3-D VIEW OF THE ANGULAR DEPENDENCE OF THE POLARIZED RADIATION SCATTERING IN PLANE-PARALLEL MEDIA

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This paper presents model investigations into the scattering of polarized radiation in anisotropic plane-parallel media. A polydisperse mixture of ice cylinders oriented mainly horizontally has been selected as a model. It has been supposed that the particles are randomly oriented in a horizontal plane. We have considered different angles of radiation incidence on a medium. The particles have been supposed to be free of admixtures and possessing homogeneous internal structure. The calculations have shown that the anisotropy of scattering media significantly affect the angular behavior of the scattering phase matrix elements with respect to both polar (θ) and azimuthal (φ) angles.

1. INTRODUCTION

The anisotropy of a medium can significantly affect the polarization of radiation propagated through it. Polarization measurements are usually much more complicated compared to measurements of intensity, but they bear more information about the microstructure of the media sounded. A lot of calculational data are accumulated to date, on the scattering of polarized radiation by ensembles of randomly oriented nonspherical particles, such as spheroids,[1] Chebyshev particles,[2] hexagonal crystals,[3,4,5] and elongated cylinders.[6,7]

The purpose of this paper is to extend the calculations[6,7] of the polarization properties of light scattering to the case of anisotropic media. The account for preferred orientation in the ensemble of nonspherical particles simulated leads to an increase in computer time necessary for calculating polarization properties of light scattering by 2 to 3 orders of magnitude as compared to the case of random orientation what makes a variety of model calculations impracticable. The bulk of output information increases approximately to the same degree. In this paper we take an ensemble of elongated cylinders randomly oriented in a fixed plane as a model in calculations. Attention to such a model is based on the results of lidar measurements which show that crystal particles take a preferred orientation with respect to the horizontal plane, owing to great difference between their big and small size.

2. CALCULATIONAL TECHNIQUE

The approximate solution to the problem on scattering of electromagnetic waves by homogeneous elongated cylinders of finite length has been given in Ref. 8. For an individual cylinder, the scattered field is written in the coordinate system (CS) $X'Y'Z'$ related to its axis of symmetry, that coincides with Z'-axis, and the wave vector of the incident field $\mathbf{k}^{(i)}$ lies in the $X'Z'$ plane at an angle β with respect to Z'-axis. The wave vector of scattered field $\mathbf{k}^{(s)}$ has an arbitrary direction set by the angles θ′ and φ′ in the $X'Y'Z'$ coordinate system. To calculate the field scattered by an ensemble of oriented cylindrical particles, it is convenient to introduce a different coordinate system, XYZ, whose Z-axis coincides with $\mathbf{k}^{(i)}$. The choice of X-axis is determined by the geometry of interaction of the incident radiation with the scattering medium. Orientation of the cylinder in this CS, i.e., the direction of its axis of symmetry is set by two Euler angles (α, β). The relation between two CSs is shown in Fig. 1. In the CS $X'Y'Z'$ related to the body, the scattered field at a distance R from the cylinder (in the far zone) is related to the incident field by the following formula:

$$\begin{pmatrix} E^{s}_{l} \\ E^{s}_{r} \end{pmatrix}_{SOA} = \frac{\exp[i(kr - \mathbf{k}^{(i)}\mathbf{z}')]}{ikR} \begin{pmatrix} A_2 & A_3 \\ A_1 & A_4 \end{pmatrix} \begin{pmatrix} E^{i}_{l} \\ E^{i}_{r} \end{pmatrix}_{IOA}, \quad (1)$$

where E^{s}_{l} and E^{s}_{r} are the parallel and perpendicular components of the scattered field in the scattering plane SOA, respectively, and, analogously, E^{i}_{l} and E^{i}_{r} are the components of the incident field in the IOA plane. The unit vectors $\mathbf{e}^{s}_{l}$ and $\mathbf{e}^{s}_{r}$ are the parallel and perpendicular vectors to the scattering plane. These vectors are selected so that $\mathbf{e}^{s}_{r}\times\mathbf{e}^{s}_{l}$ coincides with the direction of propagation. The amplitude functions A_i (i = 1, 2, 3, 4) are expressed in terms of the amplitude functions of the infinite cylinder T_i as follows[8]:

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$A_i(\theta', \varphi', \beta, a, l) =$

$$= (kl/\pi)E[kl(\cos\theta' - \cos\beta)/2]T_i(\varphi', \beta, a), \qquad (2)$$

where a and l are the radius and length of the cylinder, $k = 2\pi/\lambda$ is the wave number; λ is the wavelength of incident radiation; $E(x) = \sin(x)/x$. The process of scattering is also described as a linear transform of the Stokes parameters $[I^i, Q^i, U^i, V^i]$ of the incident field into the Stokes parameters of the scattered field with the transform matrix $\mathbf{F}$

$$\begin{bmatrix} I^s \\ Q^s \\ U^s \\ V^s \end{bmatrix}_{SOA} = \frac{1}{k^2R^2} = \mathbf{F}(\theta', \varphi', \beta, a, l) \begin{bmatrix} I^i \\ Q^i \\ U^i \\ V^i \end{bmatrix}_{IOA}, \qquad (3)$$

where the matrix $\mathbf{F}$ consists of 16 elements, each of them being real and expressed in terms of the squared amplitude functions A_1, A_2, A_3, A_4 (Ref. 9).

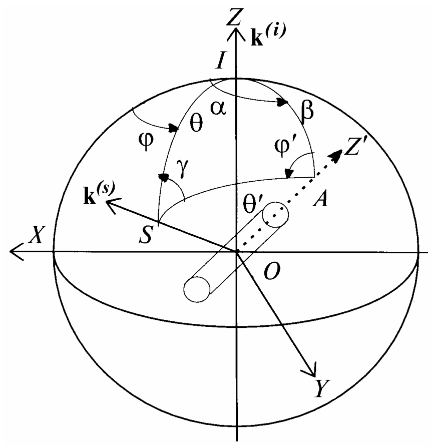

FIG. 1. Geometry of scattering by an arbitrarily oriented cylinder.

Formulas (1)–(3) describe the process of scattering by a separate cylinder. When passing to the ensemble of arbitrarily oriented cylinders, it is necessary to describe the process of scattering relative to a plane independent of each cylinder orientation. The plane containing the wave vectors of the scattered and incident fields (*SOI* in Fig. 1) is usually selected as such a plane. The Stokes parameters of the field scattered in the $S(\theta, \varphi)$ direction in the scattering plane *SOI* (in CS related to the incident field) can be obtained in the following linear process:

a) transform of the Stokes parameters of the incidence field at the turn of the incidence plane from *SOI* to *IOA*;

b) solving the scattering problem in CS related to the body, i.e. solving Eq. (1);

c) transform of the Stokes parameters of the scattered field at the turn of the scattering plane from *SOA* to *SOI*.

Mathematically this is expressed as follows

$$\begin{bmatrix} I^s \\ Q^s \\ U^s \\ V^s \end{bmatrix}_{SOI} = \frac{1}{k^2R^2} = \mathbf{Z} \begin{bmatrix} I^i \\ Q^i \\ U^i \\ V^i \end{bmatrix}_{SOI}, \qquad (4)$$

$$\mathbf{Z}(\theta, \varphi, \beta, a, l) = \mathbf{L}(-\gamma)\mathbf{F}(\theta', \varphi', \beta, a, l)\mathbf{L}(\varphi - \alpha), \quad (5)$$

where $\mathbf{L}(-\delta)$ is the matrix of the Stokes parameters transform at a turn of the scattering plane by an angle δ clockwise, if looking along the wave propagation direction

$$\mathbf{L}(-\delta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\delta & -\sin 2\delta & 0 \\ 0 & \sin 2\delta & \cos 2\delta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \qquad (6)$$

The angles θ', φ' and γ in Eqs. (2) and (5) at an arbitrary orientation (α, β) of a cylinder can be expressed using the angles θ, $\alpha - \varphi$, β

$$\cos\theta' = \cos\theta \cos\beta + \sin\theta \sin\beta \cos(\alpha - \varphi),$$

$$\cos\varphi' = [\cos\theta \sin\beta - \sin\theta \cos\beta \cos(\alpha - \varphi)]/\pm\sin\theta',$$

$$\cos\gamma = [\cos\beta \sin\theta - \sin\beta \cos\theta \cos(\alpha - \varphi)]/\pm\sin\theta',$$

where the "plus" sign denotes the case, when $0 < \alpha - \varphi < \pi$, and the "minus" sign denotes the case when $\pi < \alpha - \varphi < 2\pi$.

The aforementioned process is different than the analogous one for the arbitrarily oriented spheroids[1] in the choice of the X-axis (it is determined here by the geometry of interaction of the incident radiation with the medium) as well as in the third factor in the right-hand side of Eq. (5).

The average normalized scattering phase matrix (SPM) $\mathbf{P}(\theta, \varphi)$ of a polydisperse ensemble of cylinders, orientation of the axes of which (with respect to the incident radiation direction) is set by the weighting function $g(\alpha, \beta)$, is obtained by integrating SPM over all orientations and size

$$\mathbf{P}(\theta, \varphi) = \frac{4\pi}{k^2C_s} \int\limits_{l_1}^{l_2} \mathrm{d}l \int\limits_{a_1}^{a_2} \mathrm{d}a \int\limits_{0}^{\pi} \mathrm{d}\beta \int\limits_{0}^{2\pi} \mathbf{Z}(\theta, \varphi, \beta, a, l) \times$$

$$\times\, g(\alpha, \beta)n(a, l)\mathrm{d}\alpha, \qquad (7)$$

where $n(a, l)$ is the cylinder size distribution density, C_s is the ensemble average scattering cross section, and k is the wave number. It should be emphasized that $g(\alpha, \beta)$ and C_s for anisotropic media depend on the geometry of interaction of the incident radiation with the medium. The factor before the integrals is selected according to the condition of normalizing the scattering phase function $P_{11}(\theta, \varphi)$

$$\int\limits_0^{2\pi} \mathrm{d}\varphi \int\limits_0^{\pi} P_{11}(\theta, \varphi)\sin\theta \mathrm{d}\theta = 4\pi. \qquad (8)$$

This paper is aimed at revealing regular features in the spatial angular variation of the elements of the scattering phase matrices of the media with a selected plane of symmetry, depending on geometry of interaction of the incident radiation with such media. Cylinders are selected as a model of particles, uniformly oriented in the plane. When solving the problem on light scattering, one usually uses the coordinate system related to the incident radiation direction. The scattering geometry in such media is shown in Fig. 2. Here XZ is the incidence plane, the directions of incidence and Z-axis coincide. Cylinders are uniformly oriented in the FOA plane, where OA is the axis of an arbitrary cylinder. The angles are: $\angle IFA = \pi/2$; $\beta_0 = \angle FOI$ is the angle of incidence of the radiation on the plane; $\alpha_0 = \angle FOA$ – is the angle of orientation of the cylinder axes in the FOA plane. The angles α and β set the cylinder orientation in the XYZ coordinate system. Based on spherical trigonometry, one can obtain the relationship between these angles

$$\cos\beta = \cos\beta_0 \cos\alpha_0 , \quad \sin\alpha = \sin\alpha_0/\sin\beta. \qquad (9)$$

Since the plane and the cylinder have the mirror symmetry, it is sufficient to consider only the following ranges of variation of the angles $0 < \beta_0 \le \pi/2$, $0 \le \varphi \le \pi$, with the α_0 varying uniformly in the range from 0 to π.

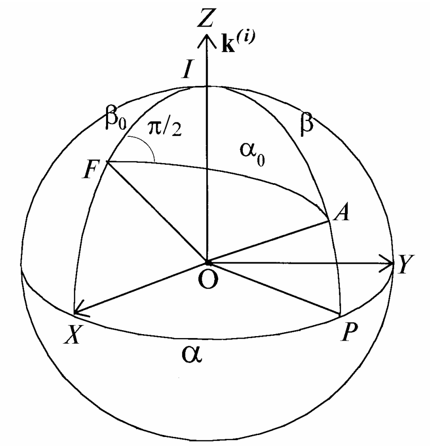

FIG. 2. Geometry of interaction of incident radiation with plane-parallel medium.

3. RESULTS OF CALCULATIONS

Five cases of the incident radiation were considered: β_0 = 90, 85, 81, 75, and 60°, as well as the ensembles of randomly oriented cylinders of similar disperse composition. Elements of the scattering phase matrix were calculated using the Monte-Carlo method. The distribution of particles over cross section radius a was simulated by lognormal with the mean geometrical radius from the resonance range of scattering (where the elements of the scattering phase matrix vary most strongly depending on the particle size) a_m = 0.5 μm and a_m = 1.0 μm, and the rms error σ_a = 0.5. The cylinder length was assumed to be uniformly distributed in the range from $8a$ to $10a$. Convergence of the integral was controlled by the results of numerical estimation for the case of the incident radiation β_0 = 90°, because in this case no dependence of the SPM elements on the angle φ occurs.

All calculations were carried out for the wavelength of incident radiation λ = 1.06 μm and the ice refractive index $n = 1.299 - i2{\cdot}10^{-4}$. Estimates were obtained for all non-zero elements of the SPM, however, due to the limited size of paper, Figures 3–5 illustrate only the calculational data on the most significant elements in the case of the cylinder ensembles with a_m = 0.5 μm. Note, that in the case of the slant incidence of radiation on the plane-parallel layer, the SPM has a symmetrical shape and consists of ten non-zero elements, as well as the fact that all the below conclusions, concerning the ensembles with a_m = 0.5 μm, also apply to the ensembles with a_m = 1.0 μm.

Let us present some peculiarities of the spatial angular behavior of the SPM elements shown in Figs. 3–5:

a) $P_{11}(\theta, \varphi)$ is the scattering phase function of unpolarized incident radiation (see Fig. 3).

The parameters β_0, $P_{11}(0, \varphi)$ and $P_{11}(\pi, \varphi)$ are constant with respect to φ at all incident angles, in spite of the fact that at all angles θ close to π they essentially depend on φ, what is natural for scattering of unpolarized light. Although the medium is discrete, the Snellius law well manifests itself, and the local maximum (the portion of radiation reflected from the plane) takes the same values at all β_0. The range of low values $P_{11}(\theta, \varphi)$ at β_0 = 90° lies in the θ range of 110–130°. When β_0 = 85°, it extends over θ and concentrates near φ equal to 90°. It gradually broadens both on θ and φ, as β_0 decreases, and becomes deeper (the values $P_{11}(\theta, \varphi)$ fall), as well as shifts to the range of larger φ. Physically this means that the portion of radiation scattered to the semi-space of the reflected beam, increases, as β_0 decreases.

b) $p(\theta, \varphi) = -P_{12}/P_{11}$ is the polarization degree of scattered radiation for the unpolarized incident radiation (Fig. 4).

In the case of random orientation of particles $p(\theta, \varphi)$ is very close to zero. If β_0 = 90°, the value p is constant on φ and has a maximum near the angle θ = 130°. If β_0 = 85°, p mainly varies on φ at the angles θ close to π, two ranges of high p values appear near the (θ, φ) points $(\pi, 0)$ and (π, π), while low values p occur near the point $(\pi, \pi/2)$. As β_0 decreases, the size of these ranges increases, as well as the maximum and minimum of p in this ranges increase in absolute value. It can be interpreted for the angles θ close to π and β_0 = 75° as follows: unpolarized light slantly incident on the plane medium is linearly polarized mainly in the incident plane.

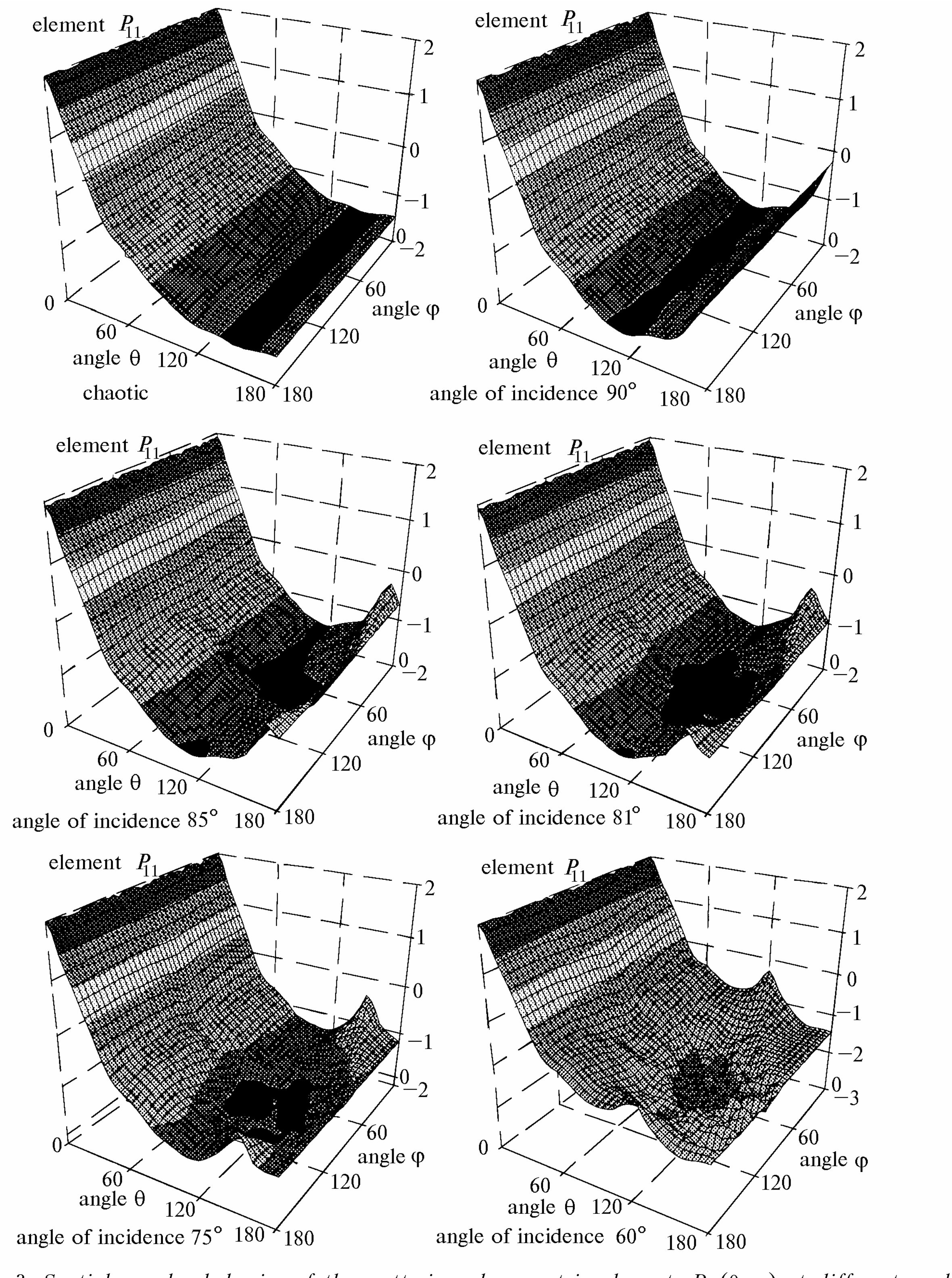

FIG. 3. Spatial angular behavior of the scattering phase matrix elements $P_{11}(\theta, \varphi)$ at different angles of radiation incidence on plane-parallel medium.

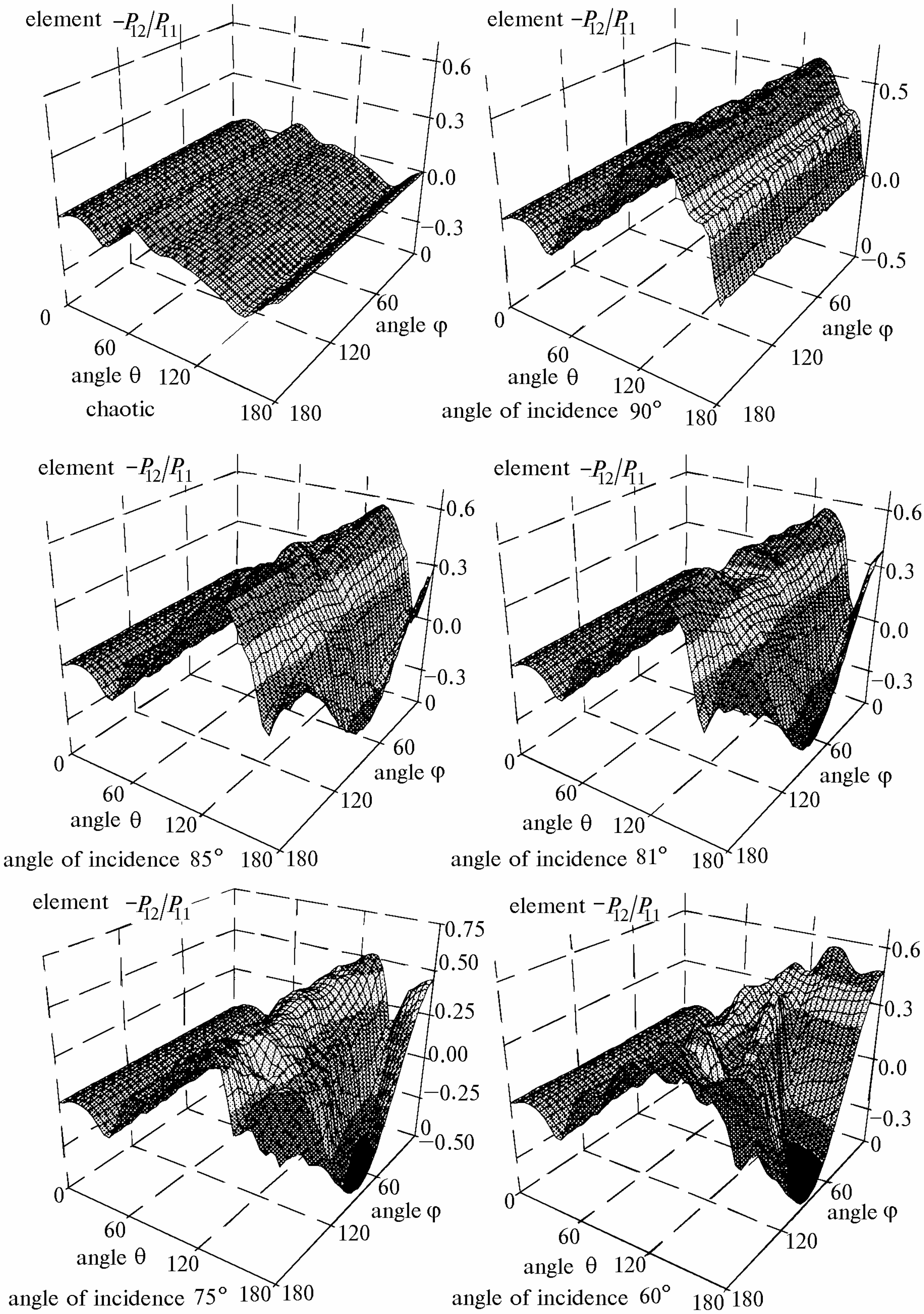

FIG. 4. The same as in Fig. 3, for $-P_{12}/P_{11}$.

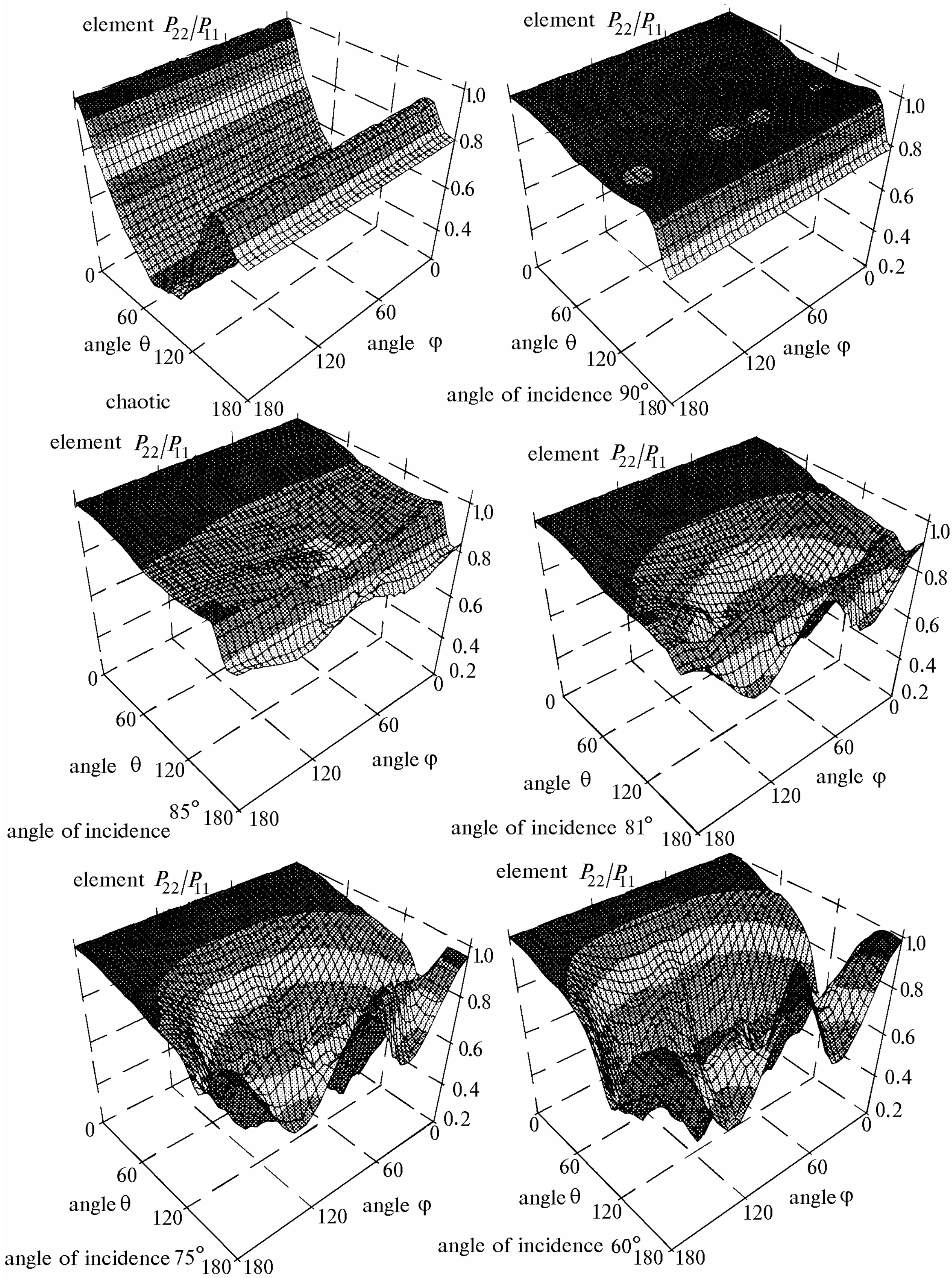

FIG. 5. The same as in Fig. 3, for P_{22}/P_{11}.

c) $d(\theta, \varphi) = P_{22}/P_{11}$ (Fig. 5). The value $(1 - d)$ is called the depolarization coefficient for the polarized incident radiation and it is the measure of particle nonsphericity. The element $d(\theta, \varphi)$ is most strongly varying with variations in β_0 among all the aforementioned elements. These variations occur in a wide range of the angles θ and φ. Let us note the most essential tendencies. At $\beta_0 = 85°$ one can observe the appearance of the range of low d values near the scattering direction $\theta = 120°$, $\varphi = 90°$ in comparison with the normal incidence ($\beta_0 = 90°$), as well as two regions of low d values near $\varphi = 45$ and 135° are observed at the angles θ close to π, and three regions of high values are observed near $\varphi = 0$, 90 and 180°. As β_0 decreases, the low values continue to decrease while high values increase, and these regions broaden.

Let us also note that in the case of backscattering the dependence of scattering phase matrix elements on φ has a harmonic behavior and $P_{ij}(\pi, \varphi) = P_{ij}(\pi, \pi - \varphi)$.

Thus, anisotropy of the scattering medium essentially affects the spatial and angular behavior of the light scattering and the polarization properties of the scattered radiation.

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