TEST ESTIMATIONS IN CALCULATIONS OF THE OPTICAL FIELD PARTIAL WAVES AMPLITUDES INSIDE THE SPHERICAL PARTICLES USING MIE THEORY

N.N. Belov, M.V. Zhuravlev, and E.E. P'yankov

L.Ya. Karpov Scientific-Research Physical Chemical Institute, Moscow

Received May 29, 1995

Asymptotic representations of the amplitudes of the partial waves in the Mie series are obtained without any special functions. These representations are useful for description of the optical fields inside spherical aerosol particles and in the plasma spherical inhomogeneities. One can use the relationships obtained in the medium range of the diffraction parameter values, namely, for the close values of the product of the diffraction parameter by the complex refractive index and of the number of a partial wave. The comparative analysis is carried out of the amplitudes calculated by the asymptotic and Mie formulas. The asymptotic formulas obtained are most convenient for testing algorithms for calculating the amplitudes constructed using the exact Mie formulas.

Calculations of the optical fields using the Mie theory make it possible to reveal the mechanisms of clearing up water aerosol by a laser beam and possible ways of development of the optical breakdown in the aerosol medium and to study the optical field intensities inside the plasma inhomogeneities. A number of difficulties connected with the problem of calculating the Rikkati-Bessel functions of the first order (RBF1) with a complex argument in the intermediate range of the diffraction parameter, namely, for $n \approx |m\rho|$, occur in the techniques for calculating the coefficients of the Mie series.[1] The standard techniques for calculating the spherical RBF1 with the complex argument, such as descending and ascending recursion, require either a large computer memory and elaborate normalizing or an extended many-digit computer word, for example, to correctly calculate the RBF1 with the imaginary part greater than 30, a 16-byte representation of complex numbers is required.[2] However, it is possible to calculate the RBF1 for calculating the partial wave amplitudes in the range $n \approx |m\rho|$ using less cumbersome procedure based on the asymptotic expressions.[3] The necessity and fruitfulness of such an approach are noted in Ref. 1.

In this paper the asymptotic representations of the partial wave amplitudes of the Mie series describing the optical fields inside the spherical particles are obtained using the asymptotic representations of the RBF1.[3] The expressions obtained make it possible to effectively test the process of calculation of the amplitudes by the exact Mie formulas and to test other algorithms (see, e.g., Refs. 2, 4, and 5) in the critical calculation range $n \approx |m\rho|$.

Writing the expressions for coefficients in the form convenient for calculations, we have[6]:

$$d_n = i^n\, m\, \frac{(2n+1)}{n\,(n+1)} \times$$

$$\times \left[m\, \xi_n(\rho)\, \psi_{n-1}(m\rho) - \left(\frac{n}{\rho}\xi_n(\rho) + \xi_n'(\rho)\right)\psi_n(m\rho)\right]^{-1}, \quad (1)$$

$$c_n = i^n\, m\, \frac{(2n+1)}{n\,(n+1)} \times$$

$$\times \left[\xi_n(\rho)\, \psi_{n-1}(m\rho) - \left(\frac{n}{m\rho}\xi_n(\rho) + \xi_n'(\rho)\right)\psi_n(m\rho)\right]^{-1}, \quad (2)$$

where m is the complex refractive index of the particulate matter, ρ is the diffraction parameter of a particle, $\xi_n(\rho)$ and $\xi_n'(\rho)$ are the Rikkati-Bessel function of the third kind and its derivative, $\psi_n(m\rho)$ is the Rikkati-Bessel function of the first kind,[7] and i is the imaginary unit. To calculate the RBF1, let us use the asymptotic Meissel series[3] describing the RBF1 for $n \approx |m\rho|$: $(m\rho - n + 1/2) = o(m\rho)^{1/3}$, $-\pi < \arg(m\rho) < \pi$. Taking into account two first terms of the series, we have for RBF1:

$$\psi_n(m\rho) = 0.56\,(m\rho)^{1/6} + 0.52\,(m\rho)^{1/6} -$$

$$-\,0.52\,(n + 1/2)\,(m\rho)^{-1/6}. \quad (3)$$

The use of asymptotic Debye formulas[7] for calculating RBF3 and its derivative in Eqs. (1) and (2) makes it possible to write them without special functions, and for $|m|<1$ we obtain:

$$d_n = i^n\, m\, \frac{(2n+1)}{n\,(n+1)}\, \xi_n^{-1}(\rho)\, [(m\, \psi_{n-1}(m\rho) -$$

$$- ((n/\rho) + \sin(\tau))\, \psi_n(m\rho)]^{-1}; \quad (4)$$

$$c_n = i^n\, m\, \frac{(2n+1)}{n\,(n+1)}\, \xi_n^{-1}(\rho)\, [(\psi_{n-1}(m\rho) -$$

$$- ((n/m\rho) + \sin(\tau))\, \psi_n(m\rho)]^{-1}, \quad (5)$$

where τ is the variable calculated from the expression $\cos(\tau) = (n + 1/2)/\rho$ and $\xi_n(\rho)$ has the form[7]:

$$\xi_n(\rho) = \exp\,(-\,i\,\rho\, f + \pi/4)/\sqrt{\sin(\tau)}\ , \quad (6)$$

where

$$f = \sin(\tau) - \tau \cos(\tau).$$

Figure 1 shows relative deviation of the partial wave amplitude calculated by Eqs. (1) and (2) (curve *1* for d_n and curve *2* for c_n) taking into account Eq. (3) from the exact value of the partial wave amplitude calculated by the recurrence formulas.[2] Calculations have been done for water aerosol, namely, for the refractive index of water at the wavelength of 1.06 μm $m = 1.39 - i\,1.49\,10^{-6}$.

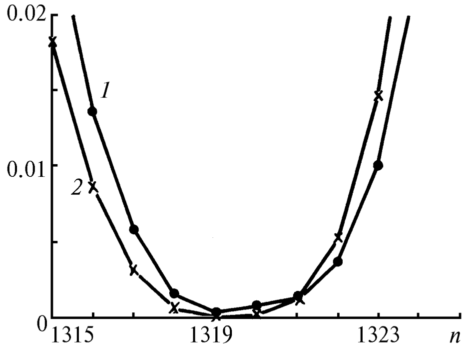

FIG 1.

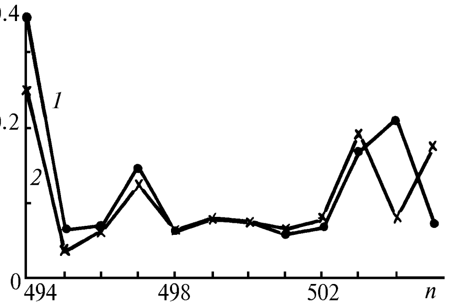

FIG. 2.

Figure 2 shows the deviation of the partial wave amplitude calculated by Eqs. (4) and (5) (curve *1* for d_n and curve *2* for c_n) taking into account Eq. (3) from the exact value of the partial wave amplitude calculated by the recurrence formulas of the descending recursion.[8] Calculations have been done for the plasma inhomogeneities with the refractive index $m = 0.5 - i\,10^{-3}$.

Thus, the relative deviation of the partial wave amplitudes from the values calculated by the recurrence formulas does not exceed 0.02 (Fig. 1) and 0.2 (Fig. 2) in the index range from 1315 to 1323 and from 484 to 504, respectively. Hence, it is possible to use the formulas obtained for testing all possible algorithms for calculating the partial wave amplitudes. To decrease the relative deviation and to extend the index range, it is necessary to take into account three and more terms in the Meissel asymptotic expansion when calculating RBF1.

ACKNOWLEDGMENTS

Authors thank the scientists of the L.Ya. Karpov Scientific-Research Physical Chemical Institute and the Scientific-Production Association "Aerosol Technology" for computers placed at our disposal for calculations.

The research presented in this paper has been done under the financial support of the International Science Foundation (Grant No. M2U000).

REFERENCES

1. D. Deirmenjian, *Electromagnetic Scattering on Spherical Polydispersions* (American Elsevier Publishing Company, Inc., New York, 1969).
2. N.N. Belov, Atmos. Oceanic Opt. **5**, No. 2, 107–109 (1992).
3. G.N. Watson. *Theory of Bessel Functions* [Russian translation] (Foreign Literature Press, Moscow, 1949), Part 1, 492 pp.
4. N.N. Belov, Atm. Opt. **4**, No. 3, 256–257 (1991).
5. A.A. Akulinin, Opt. Atm. **1**, No. 6, 112 (1988).
6. M. Kerker and D. Cooke, Appl. Opt. **12**, No. 7, 1378–1379 (1973).
7. K.S. Shifrin, *Light Scattering in the Turbid Medium* (Moscow-Leningrad, 1951),140 pp.
8. C.F. Bohren and D.R. Huffman, *Absorption and Scattering of Light by Small Particles* (Willey Inc., New York, 1983).
9. G. Grehau and G. Gousbet, Appl. Opt. **18**, No. 20, 3489–3495 (1979).