DISTRIBUTION FUNCTIONS OF THE INTENSITY FLUCTUATIONS OF A LASER BEAM IN SNOWFALL

N.A. Vostretsov, A.F. Zhukov, M.V. Kabanov, and R.Sh. Tsvyk

Institute of Atmospheric Optics,
Siberian Branch of the Russian Academy of Sciences, Tomsk
Received October 20, 1992

Distribution laws of the intensity fluctuations of a laser beam in snowfall are analyzed. It is found that for a narrow diverging laser beam propagating in snowfall without dense flakes the distribution function of the intensity fluctuations has one maximum and right–side asymmetry. In many cases the intensity fluctuations obey the gamma distribution.

INTRODUCTION

Problems associated with an assessment of potentials of laser systems intended to operate through the atmosphere and with a choice of efficient methods of stochastic signal processing motivate the urgency of the studies of distribution functions of the intensity fluctuations of laser beams propagating through the atmosphere.

At present the distribution laws of the intensity of a laser beam propagated through the turbulent atmosphere are most extensively studied. From Refs. 1 and 2 it follows that the distribution function of the weak intensity fluctuations is most commonly described by a lognormal function. There is no general solution of this problem for strong fluctuations and studies are still being continued.[1,2]

As it follows from Ref. 3, the random intensity is distributed by the lognormal law for distances $L \gg ka^2$ and by Poisson's law for $L \ll ka^2$, where $k = 2\pi/\lambda$, λ is the wavelength, and a is the particle radius.

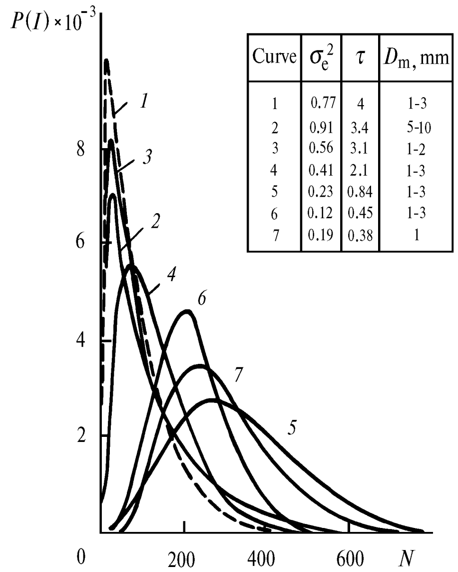
The determination of the law of the intensity distribution becomes difficult under atmospheric precipitation conditions because of concurrent fluctuations caused by scattering on both turbulent inhomogeneities and particles of precipitation. Therefore the theoretical studies in this direction are lacking completely, and experimental investigations are confined only to the results obtained in rain and snowfall in Refs. 4–8. The results of analysis of the distribution laws of the intensity fluctuations in snowfall are given in this paper.

EQUIPMENT

The scheme and measurement procedure were described in detail in our previous paper.[9] Laser radiation from an LG–38 laser ($\lambda = 0.6328$ μm) was received with a photomultiplier (FÉU) after its passage of 964–m path. A diaphragm 3 mm in diameter was placed in front of the photomultiplier. The angle of divergence of a laser beam was equal to $5\cdot10^{-4}$ rad. The signal from the FÉU was amplified and fed into an AI–1024 pulse analyzer as well as into a dispersiometer intended to measure the relative variance of the intensity fluctuations (σ_e^2). The dynamic range of the input signals of the AI–1024 was 0.2–10 V. Sampling time was 1 μs and sampling rate (f_s) was chosen with correlation time (t_c) taken into account from the condition $1/f_s \gg t_c$ to eliminate correlation of samples. The time required for the analysis was more than 20–30 s.

RESULTS OF MEASUREMENTS

We processed about 100 distributions. The typical instances of the distribution function of the intensity fluctuations in snowfall are shown in Fig. 1. One maximum, right–side asymmetry, and strengthening of asymmetry with an increase of the optical thickness τ of snowfall are characteristic of them.



Curve	σ_e^2	τ	D_m, mm
1	0.77	4	1-3
2	0.91	3.4	5-10
3	0.56	3.1	1-2
4	0.41	2.1	1-3
5	0.23	0.84	1-3
6	0.12	0.45	1-3
7	0.19	0.38	1

FIG. 1. The distribution function of the intensity fluctuations.

The analytic expression was obtained for empirical distributions by graphical representation of probabilities. To this end, the obtained experimental data were plotted on the specially lined paper designed for the selected distribution (i.e., the method of rectified diagrams was used).

Figure 2 shows an example of the test for confirmity to the lognormal distribution.

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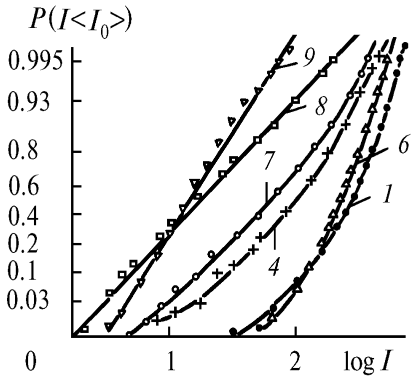

FIG. 2. *Example of the test for confirmity to the lognormal distribution law of the intensity fluctuations. For curves 1–7 the values of* σ_e^2, τ, *and* D_m *in* mm *are given in the table of Fig. 1; 8)* $\sigma_e^2 = 1.46$ *and* $\tau = 0.54$; *9)* $\sigma_e^2 = 0.98$ *and* $\tau = 0.53$ *under conditions of thick haze without snow.*

As can be seen, this model fits poorly the experimental distribution under precipitation conditions. However, in the thick haze at $\tau = 0.5$ the lognormal distribution fits fairly well the data over the entire range of signal variations, as could be expected. In our opinion, this fact confirms the validity of the employed measurement procedure and data processing. Attention is drawn to the fact that empirical distributions under precipitation conditions in the range of small $\log I$ lie much higher in these coordinates than it could be expected for the lognormal model. Such deviations were reported earlier in Ref. 5. This salient feature is characteristic for gamma distributions,[10] whose probability density for $I > 0$ is given by the formula

$$P_{\alpha,\beta}(I) = \frac{1}{\Gamma(\alpha + 1)\,\beta^{\alpha+1}}\, I^{\alpha} \exp\left(\frac{I}{\beta}\right),$$

where the parameters $\alpha > -1$ and $\beta > 0$ and $\Gamma(\alpha + 1)$ is the gamma function (being equal to $\alpha!$ for integer α).

Figures 3 and 4 show the instances of the test for confirmity to the gamma distribution. The scale of the ordinate axis was drawn by the method described in Ref. 10. Here we define more exactly that it is an analog of the well–known probabilistic coordinate grid for rectified normal (or lognormal) distribution. Analysis shows that about 90 empirical distributions from 100 are fitted better by the gamma distribution in comparison with the lognormal and Rice–Nacagami distributions. In this case, as can be seen from Figs. 3 and 4, the parameter α decreases with increase of the optical thickness. This naturally follows from the fact that smaller variances of the intensity logarithm (as well as the variances of the intensity)[10] correspond to the gamma distributions with larger power α as well as from the dependence of the variance of the intensity fluctuations σ_e^2 on the optical thickness.[9] Along with this, the background of information is still insufficient for the establishment of the dependence of the parameters α and β on weather conditions. It is necessary to perform measurements over a wider variety of weather conditions.

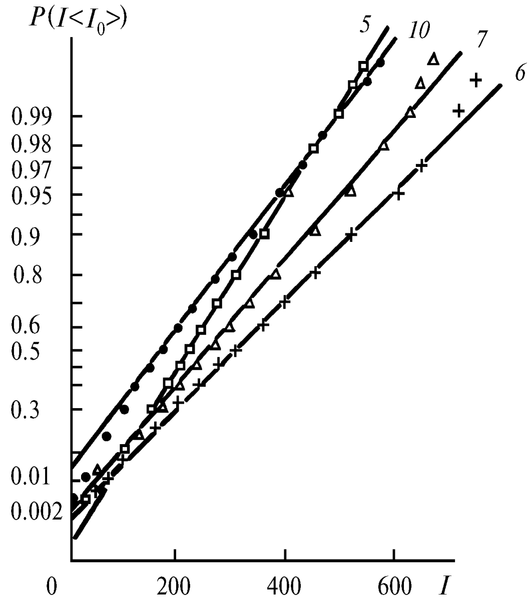

FIG. 3. *The test for confirmity to the gamma distribution for* $\alpha = 7$ *and* $D_m = (2 - 3)$ mm: *5)* $\tau = 0.84$ *and* $\sigma_e^2 = 0.23$; *6)* $\tau = 0.45$ *and* $\sigma_e^2 = 0.12$, *7)* $\tau = 0.38$ *and* $\sigma_e^2 = 0.19$, *and 10)* $\tau = 1.2$ *and* $\sigma_e^2 = 0.30$.

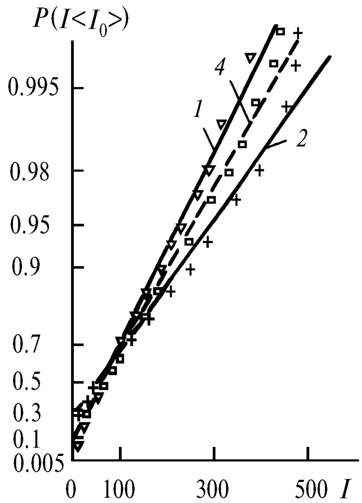

FIG. 4. *The test for confirmity to the gamma distribution for* $\alpha = 1$. *For curves 1, 2, and 4 the values of* σ_e^2, τ, *and* D_m *in* mm *are given in the table of Fig. 1.*

The signals were simultaneously recorded on an N–138 magnetograph with a narrow diverging laser beam on two paths 964 and 260 m long, as well as on 964–m path with a receiving lens 160 mm in diameter. The beam diameter in the receiving plane was 1 m. The lens was placed visually in the center of the beam and the smaller receiver was arranged next to a mount of the lens. Optical axes of the receivers were nearly parallel to each other and were spaced at a distance of 160 mm. The above–mentioned strengthening of the asymmetry with the increase of the optical thickness is clearly seen from Fig. 5, but it was found under the same atmospheric conditions. Moreover, the distribution acquires the left–side symmetry for D_r = 160 mm in contrast to the right–side symmetry for D_r = 0.3 mm.

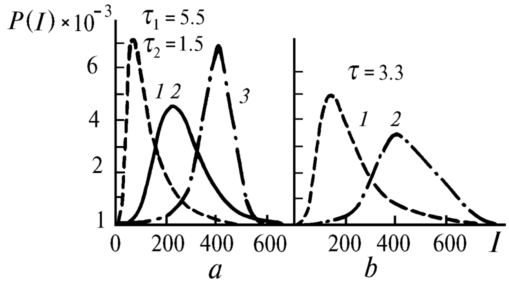

FIG. 5. The distribution function of the intensity fluctuations: a) 964 m, D_r = 0.3 *(1),* 160 mm *(3) and* 264 m, D_r = 0.3 mm *(2) and b)* 964 m, D_r = 0.3 mm *(1),* 160 mm *(2).*

ANALYSIS OF THE DATA

Experimental data are still insufficient for complete characterization of the functional form for the distribution of the intensity fluctuations under various atmospheric precipitation conditions. At the same time our analysis allows us to determine the class of functions suitable for the description of the intensity fluctuations in snowfall without flakes for narrow beam. These are functions with one maximum and right–side asymmetry. We have studied the applicability of three distributions having the above–enumerated salient features.

Naturally, it is desirable to find a logical justification for the applicability of the gamma distribution taking into account the knowledge of physics of the intensity fluctuations under precipitation conditions. One can assume that the distribution law is the mixture of several distributions. If one tentatively divides the path into the near and far zones relative to the receiver, then the mixture of the normal and Poisson distributions will be obtained for a low turbulent contribution to the fluctuations (this case was realized in our measurements at $\tau > 1$). Poisson's fluctuations appear due to screening of a narrow beam near the source by particles, whose size is larger than the beam's diameter, since the effect of each particle will be well–pronounced as in the zone adjacent to the receiver. For perceptible effect of the turbulence the weight of the lognormal functions in this mixture will increase.

However, the question whether the gamma distribution can be determined from the mixture of the lognormal and Poisson distributions, remains unsolved. And it is desirable to determine the weighting coefficients for the components of this mixture that yield the gamma distribution, if any. Roughly the weighting coefficients can be estimated from the spectral measurements.[11] All these problems can be the subject of further theoretical and experimental studies.

CONCLUSIONS

The distribution function of the intensity fluctuations of narrow diverging laser beam in snowfall without dense flakes has only one maximum and right–side asymmetry in the majority of cases and is described fairly well by the gamma distribution for weak and strong fluctuations.

Acknowledgement. In conclusion we should like to acknowledge N.P. Krivolapov for his participation in several measurements under precipitation conditions.

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