumber ω and parameters $\tau = \{\tau_1, \ldots, \tau_N\}$.The values of these parameters are found by minimizing the functional

$$q(\tau)=E\int_{-\infty}^{\infty}\left|\tilde{u}_\tau(\omega)-\tilde{u}(\omega)\right|^2 d\omega, \tag{3}$$

where E denotes the expectation value, and $\tilde{u}(\omega)$ is the spectral density of the actual (but unknown) brightness. It is assumed that possible values of $\tilde{u}(\omega)$, and likewise $\tilde{v}(\omega)$, form a statistical ensemble. As shown in Ref. 7, by varying q with respect to τ_{jj} one obtains N equations in the parameters τ_j, with coefficients that depend on the variances of the spectral densities of the actual measured signal $\tilde{k}(\omega)\tilde{u}(\omega)$ and the measurement error

$$\tilde{\delta}(\omega)=\tilde{k}(\omega)\tilde{u}(\omega)-\tilde{v}(\omega). \tag{4}$$

The specific form of these equations depends on the function $r(\omega, \tau)$ chosen.

The main difference between existing regularization techniques and the one suggested here is that in the latter, the solution we seek must be most close to the actual solution in the least squares sense, while the former techniques search for the smoothest solution. Besides greater freedom in choosing the statistical ensemble, the technique suggested in this paper has the further advantage of a definite latitude in selection of the reconstruction operator $r(\omega, \tau)$. For example, in Ref. 7 one can find the functions

$$r(\omega,\tau)=[1+\mu(\omega,\tau)]\bar{\tilde{k}}(\omega) \tag{5a}$$

$$r(\omega,\tau)=1+\mu(\omega,\tau)\bar{\tilde{k}}(\omega) \tag{5b}$$

$$r(\omega,\tau)=\bar{\tilde{k}}(\omega)/[\,|\tilde{k}(\omega)|^2+\mu(\omega,\tau)] \tag{5c}$$

where

$$\mu(\omega,\tau)=\sum_{j=1}^{N}\tau_j\omega^{2j}, \tag{6}$$

and the bar over $\tilde{k}$ denotes the complex conjugate.

Expression (5c) includes, as a special case, Tikhonov and Turchin's operators.

Numerical simulations have shown that at high noise levels (10^{-3} and higher) relative to the maximum signal, Eqs. (5a) and (5b) provide for better reconstruction, while at low noise level, Eq. (Sc) is to be preferred. Examples of reconstruction using the operator (5c) (N = 1) are presented below. It is assumed that the noise and useful signal in the measured profile can be discriminated in the spectral domain:

$$\left|\tilde{k}(\omega)\tilde{u}(\omega)\right| \gg \left|\tilde{\delta}(\omega)\right| \ \text{ for } |\omega| < \Omega,$$

$$\left|\tilde{k}(\omega)\tilde{u}(\omega)\right| \ll \left|\tilde{\delta}(\omega)\right| \ \text{ for } |\omega| > \Omega,$$

where Ω is the discrimination wave number, and $\tilde{\delta}(\omega)$ is white noise:

$$E\tilde{\delta}(\omega) = 0, \ E|\tilde{\delta}(\omega)|^2 = \varepsilon^2 = \text{const}.$$

In that case, one obtains the representation

$$r(\omega,\tau) = \tilde{k}(\omega)/[\,|\tilde{k}(\omega)|^2 + \tau\omega^2],$$

and the parameter τ is determined by the equation

$$\tau \int_0^{\Omega} \frac{\omega^4 |\tilde{v}(\omega)|^2 d\omega}{(|\tilde{k}(\omega)|^2 + \tau\omega^2)^3} = \varepsilon^2 \int_0^{\omega} \frac{\omega^2 |\tilde{k}(\omega)|^2 d\omega}{(|\tilde{k}(\omega)|^2 + \tau\omega^2)^3}$$

Here, the function $\tilde{v}(\omega)$ as well as the parameters ε and Ω are determined on the basis of a single measured profile $v(z)$.

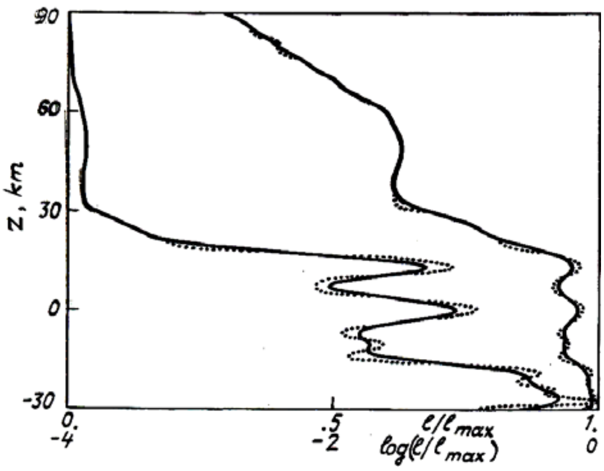

Fig. 1.

Figure 1 presents the results of numerical simulations of the smoothing process and subsequent reconstruction of the smoothed signal. The simulations have been carried out in accordance with the scheme

$$\left.\begin{array}{l} u(z) - \tilde{u}(\omega) \\ \delta(z) \to \tilde{\delta}(\omega) \end{array}\right\} - \tilde{v}(\omega) = \tilde{u}(\omega)\tilde{k}(\omega) + \tilde{\delta}(\omega) - \tilde{u}_{\tau}(\omega) - u_{\tau}(z).$$

The arrays $u(z_1)$, $\delta(z_1)$ have been defined on the lattice $z_1 = i\Delta z$, $\Delta z = 0.5$ km. Random errors $\delta(z_1)$ were modeled by a stationary uncorrelated process, with variances

$$E\delta^2(z_1) = e^2 = \text{const}.$$

It should be noted that e is not the same as the quantity ε introduced earlier. Parameter ε depends not only on e, but also on the Fourier transform algorithm used. In our calculations we used the fast Fourier cosine transform. The kernel $k(z)$ of the integral operator was modeled by a Gaussian function,

$$k(z) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left[-\frac{z^2}{2\sigma^2}\right], \quad \tilde{k}(\omega) = \exp\left[-\frac{\sigma^2\omega^2}{2}\right]. \tag{7}$$

Figure 1 illustrates reconstruction using $\sigma = 4$ km and $e/u_{max} = 3 \cdot 10^{-4}$. It can be seen from this figure that all large scale details of the profile $u(z)$ are restored, but high frequency details are lost. The high dynamic range of the reconstruction is noteworthy. It is seen, for example, that the quality of reconstruction is the same in the altitude range from 60 to 90 km as in the lower atmosphere, despite the fact that the signal is a factor of 10^3 lower than the maximum.

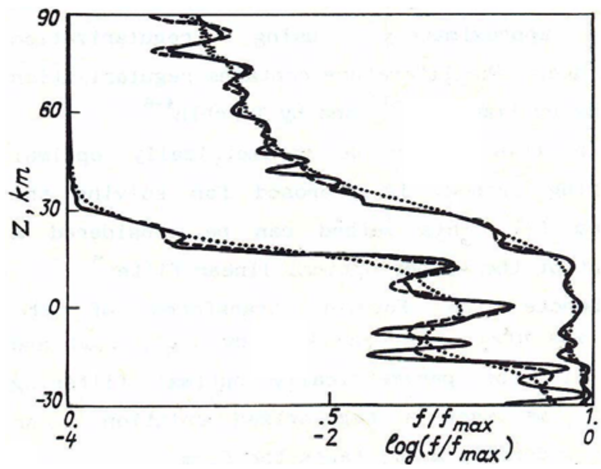

Fig. 2.

Figure 2 shows a reconstruction of the brightness of the daytime horizon at $\lambda = 1.25$ μm within the molecular oxygen emission band. The measured and smoothed profile was obtained with the FAZA teleradiometer[10]. The sensitivity of the device over its field of view is described by Eq. (7) with $\sigma = 1.6$ km. As is seen from this figure, the effect of reconstruction are more significant in the lower atmosphere, where the brightness undergoes strong variations. In the restored profile, one can also see a weak secondary maximum of oxygen emission at 80 km altitude, which is missing from the measured profile, the main maximum being at 40–60 km.

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